Terminal iterative learning control for an electrical powertrain system with backlash

Byungjun Kim, Seibum B.Choi
Department of Mechanical Engineering
KAIST, Korea Advanced Institute of Science and Technology
Daejeon, Republic of Korea
asx9608@kaist.ac.kr, sbchoi@kaist.ac.kr

***Abstract*—This paper proposes a backlash control algorithm using Terminal Iterative Learning Control (TILC) in the angle domain. The proposed method addresses the issue of control time variation in the Iterative Learning Control (ILC) method, which makes it impractical for vehicle control. By controlling backlash in the angle domain, the control interval remains the same for each iteration. The backlash impact is proportional to the velocity at the end of the backlash mode. However, by bringing the reference value nearer to zero, the impact was mitigated. Additionally, the utilization of TILC enhances its resilience to sensor noise. The proposed method is evaluated through simulations and experimental results, demonstrating its practical applicability to vehicles and high accuracy in various initial conditions. This paper provides a novel approach to backlash control in-vehicle systems, contributing to the advancement of control methods for improved ride comfort and safety.**

***Index Terms*—Electric vehicles and electric vehicle supply equipmenet, system modeling and control, intelligent trasportation**

I. Introduction

Backlash is a phenomenon that enables the backdrivability of a system. However, it can also result in impacts or deviations when the direction of rotation changes, which can have a negative impact on system performance. In the case of automotive powertrains, backlash can lead to ride comfort problems. Extensive research has been conducted to address the issue of backlash in internal combustion engine (ICE) vehicles [1], [2]. However, the slow reactivity of the ICE engine and short control intervals have limited control. In contrast, electric vehicles (EVs) use regenerative brakes, which exacerbate the ride quality problem due to backlash. Nevertheless, the fast responsiveness of the motor used in EVs provides a larger control effect than in ICE vehicles. With the growing popularity of EVs, the importance of addressing backlash issues has become increasingly apparent. Backlash problems in EVs commonly occur in Tip-in and Tip-out situations. Tip-in refers to a situation where the drive torque suddenly increases, usually when the accelerator pedal is pressed while driving. Tip-out, on the other hand, occurs when the torque decreases, such as when the driver releases the accelerator pedal. These situations are frequent during driving, and the regenerative brake can cause rapid torque changes in the case of EVs.

To control backlash, the gap size and current position of the backlash must be estimated. While the gap size can be estimated using an observer with motor and wheel speed sensors, the current position of the backlash does not satisfy observability requirements [3]. Despite the use of an observer such as a Kalman filter for estimation, achieving absolute accuracy in estimation remains challenging due to limitations in observability.

Various methods have been proposed for controllers, including PID control, nonlinear control, and model predictive control [4]–[6]. However, since the backlash system is a nonlinear system that switches models, several studies have proposed model-switching controllers [7]. One MPC proposed by Lu, which assumes that the next maneuver is known in advance, has shown good performance in pre-controlling backlash [8]. However, in most driving situations, it is difficult to know the next maneuver in advance. Additionally, since the backlash system operates in a very short time (0.1-0.15 s), excessive computation can lead to inaccurate control. Given the short control period, estimation errors, and sensor noise, applying such systems to production vehicles can be challenging.

Iterative Learning Control (ILC) is a widely used control method that effectively utilizes past control inputs and results, making it particularly useful in situations where the same task is repeated [9]. However, the practical issue is that initial conditions and references must be identical in each iteration, limiting its applicability in specific domains. In this regard, Terminal Iterative Learning Control (TILC) has been proposed, which only requires the terminal value as a reference and has the advantage of not requiring sensor values and references for all times.

Hou proposed train stop control using TILC by update the magnitude or start point of the control also, combines both methods to increase the convergence speed [10]. Inspired by this paper, we propose a backlash control algorithm using TILC in the angle domain for the first time, making it practically applicable to vehicles and reducing the amount of computation compared to existing controllers. The suggested controller exhibits exceptional performance despite variations in the initial velocity of the vehicle. Its mathematical convergence has been established, and simulation outcomes have validated its effectiveness. As a result, a proposed TILC using a constant input was introduced to enhance ride comfort and increase torque responsiveness.

In the remainder of this paper, Section 2 presents a dynamic model of the driveline backlash system. In section 3, the constant input TILC is described. Finally, the effectiveness of the proposed methods is demonstrated by numerical simulations in chapter 4.

This research was partly supported by the BK21+ program through the NRF funded by the Ministry of Education of Korea.

II. System Modeling and Problem Definition

A. System Modeling

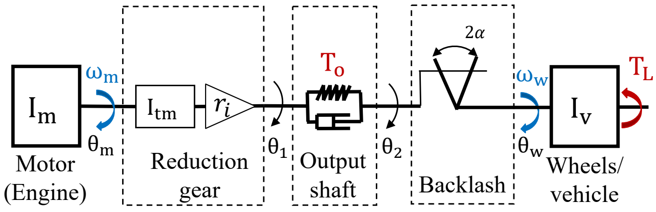

Fig. 1. Driveline model of EV with lumped backlash

Backlash is a common problem in various parts of vehicles, with the most considerable amount typically found in the ball joint between the output shaft and differential gear. In this study, the vehicle's backlash is modeled as a single lumped parameter behind the output shaft, as demonstrated in Fig. 1 The results obtained from simulation studies and previous research were consistent with the proposed model [11]. The torque balance equation of the lumped driveline backlash model is presented in (1).

$$I_m\dot{\omega}_m = T_m - \frac{T_o}{r} \tag{1a}$$
$$I_v\dot{\omega}_w = T_o - T_L \tag{1b}$$
$$T_o = k(\theta_1 - \theta_2) + c(\omega_1 - \omega_2) \tag{1c}$$
$$\theta_1 = \frac{\theta_m}{r},\ \theta_b = \theta_2 - \theta_3,\ \theta_d = \theta_1 - \theta_3, \tag{1d}$$

where $I, \omega, T,$ and r mean rotational inertia, angular velocity, torque, and gear ratio. The subscripts m, o, L and v mean motor, output shaft, road load and vehicle. In equation T_L, the road torque, representing the load on the vehicle due to external factors such as weight and road slope, is expressed by Equation (2), but this model provides only approximate values. The road torque changes over time and is challenging to measure in production vehicles.

$$T_L = r_w(m_v g\sin(\theta_r) + K_{rr}m_v g\cos(\theta_r) + \frac{1}{2}\rho v_v^2 C_d A) \tag{2}$$

The angular velocities of the motor and wheels, denoted by ω_m and ω_w, respectively, can be measured in production vehicles. Depending on the current backlash position, Powertrain systems with backlash can divide the mode into Contact mode and Backlash mode. The backlash dynamics is given by (3).

$$\dot{\theta}_b = \begin{cases} max\{0, \dot{\theta}_d + \frac{k}{c}(\theta_d - \theta_b)\} & if\ \theta_b = -\alpha \\ \dot{\theta}_d + \frac{k}{c}(\theta_d - \theta_b) & if\ |\theta_b| < \alpha \\ min\{0, \dot{\theta}_d + \frac{k}{c}(\theta_d - \theta_b)\} & if\ \theta_b = \alpha \end{cases} \tag{3}$$

During the contact mode, when $\theta_b = \pm\alpha$, the motor torque is transmitted to the wheel side via the torsional torque of the output shaft. However, during the backlash mode when $|\theta_b| < \alpha$, the motor torque is not transmitted to the wheel resulting in a zero value of T_o and decoupling of the inertia of the vehicle and motor. Consequently, there is a significant difference in angular acceleration, and the angular velocity difference becomes even more pronounced. The significant angular velocity difference causes a shock in the vehicle at the end of the backlash mode.

In backlash mode, by setting θ_b to state and set its derivative to ω_b, expression (1) can be converted to (4) using (1) and (3).

$$\begin{aligned} \omega_b &= \omega_1 - \omega_3 + (\omega_2 - \omega_1) \\ \dot{\omega}_b &= \frac{1}{I_m}T_m - \frac{1}{I_v}T_L + \frac{k}{c}(\omega_1 - \omega_2) \end{aligned} \tag{4}$$

To ensure the applicability of ILC, it is essential to maintain a consistent trial length in each iteration. However, controlling the backlash in the time domain leads to variation in the control time length, which depends on the magnitude of the applied torque, thus making it challenging to implement ILC. In order to overcome this issue, controlling the backlash in the angle domain is suggested. Remarkably, the equation (4) derived in the time domain is equivalent to the equation (5) obtained in the angle domain.

$$\frac{d\omega_b}{d\theta_b} = \frac{1}{\omega_b}(\frac{1}{I_m}T_m - \frac{1}{I_v}T_L + \frac{k}{c}(\omega_1 - \omega_2)) \tag{5}$$

A more general expression of the above formula is (6).

$$\frac{dv}{ds} = \frac{1}{v}(b \cdot u - w - g), \tag{6}$$

where v is ω_b and s is θ_b. b and u mean input gain and control input. w means iteration time-varying disturbance; in the backlash system, it represents road load (T_L). g represents an iteration invariant disturbance. In the powertrain backlash system, g represents the uncertainty of the lumped backlash model.

Expressing $\theta_1 - \theta_2$ as x and $\frac{c}{k}$ as p, the difference in angular velocity, $\omega_1 - \omega_2$, can be expressed as shown in Equation (7). In backlash mode, since $T_s = 0$, the shaft rotates in a twisted state in the opposite direction.

$$\begin{aligned} x &= -p\dot{x} \\ x &= e^{-pt + \ln x_0} \\ \dot{x} &= -pe^{pt + \ln x_0} \end{aligned} \tag{7}$$

The rate of change of x, denoted as $\dot{x}$, is tiny and exhibits a monotonic decrease. It is also apparent that x is a function of k, c, the initial deflection x_0, and time t, independent of the control input. As a result, x retains the same value at identical time instances. This property is leveraged in the following section to account for $\frac{k}{c}(\omega_1 - \omega_2)$, which is not encompassed in Equation (6). The following reasonable assumptions are made for backlash system control.

Assumption 1: The position of the backlash (θ_b) monotonically increases or decreases. This means that the direction of the backlash angle does not change within the backlash mode. In reality, the backlash period is concise, and in most driving situations, the direction of the backlash angle does not change within the backlash mode.

Assumption 2: The torque disturbance w_k is constant in the backlash mode. The road torque T_L is a function of time, such as vehicle speed and road slope. However, it can be assumed that it does not change significantly during a short backlash mode ending within 0.3s.

Assumption 3: The disturbance model $g_{k+1}(s)$ is the same as $g_k(s)$. As g is an iteration-invariant disturbance, the system has the same disturbance at the same position. This means that if the backlash moves in the same direction, the uncertainty will always be the same in the angle domain.

III. TILC-BASED DRIVELINE BACKLASH CONTROL ALGORITHMS

In typical situations, backlash impact occurs during Tip-in and Tip-out events. However, these situations are similar, except that the backlash angle moves in the opposite direction. Therefore, this paper focuses only on the Tip-out situation.

The objective of backlash control is to reduce the speed difference at the end of the backlash mode. Many studies have used a reference of 0. However, when the reference is set low, the speed of torque transmission in contact mode is slowed. Hence, this paper sets the reference as a tuning parameter that can be adjusted to suit each user's preference and vehicle properties. In particular, an arbitrary value is selected as the terminal reference.

In repeated Tip-out situations, the terminal angular velocity difference of the backlash system can be expressed using the following equation (8).

$$v_k^0 - v_0^k = \int_{\alpha}^{-\alpha} \frac{1}{v(s)}(bu_k(s) - w_k - g(s))ds + h_k, \quad (8)$$

where v_k^0 is the terminal velocity difference when the backlash position reaches the opposite side ($\theta_b = -\alpha$). v_0^k is the initial velocity difference. By utilizing equation (7), h_k can be expressed as shown in equation (9).

$$\begin{aligned} h_k = &\int_0^{t_f^k} \frac{k}{c}(\omega_1 - \omega_2)dt = e^{-pt_f^k + \ln x_0^k} - e^{\ln x_0^k} \\ =&\bar{h} + d_k^h. \end{aligned} \quad (9)$$

h_k depends on the k^{th} initial velocity (x_0^k). Therefore, it changes each iteration depending on k, and the average value is $\bar{h}$, and the difference between $\bar{h}$ and h_k is d_k^h. Like h_k, v_0^k can be represented by the sum of $\bar{v}$ and d_k^v. Finally, wtih $d_k = d_k^v + d_k^h$, terminal angular velocity at kth iteration is

$$v_k^0 = \bar{v} + \int_{\alpha}^{-\alpha} \frac{1}{v_k(s)}(bu_k(s) - w_k - g(s))ds + \bar{h} + d_k. \quad (10)$$

A. TILC-based driveline backlash control with constant input

This section employs a constant input to regulate the backlash system. Although the outcome and verification methods are akin to those presented in Hou and Wang [10], which was the source of inspiration for this study, they employed velocity domain control while this paper uses angle domain control. Because of the divergent attention and the reciprocal relationship between these domains, additional steps are necessary to establish the proof for Theorem 1. As a result, the differences in the verification method will be the primary focus of the discussion.

Theorem 1: Assuming 1-3 hold for a powertrain driveline system with backlash, and the motor input u_k follows the updated law (11), Theorem 1 guarantees that the control error is bounded as k increases and $|(1 - b\beta T_{max}\eta_k)| < 1$ satisfied, where T_{max} represents the maximum backlash period.

$$u_{k+1} = u_k + \beta \Delta v_k^0, \quad (11)$$

where $\beta > 0$ is tunning parameter.

Proof: According to equation (10), the difference between v_{k+1}^0 and v_k^0 :

$$\begin{aligned} v_{k+1}^0 - v_k^0 & \\ = &\int_{\alpha}^{-\alpha} b\Big(\frac{1}{v_{k+1}(s)}u_{k+1} - \frac{1}{v_k(s)}u_k\Big)ds \\ &- \int_{\alpha}^{-\alpha} \Big(\frac{1}{v_{k+1}(s)}(w_{k+1} + g_{k+1}) - \frac{1}{v_k(s)}(w_k + g_k)\Big) \\ &+ (d_{k+1} - d_k). \end{aligned} \quad (12)$$

Using the update law (11) and the fact that the u and w are constant during control, (12) can be rewritten as

$$\begin{aligned} v_{k+1}^0 - v_k^0 & \\ =&b\beta T_{k+1}\Delta v_k^0 + bu_k \int_{\alpha}^{-\alpha} \Big(\frac{1}{v_{k+1}(s)} - \frac{1}{v_k(s)}\Big)ds \\ &- \int_{\alpha}^{-\alpha} \Big(\frac{1}{v_{k+1}(s)}g_{k+1} - \frac{1}{v_k(s)}g_k\Big)ds \\ &+ w_k T_k - w_{k+1}T_{k+1} + (d_{k+1} - d_k), \end{aligned} \quad (13)$$

where $T_k (= \int_{\alpha}^{-\alpha} \frac{1}{v_k} ds)$ denote the total time of the kth control. Since u_{k+1} is always negative, T_k and it's maximum value T_{max} are bounded. Thus, taking the absolute value of both sides of (13) and denoting the maximum value of the disturbance as d_{max} and the maximum value of w as W_{max}, we can rewrite the equation using the mean value theorem as follows:

$$\begin{aligned} |v_{k+1}^0 - v_k^0| \le &|bu_k + g(a)| \int_{\alpha}^{-\alpha} \Big|\frac{1}{v_{k+1}(s)} - \frac{1}{v_k(s)}\Big|ds \\ &+ b\beta T_{k+1}|\Delta v_k^0| + 2W_{max} + 2d_{max} \\ =&(bu_k + g(a)) \int_{\alpha}^{-\alpha} \frac{|v_{k+1}(s) - v_k(s)|}{v_{k+1}(s)v_k(s)}ds \\ &+ b\beta T_{k+1}|\Delta v_k^0| + 2W_{max} + 2d_{max}, \end{aligned} \quad (14)$$

where $-\alpha < a < \alpha$, $bu_k + g(a) > 0$, and $v_{k+1}(s)v_k(s) \ge 0$.

Let

$$M \triangleq \inf_{s \in [-\alpha, \alpha], i \in \mathbf{Z}^+} \frac{bu_i + g(a)}{v_i^2(s)} ds, \quad (15)$$

with (15), and applying Gronwall ineqaulity to (14):

$$\begin{aligned} |{v^0}_{k+1} - v_k^0| & \\ &\le (b\beta T_{max}|\Delta v_k^0| + 2W_{max} + 2d_{max}) exp\Big(\int_{\alpha}^{-\alpha} M ds\Big) \\ &= (b\beta T_{max}|\Delta v_k^0| + 2W_{max} + 2d_{max}) exp(2M\alpha). \end{aligned} \quad (16)$$

Defining $exp(2M\alpha) = \varphi$, $W_{max} + d_{max} = D_{max}$ (16) becomes

$$|v_{k+1}^0 - v_k^0| \le b\beta T_{max}\varphi|\Delta v_k^0| + 2\varphi D_{max} \quad (17)$$

There exists η_k that satisfy (18), with $0 \le \eta_k \le \varphi$

$$|v_{k+1}^0 - v_k^0| = \beta T_{max}\eta_k|\Delta v_k^0| + 2\eta_k D_{max}. \quad (18)$$

The terminal error at $k+1$ iteration is

$$\Delta v_{k+1}^0 = \Delta v_k^0 - (v_{k+1}^0 - v_k^0). \quad (19)$$

Using input update law, the absolute value of the terminal error is

$$|\Delta v_{k+1}^0| = |(u_k - u_{k+1})/\beta - (v_{k+1}^0 - v_k^0)|. \quad (20)$$

If $u_{k+1} > u_k$, then $v_{k+1}^0 < v_k^0$, which is obvious because in regenerative situations, the motor torque close to zero makes the final velocity small. By using the fact that $\big((u_k - u_{k+1})/\beta\big)(v^0k+1-v^0k) > 0$ and (18), we can rewrite (19) as follows:

$$\begin{aligned} |\Delta v_{k+1}^0| &= ||\Delta v_k^0| - |(v_{k+1}^0 - v_k^0)|| \\ &= |(1 - b\beta T_{max}\eta_k)|\Delta v_k^0| - 2\eta_k D_{max} \\ &\le |1 - b\beta T_{max}\eta_k||\Delta v_k^0| + 2\eta_k D_{max}. \end{aligned} \quad (21)$$

The convergence performance of the algorithm is determined by $|(1 - b\beta T_{max}\eta_k)|$, and therefore, its magnitude needs to be investigated to ensure the effectiveness of the control. The details of this investigation are the same as those in [10] and are therefore omitted here. The resulting conclusion is that the control error is bounded as shown in Equation (22) for all values of k.

$$|\Delta v_k^0| < |\Delta v_1^0| + \frac{2\varphi D_{max}}{1-\bar{\rho}}, \quad (22)$$

where $0 < \rho < 1, 1-\rho < b\beta\varphi$. If $D_{max} = 0$, then the monotonic convergence is ensured.

IV. Simulation Results

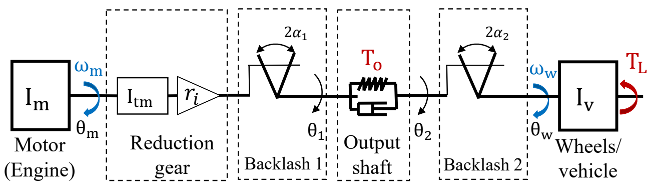

Fig. 2. Simulation driveline model of EV

The simulation was implemented using SIMULINK's SIMDRIVELINE and involved a repeated regenerative brake scenario. The SimDriveline was configured according to Equation 1, and to introduce some uncertainty, the backlash was also incorporated into the reduction gear in front of the output shaft, as illustrated in Fig. 2.

The vehicle's weight remained constant throughout the simulation, and two scenarios were considered: one where the vehicle's initial speed was fixed and the other where it was randomly set between 30 and 70 km/h. The backlash gap α_1 is set as 0.1rad, and α_2 is set as 0.3rad. Assuming that the starting point of backlash can be accurately measured using an observer based on previous studies. Even though the control is in the angle domain, the input is constant, requiring input of the same magnitude during the backlash period. The simulation used a learning gain β of 2, and the terminal reference was set to -0.2.

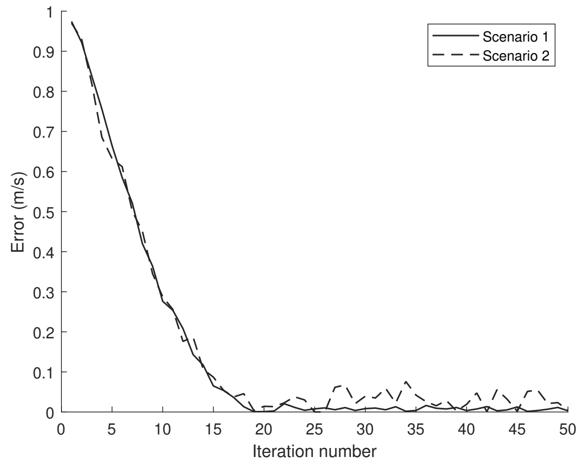

Fig. 3. Terminal error along the iteration axis (Scenario 1 & 2)

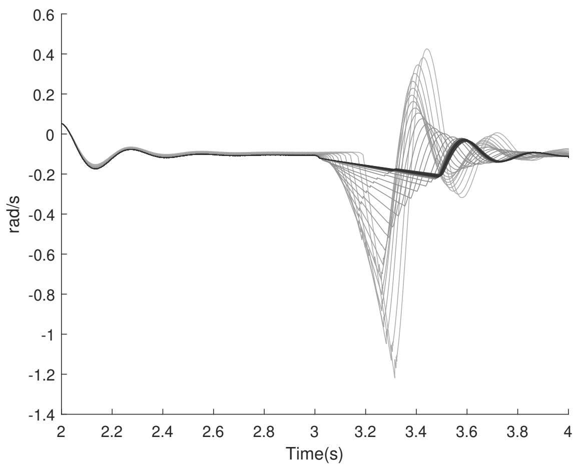

Fig. 4. Speed difference (Scenario 1)

Fig. 3 demonstrates the convergence of the error to zero with an increasing number of iterations. However, for scenario 2, it is observed that the error bound is more significant than 1, which can be attributed to the more considerable value of D_{max} in (22) resulting from the difference in initial velocities and immense value of W_{max}.

The speed difference results are presented in Figs. 4 and 6, where the line color darkens with an increase in the number of iterations. It can be observed that scenario 2 converges to a broader range compared to scenario 1, similar to Fig. 3. The figures in Figs. 5 and 7 illustrate the results for the control input and backlash section. As the number of iterations increases, it converges to a specific control input, and it can be observed that the duration of the backlash section increases with a more extensive control input. This

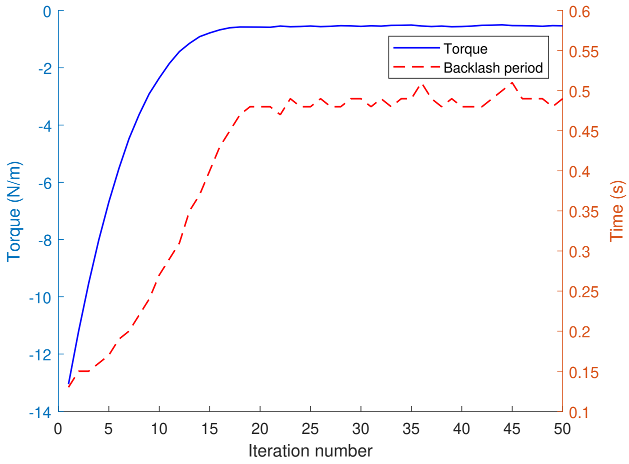

Fig. 5. Input and backlash period along the iteration axis (Scenario 1)

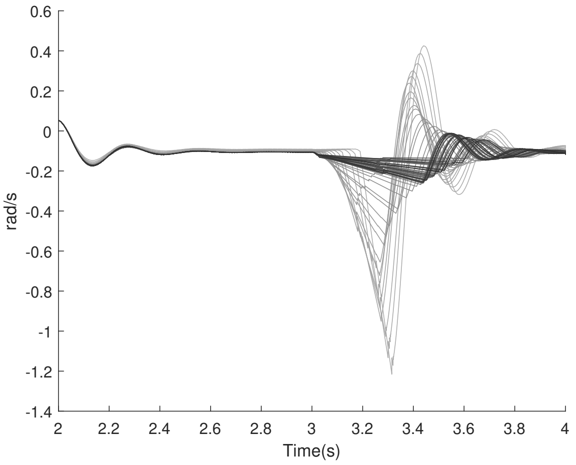

Fig. 6. Speed difference (Scenario 2)

is because using a smaller input value takes longer to reach the opposite side of the backlash.

In summary, the simulation results confirmed the proposed method converged effectively to the terminal target reference. Furthermore, it was observed that there was no significant difference in convergence even when the initial velocity was varied.

V. Conclusion

This paper presents a terminal iterative learning control (TILC) method for regenerative braking systems with backlash. The proposed ILC operates in the angle domain, and its convergence has been mathematically proved. Simulation results show that the proposed method can precisely control the regenerative braking system, even when two backlashes are present in the powertrain. The method is effective for both constant and random initial velocities. The results demonstrate the effectiveness of the proposed ILC method for controlling regenerative braking systems with backlash in the angle domain. The current study focused on a constant terminal reference, and further development is required to apply the proposed method when the terminal reference changes in each iteration. Additionally, it would be valuable to extend the method to demonstrate convergence in various scenarios and situations. Future research can focus on these aspects to further improve the effectiveness and applicability of the proposed method.

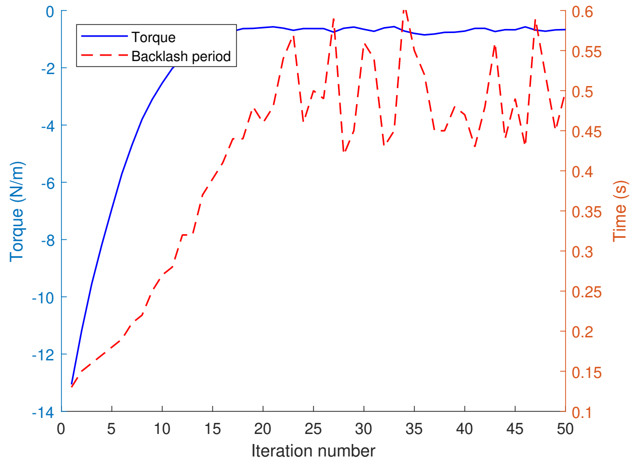

Fig. 7. Input and backlash period along the iteration axis (Scenario 2)

References

[1] Lagerberg, Adam, and Bo Egardt. "Evaluation of control strategies for automotive powertrains with backlash." AVEC'02, Intl. Symp. on Advanced Vehicle Control 2002. 2002.

[2] Pham, Truc, et al. "Nonlinear flatness-based control of driveline oscillations for a powertrain with backlash traversing." IFAC-PapersOnLine 49.11 (2016): 749-755.

[3] Lagerberg, Adam, and Bo Egardt. "Backlash estimation with application to automotive powertrains." IEEE Transactions on Control Systems Technology 15.3 (2007): 483-493.

[4] Akbar, Zeeshan Ali, et al. "Pid based control of electric vehicle drive line with backlash." 2019 IEEE 10th International Conference on Mechanical and Aerospace Engineering (ICMAE). IEEE, 2019.

[5] Lv, Chen, et al. Zhang, Jianwu, Benben Chai, and Xingyang Lu. "Active oscillation control of electric vehicles with two-speed transmission considering nonlinear backlash." Proceedings of the Institution of Mechanical Engineers, Part K: Journal of Multi-body Dynamics 234.1 (2020): 116-133.

[6] Lagerberg, Adam, and Bo Egardt. "Model predictive control of automotive powertrains with backlash." IFAC Proceedings Volumes 38.1 (2005): 1-6.

[7] Lv, Chen, et al. "Mode-switching-based active control of a powertrain system with non-linear backlash and flexibility for an electric vehicle during regenerative deceleration."

[8] Lu, Xingyang, Tongli Lu, and Benben Chai. "Mode-switch model predictive controller with "pre-contact" method for alleviating driveline vibration of electric vehicles considering backlash." Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering 234.8 (2020): 2176-2194.

[9] Bristow, Douglas A., Marina Tharayil, and Andrew G. Alleyne. "A survey of iterative learning control." IEEE control systems magazine 26.3 (2006): 96-114.

[10] Hou, Zhongsheng, et al. "Terminal iterative learning control based station stop control of a train." International Journal of Control 84.7 (2011): 1263-1274.

[11] Darokar, Kaushal Kumar. Automotive Driveline Backlash State and Size Estimator Design for Anti-Jerk Control. Diss. Michigan Technological University, 2019.