Adaptive Collision Avoidance Using Road Friction Information

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Abstract—Technical development with the goal of achieving zero accidents and zero fatalities is ongoing. The autonomous emergency braking systems that debuted in the late 2000s have proven their value regarding improved safety. However, the technology still presents many challenges because it is not easy to ensure that the system will operate as intended in any environment and at any time. Any system that is unaware of its environment is prone to be excessively conservative, which could adversely affect the efficacy of said system. Situation awareness is a key to resolving this problem. The present study suggests the use of warning braking to gain an awareness of the level of road friction, which is one of the major uncertainties faced on the road. During warning braking, the tire-road maximum friction coefficient is estimated in real time, and a threat assessment is performed adaptively based on the friction information. Because warning braking is momentary and applied with limited dynamics due to issues related to human factors, this study discusses the major considerations and requirements for the key parameters related to warning braking. The performance of the suggested adaptive collision avoidance scheme is verified by means of simulation and experiments.

Index Terms— Collision avoidance, threat assessment, situation awareness, autonomous emergency braking.

NOMENCLATURE

- *WB* Warning braking.
- *PB* Partial braking.
- *EB* Emergency braking.
- A_b Automated deceleration for threat assessment $[m/s^2]$.
- J_b Limited jerk of automated braking $[m/s^3]$.
- T_d Time delay [s].
- J_{wb} Jerk of warning braking $[m/s^3]$.
- D_{wb} Duration of warning braking [s].
- A_{wb} Peak deceleration of warning braking $[m/s^2]$.
- t_h Prediction horizon [s].
- v_{ego} Speed of ego vehicle [m/s].
- v_{tar} Speed of target vehicle [m/s].
- v_r Relative speed between ego and target vehicle [m/s].

Manuscript received May 25, 2017; revised December 27, 2017; accepted February 25, 2018. Date of publication April 23, 2018; date of current version December 21, 2018. This work was supported in part by the Korea Ministry of Science, ICT and Future Planning under Grant R-20160302-003057. The Associate Editor for this paper was M. Brackstone. (*Corresponding author: Seibum B. Choi.*)

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Digital Object Identifier 10.1109/TITS.2018.2816947

Distance between ego and target vehicle [m]. C_{et} C_O Minimum required distance [m]. T_{me} Predicted time at which deceleration reaches $A_h[s]$. T_{se} Predicted time at which v_{ego} becomes zero [s]. Predicted time at which v_{tar} becomes zero [s]. T_{st} T_{eq} Predicted time at which v_r becomes zero [s]. T_{wb} Trigger time of WB[s]. T_{eb} Trigger time of EB[s]. Tdiff Time difference between T_{wb} and T_{eb} [s]. \mathcal{M} Time margin for $D_{wb}[s]$. Tire-road maximum friction-coefficient [-]. μ_m Longitudinal friction-coefficient of front(rear) $\mu_{xf(r)}$ wheels [-]. Lateral friction-coefficient of front(rear) $\mu_{yf(r)}$ wheels [-]. Longitudinal wheel slip of front(rear) wheels [-]. $S_{xf(r)}$ т Vehicle mass [kg]. G Acceleration due to gravity $[m/s^2]$. C_{lt} Load transfer coefficient [kg]. h_{cg} Height of center of gravity relative to ground [m]. L Vehicle wheelbase [*m*]. $L_{f(r)}$ Distance from center of gravity to front(rear) axle [m]. $F_{xf(r)}$ Longitudinal tire force of front(rear) wheels [N].

Acceleration of ego(subject) vehicle $[m/s^2]$.

- $F_{yf(r)}$ Lateral tire force of front(rear) wheels [N].
- $F_{zf(r)}$ Normal tire force of front(rear) wheels [N].
- R_x Tire rolling resistance [N].
- F_a Aero-drag force [N].
- I_w Wheel inertia $[kg \cdot m^2]$.
- $\omega_{f(r)}$ Front(Rear) wheel speed [rad/s].
- r_e Effective tire radius [m].
- $T_{bf(r)}$ Braking torque of front(rear) wheels [Nm].
- η_b Braking torque distribution ratio [-].
- K_o Positive tire force observer gain [-].
- K_f Positive tire force estimation gain [-].
- \mathcal{F}_n Friction space with *n* sub-spaces.
- Θ_i Friction sub-space of index *i*.
- λ Forgetting factor for friction estimation [-].

I. INTRODUCTION

THE intelligent vehicle market is growing quickly because of customer demand and the efforts being made by the industry to improve road safety. In particular, the incorporation of autonomous emergency braking systems is increasing drastically due to the demands of the New Car Assessment

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Program (NCAP) and the regulations of each country. Thus, car makers and suppliers are struggling to perfect their technologies such that products can be offered to the market.

The success of an autonomous emergency braking system can be defined technically as whether the system operates at an appropriate time. If this cannot be assured, then the system would not be practical and the false reliance of drivers on the system could lead to unnecessary accidents. Here, threat assessment takes the place of the human brain in identifying the appropriate timing.

Threat assessment for collision avoidance has gained the attention of researchers over the past several decades. The classical deterministic approach utilizes the kinematics and dynamics of vehicles [1]–[7]. Because both the ego and target vehicles exhibit uncertainty in their future motion, stochastic approaches [8], [9] are now popular. Some research [10] integrates threat assessment into the control algorithm. Threat assessment involves predicting the future from data acquired with sensors. A problem arises in that uncertainties are introduced into this process. Because uncertainty arises mostly from the perception of the environment, precise situation awareness is required to overcome that uncertainty.

The road status is a typical major uncertainty because it significantly affects the braking distance and the stability of the vehicle during emergency braking. According to the accident statistics for the South Korean rainy season between 2012 and 2014 [11], there were a total of 15,862 casualties, while the rate of highway fatalities increases by more than 60% on rainy days. Many such accidents could be a result of human drivers misjudging the situation, which is analogous to an autonomous system. If a system were to trigger emergency braking without any information about the road, the result would vary greatly depending on the road status. For example, the trigger time on a wet road should be shifted ahead relative to that for a dry road. In addition, emergency braking on slippery roads could lead to unnecessary secondary accidents. It is obvious that a situaional awareness of the road would improve the efficacy of autonomous emergency braking systems.

Various techniques for estimating the degree of road friction have been introduced in the literature. A common technique involves utilizing longitudinal dynamics during vehicle acceleration or deceleration [12]–[15]. The lateral dynamics can also be harmonized to reinforce the estimation [16], [17]. Adaptive threat assessment with a friction estimate was introduced in [18]. However, relying on the results of an occasional estimate cannot guarantee that the result will be valid at the instant of the threat assessment. For application to autonomous emergency braking, the primary requirements are that the road friction information should be available before the trigger of emergency braking and should be homogeneous from its estimation until the end of the emergency braking.

A. Outline of This Paper

The present study examines the utilization of warning braking to accomplish the goals described above. Warning braking is a kind of haptic measure for collision warning, as recommended in the international standard [19], and has



Fig. 1. Operation cascade concept with the benefit of warning braking.

already been commercialized by some OEM in Europe. In addition, the Euro-NCAP favors the incorporation of a warning braking feature. In the literature [20]–[24], the authors considered the various configurations of warning braking in the experiments and discussed the results from the perspective of human factors, particularly in terms of the coordination between the efficacy and driver's acceptance. In addition to warning braking being an effective warning measure in itself, the present study additionally focuses on the possibility of awareness of the road status, immediately before the triggering of emergency braking. The tire-road friction-coefficient can be estimated during the warning braking and the system can then adjust the trigger time according to the results of the estimation. In other words, the system can conduct a more precise threat assessment with situation awareness of the road due to the application of the warning braking.

Figure 1 depicts a conceptual operation cascade with the benefit of warning braking for forward collision avoidance. When the ego vehicle, which is the following vehicle equipped with an autonomous braking system, gets closer to the target vehicle, warning braking is activated at T_{wb} . Here, TR_1 represents the trajectory of the ego vehicle when the emergency braking is triggered at T_{eb} on a dry road, resulting in collision avoidance, whereas TR_2 is the case where the emergency braking is triggered on a slippery road, resulting in a collision due to the lack of tire-road adhesion. Here, TR_3 represents the trajectory when the time of triggering is shifted to T_{eb}^* according to the friction estimation during the warning braking, resulting in collision avoidance.

In this paper, we discuss the following five topics. First, when the warning/emergency braking should be triggered (Section II). A classical threat assessment algorithm incorporating the nonlinear characteristic of the braking system is introduced in this topic. One of the major parameters of the proposed threat assessment algorithm is the road friction. Second, we address how the warning braking should be operated (Section III). The background to the usage, basic requirements, detailed implementation, and key parameters of the warning braking are introduced in this topic. Third, we consider how the road friction can be estimated (Section IV). A quantized slip-slope method combined with the curvematching algorithm, which is robust and suitable for collision avoidance applications, is proposed herein. Fourth, the benefits of warning braking are verified by experiment (Section V). Finally, the discussion is extended to some exceptional cases (Section VI). We made the following basic assumptions for this study,

Assumption 1: The tire-road friction-coefficient is homogeneous between the warning braking and the subsequent autonomous braking.

Assumption 2: The tire-road friction-coefficient is homogeneous between the four wheels.

Assumption 3: The operation cascade is composed of the warning braking, followed by the emergency braking, as depicted in Fig. 1.

Assumption 4: Straight flat road driving is assumed for simplicity.

II. THREAT ASSESSMENT USING ROAD INFORMATION

A threat assessment that incorporates road uncertainty and nonlinear vehicle motion is presented in this section. Although generalized approaches that cover arbitrary collision cases have been introduced in the literature [3], [4], the algorithm suggested in the present study is simplified to consider only a rear-end collision on a straight road, which is a simple but representative case, because the focus of the present study is the coordination between the threat assessment and the road friction information.

A. Vehicle Motion Prediction

The motion prediction is based on the kinematics of the ego and target vehicles, incorporating the nonlinear brake input of the ego vehicle [3], [5] with parameter set p_b of the brake system as:

$$\boldsymbol{p}_b = [A_b \ J_b \ T_d] \tag{1}$$

where A_b denotes the automated deceleration level, J_b denotes the limited jerk, and T_d denotes the delay. The limited jerk of the brake response becomes considerable at low speeds because the dwell time of the maximum deceleration is short. Figure 2 represents the predicted motion of the ego vehicle during the prediction horizon t_h when automated braking is triggered at $t_h = 0$. As illustrated in Fig. 2, the deceleration of the ego vehicle increases with the limited jerk J_b after T_d . The initial acceleration of the ego vehicle is neglected.

For the emergency braking, the predicted speed of the ego vehicle for $t_h \in (T_{me}, T_{se}]$ is calculated as:

$$v_{ego,h}(t_h, \hat{\mu}_m) = v_{ego,h}(T_{me}) + A_{b,eb}(\hat{\mu}_m)(t_h - T_{me})$$
(2)

where v_{ego} and $v_{ego,h}$ denote the speed of the ego vehicle and its predicted value. Here, $\hat{\mu}_m$ denotes the estimated value of maximum tire-road friction-coefficient μ_m at $t_h = 0$. In addition, A_b for the emergency braking is set to $\hat{A}_{b,eb}$, which denotes the estimated maximum achievable deceleration. Note that $\hat{A}_{b,eb}$ is the function of the road friction, and the road friction information is the time variable that should be estimated in real-time, not at arbitrary intervals.

To estimate $\hat{A}_{b,eb}$, the following longitudinal vehicle force balance is considered first:

$$mA_b = \mu_m mG + \gamma \left(v_{ego} \right) \tag{3}$$



Fig. 2. Predicted motion of ego vehicle with limited slew rate and delay when emergency braking is triggered at $t_h = 0$. $a_{ego,h}$ denotes predicted acceleration of ego vehicle.

where *m*, *G*, and γ denote the mass of the vehicle, the acceleration due to gravity [m/s²], and additional forces [N] such as rolling resistance and aerodynamic force, respectively. Then, $\hat{A}_{b,eb}$ can be approximated from the first-order Taylor expansion, as follows:

$$A_{b,eb}(\hat{\mu}_m) = A_{b,o}(\mu_{m,o}) + G(\hat{\mu}_m - \mu_{m,o})$$
(4)

where $\mu_{m,o}$ and $A_{b,o}$ denote the nominal value of the maximum tire-road friction-coefficient and the maximum achievable deceleration, respectively. Moreover, $\partial A_b/\partial \mu_m = G$ from (3). In the present study, $\hat{\mu}_m$ is bound to the operation range $\mathcal{U}_{op} \sim [\mu_{m,min} \ \mu_{m,o}]$ so that the $\hat{A}_{b,eb}$ is also bound to $\hat{A}_{b,eb}(\mathcal{U}_{op})$. With the estimated maximum achievable deceleration, the trigger time of emergency braking is adapted (shifted) to reflect the road status:

$$T_{eb}(A_{b,o}) \to T^*_{eb}(\hat{A}_{b,eb})$$
 (5)

where T_{eb}^* denote the shifted trigger time of the emergency braking, respectively. In this case, the delay consists only of the system delay $T_{d,s}$, i.e., $T_d = T_{d,s}$.

For the warning braking (collision warning), the parameter set p_b for the braking system is configured to mimic the behavior of a human driver. To attain this, A_b is set to a constant $A_{b,wb}$ considering the relatively moderate braking of human drivers when attempting to avoid a collision. The lower bound $\mu_{m,min}$ in \mathcal{U}_{op} is tuned so that $\hat{A}_{b,eb}(\mu_{m,min}) \ge A_{b,wb}$ in this study. Note that the human delay $T_{d,h}$ is appended to the system delay $T_{d,s}$ for the warning braking, i.e., $T_d = T_{d,s} + T_{d,h}$. Even though the road friction information is not available during threat assessment for the warning braking, the road friction usually does not affect the warning braking because the actuation of the brake system is limited, as will be discussed later.

The target vehicle is simply assumed to maintain its current deceleration, ultimately stopping at T_{st} . The details of the kinematics for the motion prediction are omitted from this paper.

B. Threat Assessment

A braking-distance-based threat assessment is suggested in this section. It can be regarded as the extended version of the Time-To-Brake (TTB) approach [6], with consideration



Fig. 3. Case I: Ego vehicle stops later than the target vehicle.

given to the nonlinear characteristics of the braking system, as depicted in Fig. 2. Assuming that the autonomous braking is triggered at this instant, the following condition should be satisfied to enable collision avoidance:

$$\min_{\forall t_h \in \mathcal{H}_{se}} c_{et,h}(t_h, \hat{\mu}_m) \ge c_o, \quad \mathcal{H}_{se} \sim (0, T_{se}]$$
(6)

where $c_{et,h}$ denotes the predicted distance between the ego and target vehicle and $c_o > 0$ denotes the minimum distance, which is a tuning parameter. If (6) is not satisfied, the system triggers autonomous braking. However, it is computationally demanding to assess (6) for every sample time. Instead, we consider the following equivalent condition for the threat assessment:

$$c_{et,h}(T_{eq},\hat{\mu}_m) \ge c_o \tag{7}$$

where $T_{eq} \in \mathcal{H}_{se}$ denotes the time in the prediction space at which the speed of the ego vehicle is predicted to be equal to that of the target vehicle, i.e., the relative speed becomes zero, such that:

$$v_{ego,h}(T_{eq}, \hat{\mu}_m) - v_{tar,h}(T_{eq}, \hat{\mu}_m) = 0$$
 (8)

The searching for T_{eq} is divided into two cases, as follows. Only the case of $v_{ego} > v_{tar}$ is considered, which is true for the potential collisions.

1) Case I ($T_{st} \leq T_{se}$): In this case, the calculation of T_{eq} is straightforward. This is simply the time at which the ego vehicle comes to a stop, i.e. $T_{eq} = T_{se}$, as illustrated in Fig. 3. Then, T_{eq} can be found by solving the following linear equation:

$$v_{ego,h}(T_{me}, \hat{\mu}_m) + A_b(T_{eq} - T_{me}) = 0$$
 (9)

2) Case II $(T_{st} > T_{se})$: In this case, piece-wise solving is required to find T_{eq} by solving (8) sequentially for $t_h \in (T_d, T_{me}]$ and $t_h \in (T_{me}, T_{se})$. Here, T_{se} is excluded because it contradicts the condition of this case.

If a solution T_{eq} is found, then examine (7) and determine whether the autonomous braking should be triggered.

Claim 1: If $v_{ego} > v_{tar}$ and $T_{st} > T_{se}$, there is always a unique solution to $T_{eq} \in (0, T_{se})$ in (8).

Proof: Because $v_{ego,h}(0^+) > v_{tar,h}(0^+)$ and $v_{ego,h}(T_{se}^-) < v_{tar,h}(T_{se}^-)$ for $T_{se}^- > 0^+$, a unique solution T_{eq} always exists in $(0, T_{se})$ as illustrated in Fig. 4.

Claim 2: If (7) is satisfied and $v_{ego} > v_{tar}$, collision avoidance is guaranteed for $\forall t_h \in \mathcal{H}_{se}$.



Fig. 4. Case II: Ego vehicle stops earlier than the target vehicle.

Proof: As illustrated in Fig. 3 for Case I, and in Fig. 4 for Case II, $c_{et,h}$ is the convex function with a minimum at the unique $T_{eq} \in \mathcal{H}_{se}$. Hence, collision avoidance is guaranteed if (7) is satisfied.

III. WARNING BRAKING

A. Basic Considerations

Besides the basic advantages such as congruent stimulusresponse mapping and speed reduction, a further possible advantage of the warning braking is addressed in this study. That is, real-time tire-road friction estimation could make the subsequent emergency braking more effective. There are two basic considerations for warning braking in terms of the maximum tire-road friction estimation, as follows,

- Securing sufficient vehicle dynamics for accurate friction estimation.
- Appropriate time in which the friction estimation can be completed before the triggering of the emergency braking.

The first consideration implies that larger braking dynamics can enhance the road friction estimation. It is also known that the larger braking dynamics ensure better driver responses such as the accelerator release time and the stopping distance [24]. However, the braking dynamics for the warning braking are inevitably limited by the intrinsic considerations, from the perspective of human factors.

B. Parameterization

The key parameters for the warning braking are the duration and deceleration (or jerk) level, which are related to the considerations discussed in the previous section. Figure 5 shows an illustrative example of the warning braking pulse with parameters:

$$\boldsymbol{p}_{wb} = [J_{wb} \ D_{wb}] \tag{10}$$

where J_{wb} and D_{wb} denote the jerk and duration of the warning braking, respectively. Increasing the value of p_{wb} would result in larger braking dynamics. However, the parameter values are inevitably limited, as discussed above. To overcome this, a strategy is required to secure sufficiently high braking dynamics with the limited values of p_{wb} . The basic idea presented in the present study involves measuring the braking dynamics at the rear wheels. Basically, the purpose of this is to incorporate the load-transfer effect. In addition, the advantage of no interference from the tractive force can be anticipated by assuming front-wheel drive.



Fig. 5. Illustrative example of warning braking pulse.



Fig. 6. Longitudinal bicycle model. Here, m, L, $L_{f(r)}$, and h_{cg} are all assumed to be known constants.

1) Vehicle Parameters: Before we discuss the warning braking parameters, we introduce the fundamental vehicle dynamics model and parameters in this section [25], [26]. From the bicycle model depicted in Fig. 6, the normal tire force acting on each wheel can be estimated as follows:

$$F_{zf(r)} = \frac{mGL_{r(f)}}{L} + \Delta F_{zf(r)}$$

$$\Delta F_{zf} = C_{lt}|a_{ego}|, \quad \Delta F_{zr} = -C_{lt}|a_{ego}|$$
(11)

where $\Delta F_{zf(r)}$ denotes the longitudinal load transfer during deceleration, and $C_{lt} = mh_{cg}/L > 0$ denotes the load transfer coefficient. The aero-drag force is neglected here for simplicity. The longitudinal vehicle force balance is given by a generalization of (3), as follows:

$$ma_{ego} = F_{xf} + F_{xr} - R_x - F_a + \gamma \tag{12}$$

The rotational dynamics for each wheel is given as:

$$I_{w}\dot{\omega}_{f(r)} = -T_{bf(r)} - r_{e}F_{xf(r)}$$
(13)

where I_w , $\omega_{f(r)}$, and r_e denote the wheel inertia, front (rear) wheel speed, and effective tire radius, respectively. For a braking maneuver, the braking torque of the front (rear) wheel $T_{bf(r)}$ is defined as:

$$\begin{bmatrix} T_{bf} \\ T_{br} \end{bmatrix} = \begin{bmatrix} (1 - \eta_b) T_b \\ \eta_b T_b \end{bmatrix}$$
(14)

where $0 \le \eta_b \le 1$ denotes the braking torque distribution ratio.

2) Deceleration (Excitation): The ability to distinguish between road surfaces, especially between high and medium levels of adhesion, as in the case of dry and wet asphalt, requires sufficient *excitation*, which refers to the variation in the friction-coefficient during braking. In such a case, a larger *excitation* statistically guarantees a more reliable friction estimate [12], [13].

Claim 3: The longitudinal *excitation* at the rear wheels during the warning braking can be approximated by the following inequality equation, provided there is no additional force at the rear wheels,

$$\Delta \mu_{xr} \ge \Delta \mu_{ro} + C'_{lt} \mu^{avg}_{xr} \tag{15}$$

where $\Delta \mu_{xro} = (\eta_b A_{wb}/G)(L/L_f)$ denotes the *excitation* without any load transfer effect, μ_{xr}^{avg} denotes the average friction-coefficient at the rear wheels during the *excitation*, and $C'_{lt} = (C_{lt}/m)(A_{wb}/G)(L/L_f)$ denotes the normalized load transfer coefficient.

Proof: The tire-road friction-coefficient at the rear wheels is defined as:

$$\mu_{xr}(t) = \frac{F_{xr}(t)}{F_{zr}(t)} \tag{16}$$

From (16), the *excitation* at the rear wheels during the warning braking can be approximated to the following gradient:

$$\Delta \mu_{xr} = \int_0^{D_{wb}} \left(\frac{\partial \mu_{xr}}{\partial F_{xr}} \frac{dF_{xr}}{dt} + \frac{\partial \mu_{xr}}{\partial F_{zr}} \frac{dF_{zr}}{dt} \right) dt \qquad (17)$$

Because $\partial \mu_{xr} / \partial F_{xr} = 1/F_{zr}$ and $\partial \mu_{xr} / \partial F_{zr} = -\mu_{xr}/F_{zr}$ from (16), (17) can be rewritten as:

$$\Delta \mu_{xr} = \int_0^{D_{wb}} \frac{1}{F_{zro} - |dF_{zr}|} \left(\frac{dF_{xr}}{dt} + \mu_{xr} \left|\frac{dF_{zr}}{dt}\right|\right) dt \quad (18)$$

In (18), $F_{zr} = F_{zro} - |dF_{zr}|$ where F_{zro} denotes the initial normal force at the rear wheels at the start of warning braking, which can be retrieved from (11), and dF_{zr} denotes the variation in the normal force at the rear wheels from the start of warning braking. From the assumption of a monotonic decrease of F_{zr} during the warning braking, (18) becomes the following inequality condition:

$$\Delta \mu_{xr} \ge \underline{\Delta \mu_{xr}} = \int_0^{D_{wb}} \frac{1}{F_{zro}} \left(\frac{dF_{xr}}{dt} + \mu_{xr} \left| \frac{dF_{zr}}{dt} \right| \right) dt \quad (19)$$

where $\Delta \mu_{xr}$ denotes the lower bound of the *excitation* level. Finally, from (11), (12), and (13), neglecting the variation of trivial forces and wheel inertia, $\Delta \mu_{xr}$ can be rewritten as:

$$\underline{\Delta\mu_{xr}} = \frac{1}{F_{zro}} \left(m\eta_b A_{wb} + C_{lt} A_{wb} \mu_{xr}^{avg} \right) \\
= \frac{A_{wb}}{G} \frac{L}{L_f} \left(\eta_b + \frac{C_{lt}}{m} \mu_{xr}^{avg} \right)$$
(20)

If the average friction-coefficient is defined roughly as $\mu_{xr}^{avg} = 0.5 \Delta \mu_{xr}$, then the relationship in (15) can be rearranged as follows:

$$\Delta \mu_{xr} \ge \underline{\Delta \mu_{xr}} = \frac{\Delta \mu_{xro}}{1 - 0.5C'_{lt}} \tag{21}$$



Fig. 7. Simulation results of *excitation* for various braking torque distribution ratios. Here, $A_{wb,0.4}$ denotes the required A_{wb} for the *excitation* at each axle be 0.4. The solid line represents the values calculated from $\Delta \mu_{xr}$ in (21). The crosses and circles represent the simulation results for the front and rear axles, respectively.

Note that the excitation at the rear wheels can be elevated with the normalized load-transfer coefficient C'_{lt} as shown in (21). A previous study [13] recommended that the *excitation* be larger than 0.4 for the distinction of slip- μ slopes between different types of road surfaces. Figure 7 represents the simulation results of excitation for a range of braking torque distribution ratios η_b . The simulation is done with the veDYNA[®] commercial high-precision vehicle dynamics platform that embodies a sophisticated tire model. The level of A_{wb} is limited to 0.5G, based on the recommendation given in the ISO standard [19]. As depicted by the circles in Fig. 7, the simulation results for the rear axle are in good agreement with the approximation in (21). It is noteworthy that $A_{wb,0,4}$ for the rear axle is only about 0.14G if the warning braking is applied only to the rear wheels. This fact is important from the viewpoint of attaining coordination with the human factor issues. For example, the friction estimate can be reliable with a configuration of $\eta_b = 0.68$ for the recommended value of $A_{wb} = 0.2G$ in [24]. Vehicle parameters L, L_f , and C_{lt} in the simulation are 2.57 m, 1.25 m, and 270.29, respectively.

Based on the above discussion, the autonomous emergency braking system can be configured in such a way that the braking torque is concentrated on one axle to attain reliable friction estimation during warning braking, but distributed to both axles to attain maximum road adhesion during emergency braking.

Even though the present study assumed straight road driving, it would be useful to investigate how the the lateral dynamics in a normal conflict situation affect the longitudinal *excitation*. Assuming a friction circle, the longitudinal *excitation* during warning braking is limited as follows:

$$MAX(\Delta\mu_{xr}) = \sqrt{\mu_m^2 - \mu_{yro}^2}$$
(22)

where $\mu_{yro} = F_{yro}/F_{zro}$ is the initial lateral component of the friction-coefficient at the start of the warning braking, and the additive *excitation* of lateral component $\Delta \mu_{yr}$ during warning braking is neglected because the warning braking is a longitudinal maneuver.

According to [27], the 50th and 90th percentile of the lateral acceleration a_y for curves with small radii of less than 70 m are



Fig. 8. Simulation of combined *excitation* in rear wheels for various lateral accelerations a_y for a curve of radius 20m. The solid line with circles denotes the maximum combined *excitation* during the warning braking when a_y is kept constant.

around 3 and $5 m/s^2$. Figure 8 shows the simulation results of the combined excitation during warning braking for various lateral accelerations with reference to [27]. The maximum value of combined *excitation* is observed in the rear wheels while the warning braking is triggered with $A_{wb} = A_{wb,0.4}$, i.e., while the sufficient longitudinal excitation is attained for reliable friction estimation, and η_b is set to 1. In Fig. 8, the combined excitation is about 0.6 if a_y is maintained at $5 m/s^2$, which is affordable for the friction estimation for the dry or wet road surfaces. This fact implies that the longitudinal excitation, required for reliable friction estimation during warning braking, can be assured in the majority of conflict situations. If the lateral acceleration is set to an average level, the effect of lateral dynamics becomes insignificant as depicted in Fig. 8. Moreover, warning braking in a normal conflict situation is usually triggered when drivers are distracted in situations in which the lateral dynamics are relatively gentle and the sensor system has sufficient opportunity to recognize the hazardous situation.

3) Duration: Another important consideration is that there should be a sufficient time margin between the friction estimation and the triggering of autonomous braking. This can be interpreted as a requirement for the duration of the warning braking, as follows:

$$D_{wb} \le \min\{\mathcal{M}(\mu_m, v_r) | \forall \mu_m \in \mathcal{U}_{op}, \quad \forall v_r \in \mathcal{V}_{op}\}$$
(23a)

$$\mathcal{M}(\mu_m, v_r) = T_{diff}(v_r) - \Delta T_{eb}(\mu_m, v_r)$$
(23b)

where $T_{diff} = T_{eb} - T_{wb}$ is the time difference between the triggering of the warning braking and the following emergency braking under nominal road conditions, while $\Delta T_{eb} = T_{eb} - T_{eb}^*$ is the amount of time by which the emergency braking is shifted according to the friction estimation results. Here, v_r denotes the relative speed, and $\mathcal{V}_{op} \sim [v_{r,min} v_{r,max}]$ is the operation range of relative speed, which would be at the manufacturer's discretion. The parameters p_b and p_{wb} are omitted from (23b). The use of too conservative warning or a lack of adhesion would result in the violation of the condition in (23), such that a collision could occur. However, the proposed method is still effective because the collision speed can at least be reduced with the estimated friction information,

TABLE I Collision Avoidance Strategies [28]

Car Maker	Alert Cascade	$T_{diff}[\mathbf{s}]$
A	$CW - PB^* - EB$	1.2
B	$CW - PB^*$	3.3
C	$CW - EB^*$	1.4
D	$CW - WB - PB1 - PB2^* - EB$	1.1

i.e., the collision can be mitigated. The value of $\mathcal{M}(\mu_m, v_r)$ depends on the actuation gap, $\mathcal{O}(\mu_m) = \hat{A}_{b,eb}(\mu_m) - A_{b,wb} \ge 0$. If $\mu_{m,min}$ is set to $\mathcal{O}(\mu_{m,min}) > \epsilon$, where $\epsilon \in \mathcal{R}$ is a positive small value, the following properties hold:

$$\frac{\partial T_{diff}}{\partial v_r} > \frac{\partial (\Delta T_{eb})}{\partial v_r} > 0, \quad \frac{\partial (\Delta T_{eb})}{\partial \mu_m} < 0 \tag{24}$$

Thus, from (23b), it follows that:

$$\frac{\partial \mathcal{M}(\cdot)}{\partial v_r} > 0, \quad \frac{\partial \mathcal{M}(\cdot)}{\partial \mu_m} > 0 \tag{25}$$

That is, $\mathcal{M}(\mu_m, v_r)$ is a monotonically increasing function for the relative speed v_r and μ_m in this case. Thus, $\mathcal{M}(\mu_m, v_r)$ is expected to be minimized at the minimum operating values of $v_{r,min}$ and $\mu_{m,min}$. However, $\mathcal{M}(\mu_m, v_r)$ becomes nonlinear at extremely low speeds and reaches a minimum at a certain critical speed. This is because the speed reduction Δv_{wb} by warning braking becomes dominant. The simulation results, presented later, show that the critical speed is around 12 km/h. On the other hand, if $\mu_{m,min}$ is set to $\mathcal{O}(\mu_{m,min}) \leq \epsilon$, the minimum of $\mathcal{M}(\mu_m.v_r)$ is approximately a function of parameters p_b and p_{wb} , as follows:

$$D_{wb} \le \mathcal{M}(\boldsymbol{p}_b, \boldsymbol{p}_{wb}) \approx T_{d,h} + \frac{\Delta v_{wb}}{A_{b,wb}}$$
(26)

where the last term on the right side represents the additional time margin resulting from the speed reduction by the warning braking.

The time margin depends on the characteristics of the associated threat assessment algorithm and the strategy adopted for the operation cascade. Car makers all have their own strategies for collision avoidance, as shown in Table I. In Table I, *WB*, *PB*, and *EB* are the abbreviations for warning braking, partial braking, and emergency braking, respectively. Some car makers utilized warning braking in their operation cascade. In Table 1, T_{diff} is the time between the collision warning (*CW*) and the asterisked autonomous braking stage in the operation cascade. All the results in Table I are obtained from a scenario in which an ego vehicle travelling at 100 km/h approaches a slower target vehicle at 60 km/h under nominal (dry) road conditions.

IV. FRICTION ESTIMATION

A. Slip-Slope Method

The warning braking is momentary and is operated with limited jerk/deceleration, considering the human factors described in Section III. Thus, we apply a slip-slope based method to



Fig. 9. Friction space for rear wheels.

estimate μ_m because a better estimation performance can be ensured even with the small amount of wheel slip [12], [13].

B. Quantized Friction Estimation

The actuation level of the braking system is adapted according to the friction estimation results. Some important considerations are raised by the use of this configuration. First, the system performance should be predictable to the driver and be robust. Second, the system should be manageable for various actuation levels. From these points of view, a quantized slip-slope method combined with the curve-matching (CM) algorithm is suggested for estimating μ_m . In the quantized slip-slope method, the representative slip-slopes are predefined for different tire-road adhesion levels. The optimal slip-slope, that is, the one producing the best match with the trajectory of the samples $(s_{xr,k}, \mu_{xr,k})$, is searched in realtime, where a subscript k denotes the k-th sample. Hereafter, the subscript represents the rear wheel case. Therefore, the road friction is estimated only for the quantized road adhesion levels in \mathcal{U}_{op} .

Consider a 2-dimensional friction space $\mathcal{F}_n \in \mathcal{R}^2$ that consists of every possible combination of (s_{xr}, μ_{xr}) , where *n* denotes the number of sub-spaces Θ_i that classifies the level of tire-road adhesion, giving:

$$\mathcal{F}_n = \bigcup_{i=1}^n \Theta_i(K_{s,i}, \delta_i) \tag{27}$$

where $\Theta_i \cap \Theta_j = \emptyset$ for $i \neq j$, and each sub-space Θ_i is characterized with a representative slip-slope $K_{s,i}$. In (27), δ_i denotes the conservativeness factor, which is the gap between $K_{s,i}$ and the boundary of sub-space, as illustrated in Fig. 9. A larger value of δ_i corresponds to a more conservative approach. Here, the term *conservative* means that the samples tend to fit a steeper slip-slope. In Fig. 9, A_i denotes the line of representative slip-slope $K_{s,i}$, which defines the linear relationship between the wheel slip s_{xr} and the road friction μ_{xr} , and which is defined as:

$$A_i: K_{s,i} s_{xr} - \mu_{xr} = 0 \tag{28}$$

Furthermore, the distance between each sample point P_k and each line A_i is defined as the Euclidian distance between P_k



Fig. 10. Friction space with various levels of road adhesion. The horizontal length of each sub-space Θ_i indicates the divergence in the observed slip-slopes.

and the projected point P'_k , as follows:

$$j_d(A_i, P_k) \sim ||P_k - P'_k||_2$$
 (29)

This is an incremental cost obtained from the distance metric. Then, the search for the optimal slip-slope becomes a problem of minimizing the sum of the incremental costs:

$$J_{i} = \sum_{k=1}^{m} \lambda^{m-k} j_{d}(A_{i}, P_{k})^{2} = \sum_{k=1}^{m} \lambda^{m-k} \frac{\|K_{s,i}s_{xr,k} - \mu_{xr,k}\|^{2}}{1 + K_{s,i}^{2}}$$
(30)

where *m* denotes the number of samples, and λ denotes the forgetting factor, which is a tuning parameter. Because the distinction between two adjacent slip-slopes improves for a larger *excitation*, the forgetting factor would enhance the estimation of the slip-slope. This will be verified with the experimental results, later. Then, the optimal slip-slope $K_{opt,k}$ at the current sample time can be obtained as follows:

$$K_{opt,k} = K_{s,i^*} | i^* = arg \min J_{i,k}$$
 (31)

Finally, the $\hat{\mu}_m$ is determined from $K_{opt,k}$. Some previous research used linear equations such as $\hat{\mu}_m = c_1 K_s + c_2$ to determine the friction-coefficient from the estimated slipslope. However, the slip-slope varies within a range even for the same tire type or the same road surface [12], [13]. Thus, a systematic approach based on the quantized slip-slope method is suggested as a means of enabling robust friction estimation. As illustrated in Fig. 10, it is first necessary to divide the friction space \mathcal{F}_n into *n* independent sub-spaces Θ_i , based on the results of experimental observations. Next, it is necessary to determine the representative slip-slopes $K_{s,i}$ by tuning the conservativeness factors δ_i . The tuning of δ_i can be done either statistically or mathematically, according to the designer's choice. For example, the value of δ_i could be tuned mathematically for consecutive sub-spaces, as follows:

$$\delta_i > \frac{K_{s,i+1} + K_{s,i}}{2} \tag{32}$$

This could lead to the obtaining of conservative results.

Each sub-space Θ_i matches a specific maximum tire-road friction-coefficient $\hat{\mu}_m(\Theta_i)$, as represented by Fig. 10. Thus, $\hat{\mu}_m(\Theta_i)$ can simply be obtained from a one-dimensional lookup table (LUT) such as,

$$\hat{\mu}_m(\Theta_i) = \text{LUT}(K_{opt,k}) \tag{33}$$

The input to the LUT is $K_{opt,k}$, the best-matched $K_{s,i}$.



Fig. 11. Hydraulic power unit.

TABLE II Test Vehicle Specifications

Parameter	Original	HPU installed
Mass(kg)	1525	1584
Wheelbase(m)	2.845	2.845
$L_f(m)$	1.105	1.209
$C_{lt}^{\prime}[-]$	0.1	0.091

Although only the distance metric is considered in this study, additional metrics, e.g., the direction metric [29], can also be considered to refine the matching process. A direction metric can be beneficial when there is a bias in the origin. As mentioned above, this quantized approach can be advantageous for the collision avoidance application in terms of system management because the system is calibrated only for a few pre-defined actuation levels. In addition, the driver's acceptance can be improved because the driver can predict the performance of the system, while the operation characteristics of the system can easily be tuned with parameters such as $K_{s,i}$ and δ_i .

V. EXPERIMENTS

A. Experiment Set-Up

1) Test Vehicle: As a test vehicle, we used a Hyundai Grandeur HG240. To attain the warning braking function, we developed an auxiliary hydraulic power unit (HPU) and installed it in the trunk of the test vehicle as shown in Fig. 11. The HPU mainly consists of an accumulator, motor/pump, and servo-valve. The warning braking is applied by the control of hydraulic servo-valve. The servo-valve used in the experiments was a MOOG G761. Table II lists the specifications of the test vehicle. The vehicle was also equipped with an RT3003 inertial and GPS navigation system, produced by Oxford Technical Solutions, to measure the distance to the target vehicle, and AutoBox, manufactured by dSPACE, for the real-time threat assessment. The longitudinal acceleration and wheel speeds are retrieved via CAN communication to estimate the road friction.

2) Warning Braking Parameters: As discussed previously, the braking dynamics should be maximized with limited

actuation levels, given the human factors issue. To maximize the effect of braking torque distribution, warning braking is applied only to the rear wheels in the experiments, i.e., $\eta_b = 1$ during the warning braking. The p_{wb} is set as:

$$\boldsymbol{p}_{wb} = [-0.32G/s \ 0.65s] \tag{34}$$

which results in a speed reduction of about 0.8 m/s. The values in (34) are in line with the recommendations produced by from the human factors perspective results in [24]. Of course, the values may vary with the characteristics of the algorithm and the strategies. For example, the value of D_{wb} may be reduced for a smaller time margin. The theoretical expectation of *excitation* for an HPU-installed test vehicle is about 0.53, as obtained from (21).

3) Estimation of Tire Forces: The tire forces should be estimated for the calculation of tire-road friction-coefficient μ_{xr} in (16). The normal tire force F_{zr} is estimated from (11). The longitudinal tire force F_{xr} can be estimated from the force balance in (12) by adopting an observer design, as follows:

$$m\dot{\hat{v}}_{ego} = \hat{F}_{xr} - R_x - F_a + K_o(v_{ego} - \hat{v}_{ego})$$
 (35)

where \hat{v}_{ego} , \hat{v}_{ego} , and \hat{F}_{xr} denote the estimated speed of the ego vehicle, the derivative of \hat{v}_{ego} , and the estimated longitudinal tire force, respectively. The value of K_o in (35) denotes the positive observer gain. Because warning braking is applied only to the rear wheels, the longitudinal tire force at the front wheels is neglected and v_{ego} is obtained from the front wheel speed while warning braking is being applied. Based on the observer design described in (35), F_{xr} is estimated from the following adaptive scheme [14]:

$$\hat{F}_{xr} = K_f (v_{ego} - \hat{v}_{ego}) \tag{36}$$

where \hat{F}_{xr} denotes the derivative of \hat{F}_{xr} and K_f denotes the positive force estimation gain. For the experiments, the values of K_o and K_f were set to 12,000 and 300,000, respectively, which were tuned in the simulations.

Regarding the robustness issue, if there is a difference in the sensitivity to the mass change between the estimated value of F_{xr} and F_{zr} , the effect of mass change becomes significant in the estimation of the friction-coefficient, as discussed in [14]. However, as denoted in (11) and (35), the vehicle mass affects both F_{xr} and F_{zr} in the present study. Thus, it would be reasonable to expect that the friction estimation is robust, regardless of the mass change. Figure 12 shows the results of the μ_{xr} estimation for a mass change of -20% to 20% relative to the original value, with the simulation settings and parameters as described in Section III. Figure 12 shows that the friction estimate is robust to the mass change, which is in good agreement with the expectation described above.

B. Experiment Results

1) Construction of Friction Space: The slip-slopes were measured for two adhesion levels, dry and wet asphalt, i.e., n = 2. The experiments were conducted with the test vehicle on the low-friction track of the proving grounds of the Korea Automobile Testing & Research Institute (KATRI). The specification of μ_m of wet asphalt lane was 0.7 as shown



Fig. 12. Simulation results of μ_{xr} estimation with mass change.



Fig. 13. Wet asphalt lane of KATRI low-friction track.



Fig. 14. Test trials for construction of sub-spaces. (a) Implementation of warning braking pulses. The solid line is the reference profile for the warning braking pulse. (b) μ -slip curves of test trials. The blue lines correspond to the wet asphalt while the red lines are for dry asphalt.

in Fig. 13. We conducted 50 trials for each road adhesion level, giving a total of 100 trials. Half of the trials conducted for each road adhesion level, depicted in Fig. 14, were dedicated to the construction of sub-spaces, while the remaining half were used for the tuning of the representative slip-slopes. Table III represents the slip-slope measurements obtained for the construction of sub-spaces. The top and bottom outliers

TABLE III Results of Slip-Slope Measurements

	Dry Asphalt	Wet Asphalt
Median	19.41	15.35
σ	1.41	1.31
Maximum	21.96	17.66
Minimum	16.77	12.38
$\Delta \mu_{xr}$ (avg.)	0.49	0.50



Fig. 15. Statistical results obtained from confusion matrix.

were excluded from the statistics. Even though the *excitation* in the experiments are only estimates, their average is close to the theoretical value of 0.53, as represented by Table III.

The boundary slip-slope between two sub-spaces was set to 16.8, which is the minimum slip-slope for dry asphalt. Therefore, the overlapped range is truncated at the lower adhesion side. The value of $K_{s,2}$ is fixed to the median value of 19.41, and $K_{s,1}$ is tuned from the statistical results. Figure 15 represents the statistical results of estimating the road friction from the confusion matrix defined in Table IV for different values of conservativeness factor δ_1 . The precision and false-positive rate are comparatively flat over δ_1 . However, the true-positive rate sharply drops at $\delta_1 = 2.0$. Considering the efficacy of the system, δ_1 is thus set to 2.0, which results in $K_{s,1} = 14.8$. The forgetting factor λ in (30) was set to 0.91 in the experiments.

In consequence, the friction space \mathcal{F}_2 can be constructed from the experiment results as shown in Fig. 16. The truepositive rate is related to the efficacy of the system, while the false-positive rate is related to the driver's acceptance. The system must be optimized somewhere between these two rates. The estimated maximum friction-coefficients are empirically set as $\hat{\mu}_m(\Theta_1) \sim 0.7$ and $\hat{\mu}_m(\Theta_2) \sim 1.0$ for the wet and dry asphalt, respectively.

Figure 17 shows an experiment result for the friction estimation obtained from a sample trial. As shown in Fig. 17(b), the estimated costs are reversed at t = 7.02 s and the estimated maximum tire-road friction-coefficient changes from the default value of $\hat{\mu}_m(\Theta_1) \sim 1.0$ to 0.7, which is the final estimated value. Note that the estimations in the early stages are not distinguishable between different road adhesion



Fig. 16. Friction space construction (n = 2).



Fig. 17. Experiment results of friction estimation in the time domain. (a) Results of road friction estimation for wet asphalt in time domain. The solid and dotted lines represent the estimation results with forgetting factors of 0.91 and 1.0, respectively. (b) Calculated cost J_i during estimation with forgetting factor of 0.91.

TABLE IV Confusion Matrix

		Predicted	
		Wet	Dry
True	Wet	True	True
		Positive	Negative
	Dry	False	False
		Positive	Negative

levels. This is because the slip-slopes for a low *excitation* are not distinctive, as depicted in Fig. 14(b). Also note that the estimation is accomplished at an earlier stage if the forgetting factor is applied, as shown in Fig. 17(a).

2) Threat Assessment With Friction Information: With the constructed friction space \mathcal{F}_2 , the effect of the friction information on the threat assessment is verified in the experiments. The experiments were performed on the wet asphalt lane of KATRI low-friction track. A stopped vehicle scenario is used for the experiments. An ego vehicle travelling at 50 km/h approaches an imaginary stopped target on a straight/flat road.



Fig. 18. Simulation results of time margin vs. relative speed.

The imaginary target is placed somewhere in the proving ground and the distance to imaginary target c_{et} and v_{ego} are measured from the RT3003 GPS. The operation cascade is composed of the warning braking and the following emergency braking as *Assumption* 3. The minimum distance c_o is set to 0.5 m. The value of p_{wb} is set as given by (34) and p_b is set as follows, reflecting the characteristics of the test vehicle braking system:

$$\boldsymbol{p}_{b} = \begin{cases} [-0.4G - 2G/s \ 0.86s], & for \ WB \\ [-G - 2G/s \ 0.2s], & for \ EB \end{cases}$$
(37)

where the value of A_b for the emergency braking represents the nominal value $A_{b,o}$, and T_d for the warning braking is the sum of $T_{d,s}$ for 0.2 s and $T_{d,h}$ for 0.66 s. The value of $T_{d,h}$ is based on the reference report [30], which is the median value of the reaction time of human drivers. Figure 18 shows the results of simulating the time margin $\mathcal{M}(\mu_m, v_r)$ versus the relative speed v_r for $\mu_m \in \mathcal{U}_{op} \sim [0.41.0]$, with parameters p_b and p_{ub} of (34) and (37), respectively. From the simulation results, we obtain $T_{diff}(v_r = 50 \, km/h)$ and $\mathcal{M}(\mu_m = 0.7, v_r =$ 50 km/h values of 1.7 s and 1.49 s, respectively. Of course, the time margin may be made shorter for a more conservative threat assessment. If the threat assessment for the warning braking is too conservative to assure a sufficient duration, one possible means of overcoming this is to reduce the setting of parameter D_{wb} to a lower value because only a few hundreds of milliseconds are needed for the friction estimation. However, this requires an increase in the designed jerk J_{wb} of the warning braking, and the decision should be made based on a consideration of the human factors. In Fig. 18, the simulation result for the case of $\mu_m = 0.4$ conforms to the discussion in (26).

The test vehicle approached the imaginary stopped target both with and without friction estimation. Figure 19(a) represents the experiment results obtained without any friction information. Because the threat assessment is done with the default value of maximum adhesion, the collision occurs at a reduced speed, that is, the collision is mitigated. On the other hand, the collision is completely avoided with the friction information, as depicted in Fig. 19(b). The final distance is about 0.38 m. Comparing this with the results in Fig. 19(a) shows that the emergency braking trigger is shifted forward as in (5), because the proposed adaptive threat assessment algorithm is notified of the friction information. Figure 20 represents the experiment results for the maximum achievable deceleration estimation for the case shown in Fig. 19(b). As soon as the warning braking is completed, the maximum tire-road friction-coefficient $\hat{\mu}_m$ is estimated and $\hat{A}_{b,eb}(\hat{\mu}_m)$ is calculated from the nominal value of $A_{b,o}$, as in (4).

VI. DISCUSSION OF EXCEPTIONAL CASES

A. Split-µ During Warning Braking

In the case of the road adhesion being maintained for both the left and right wheels during split- μ , the road friction can be estimated for each wheel while attaining the required adhesion for both sides. Herein, the suggested scheme and discussion also can be applied to the individual wheels without any modification based on *Assumption 4*. Finally, the maximum achievable deceleration $\hat{A}_{b,eb}$ that is accounted for the adaptation of the trigger time for the emergency braking would be determined as follows:

$$\hat{A}_{b,eb} = min(\hat{A}_{b,eb,i}), \quad i = fl, fr, rl, rr$$
(38)

where $\hat{A}_{b,eb,i}$ denotes the estimated maximum achievable deceleration for each wheel. The strategy described in (38) prevents unstable motion during emergency braking in a prognostic manner, unlike existing stability control systems that are aware of the road status only after the emergency braking is triggered.

Figure 21 shows the results of simulating this case under the split- μ condition, using the simulation settings given in Section III, $\eta_b = 1$, and p_{wb} in (34). In this simulation, the friction space is empirically composed of two sub-spaces, as follows:

$$\begin{bmatrix} K_{s,1} \\ K_{s,2} \end{bmatrix} = \begin{bmatrix} 15.70 \\ 18.73 \end{bmatrix}, \quad \begin{bmatrix} \hat{\mu}_m(\Theta_1) \\ \hat{\mu}_m(\Theta_2) \end{bmatrix} = \begin{bmatrix} 0.7 \\ 1.0 \end{bmatrix}$$
(39)

The left wheels of the vehicle are run on the low- μ side while the forgetting factor λ is set to 1.0. As represented in Fig. 21, the road friction of each side of the surface is successfully estimated without any loss of adhesion. Because the road adhesion is attained on both sides of the split- μ during the warning braking, the yaw-rate remains stable throughout the actuation of the warning braking, as shown in Fig. 21(c).

If road adhesion is lost for one side of the wheel during split- μ , unstable motion such as yawing must be prevented because the warning braking is just the means of haptic warning for a distracted driver. This can be done by the active intervention of a system with a higher priority such as the anti-lock braking or stability control system. Such a legacy system would regulate the braking force as soon as saturation in the tire force is observed during *excitation*. Finally, the strategy described in (38) can be applied to prepare for the subsequent emergency braking, with the road friction information estimated before saturation. The estimation of the friction on extremely slippery surfaces will be described in detail later.



Fig. 19. Experiment results of threat assessment obtained with friction information. (a) Collision mitigation without friction information. (b) Collision avoidance with friction information. The solid line represents the distance between the ego vehicle and the stopped imaginary target, while the dotted line represents the speed of the ego vehicle. The dash-dot and dashed lines represent the triggers for the warning braking and emergency braking, respectively.



Fig. 20. Estimation of maximum adhesion level after warning braking. The solid line represents the estimated maximum achievable acceleration, while the dotted line represents the actual acceleration of the ego vehicle.

B. Transient-µ During Warning Braking

This is a very rare case because the duration of the warning braking is only a few hundreds of milliseconds. In this case, the detection algorithm for the surface change should be incorporated. Gustafsson [12] suggests the use of the CUSUM algorithm for detecting a surface change and issuing a warning to the driver. The covariance matrix of the Kalman filter is adjusted such that the time constant becomes shorter as a result of detecting the surface change. Choi *et al.* [16] suggest the use of the adaptive forgetting factor for the estimation based on the recursive least square method.

In the present study, the performance of the suggested scheme in the transient- μ situation is verified using the CUSUM detector for the surface change during warning braking, as follows [12]:

$$g_{0} = 0$$

$$g_{k} = g_{k-1} + e_{k} + \nu$$

$$g_{k} = MAX(g_{k}, 0)$$
(40)

where ν is a design parameter, and e_k denotes the estimation error for the wheel slip in the current step, which is defined



Fig. 21. Friction estimation for split- μ surface. (a) Calculated costs J_i (left axis) and estimated road friction (right axis) for rear-left wheel. (b) Calculated costs J_i (left axis) and estimated road friction (right axis) for rear-right wheel. (c) Yaw-rate during warning braking.

as:

$$e_k = \mu_{xr,k} K_{opt,k}^{-1} - s_{xr,k}$$
(41)

where the overestimation of slip-slope results in a positive value of e_k . Here, an adjustment rule for the forgetting factor based on [12] is suggested, as follows:

$$if \ g_k > h$$
$$g_k = 0; \quad \lambda = \lambda_o - \lambda_g e^{-\tau^{-1}}t$$
(42)

where λ_o , λ_g denote the original and marginal values of the forgetting factor, respectively, and *h* is a positive threshold value.



Fig. 22. Costs with the detection of surface change. (a) Calculated costs J_i during warning braking. (b) Forgetting factor λ (solid line, left axis) and the estimation error of wheel slip (dotted line, right axis).

Figure 22 represents the simulation results of the cost J_i with the CUSUM detector during warning braking under transient- μ conditions. The friction-coefficient of the road surface suddenly changes from 1.0 to 0.7 at t = 12.58 s. The friction space and simulation parameters are set to the same values as in the split- μ case. The values of λ_o and λ_g are set to 1.0 and 0.15 in the simulation, respectively. In other words, the forgetting factor is reduced to 0.85 instantaneously and then reverts back to the original value of 1.0 exponentially for the stable estimation. As represented in Fig. 22, the costs are reset when $g_k > h$ and the result of friction estimation is eventually reversed. The ν , h, and τ values are set to 0, 0.15, and 0.05 s, respectively, in this simulation.

C. Extremely Slippery Surfaces

Because the slip-slopes of the extremely slippery surfaces, such as snow or ice, are more easily distinguishable from the nominal higher one, the suggested scheme would perform well in the low-slip linear region.

However, if the system actuates the braking system upon attaining $A_{wb,0.4}$ on snowy or icy surfaces without any prior knowledge of the road friction, the longitudinal tire force will start to saturate before the *excitation* reaches the target value. In such a case, the braking force should be regulated by the operation of the legacy systems as discussed above. At the same time, it is recommended that a decision be made regarding the road friction, terminating the friction estimation with the suggested scheme as soon as saturation is detected. If the friction estimation using the suggested scheme is continued even after saturation, the data samples would be fitted to the lowest friction sub-space, which may result in an increased degree of error. If necessary, further estimation after the saturation can be done using popular tire models [16], [17], for example.

Figure 23 shows the simulation results for the μ -slip curve and cost J_i for the rear wheels. The simulation was conducted



Fig. 23. Simulation results of friction estimation for $\mu_m = 0.3$. (a) μ -slip curve during saturation. (b) Cost J_i during saturation where J_i represents the cost for Θ_i .

with $\eta_b = 1$, $\lambda = 1$, and $\mu_m = 0.3$, where the vehicle triggers a warning braking at with $\boldsymbol{p}_{wb} = [A_{wb,0.4} \ 0.65 \ s]$. The friction space is composed of three sub-spaces as follows:

$$\begin{bmatrix} K_{s,1} \\ K_{s,2} \\ K_{s,3} \end{bmatrix} = \begin{bmatrix} 8.56 \\ 9.67 \\ 15.70 \end{bmatrix}, \begin{bmatrix} \hat{\mu}_m(\Theta_1) \\ \hat{\mu}_m(\Theta_2) \\ \hat{\mu}_m(\Theta_3) \end{bmatrix} = \begin{bmatrix} 0.2 \\ 0.3 \\ 0.7 \end{bmatrix}$$
(43)

As expected, saturation occurs during the *excitation* due to the lack of adhesion, as shown in Fig. 23(a). As shown in Fig. 23(b), the distinction between the slip-slopes is well maintained in the low-slip linear region. However, as saturation starts at about $s_{xr} = 0.02$, the gap between the costs starts to decrease such that, ultimately, the costs are reversed at about $s_{xr} = 0.06$ (at $t = 6.12 \ s$). The simulation results are in good agreement with the discussion above.

VII. CONCLUSION

The friction estimation in this study was based on the slipslope method, which uses the slope of the μ -slip curve to determine the road adhesion level. However, the slip-slope can vary even for the same type of tire or the same road surface. In addition, the system must be easy-to-manage for various load adhesion levels. Thus, a quantized slip-slope method combined with a curve-matching algorithm is introduced in this study. An important feature is that the suggested framework and considerations are valid regardless of the limitations in the friction estimation, while the performance of the system can be guaranteed using advanced friction estimation technology. Moreover, friction-estimation techniques other than the slipslope method can be incorporated into the suggested adaptive collision avoidance scheme.

Future work would include the the reinforcement of the friction estimation to suppress the false-positive and to attain the required level of performance even on the non-homogeneous road surfaces, and the extension of the scheme to a wide range of conflict scenarios in more detail involving the possibility of lateral evasive maneuvering.

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