# Real-Time Individual Tire Force Estimation for an All-Wheel Drive Vehicle

Hojin Jung <sup>D</sup> and Seibum B. Choi, *Member, IEEE* 

Abstract—Due to demands for active control systems that can enhance driving performance, active-type all-wheel drive (AWD) systems have drawn interest recently. Equipped with an electronically actuated motor to control a wet clutch in a transfer case, the active-type AWD facilitates the variable distribution of torque from the main driveshaft to the subdriveshaft. The conventional approaches to the estimation of tire force, developed to implement chassis control, have only focused on the two-wheel drive vehicle dynamics model. In this paper, an individual tire force estimation algorithm that is particularly designed for AWD vehicles is proposed. Using the interacting multiple model filter method, the suggested algorithm can help avoid the chattering response caused by immediately switching between vehicle dynamic models. Data obtained from the controller area network of production vehicles were used for the real-time application of the proposed estimator to chassis control. Then, the proposed estimator was validated using an AWD vehicle in various driving scenarios.

*Index Terms*—All-wheel drive (AWD), discrete-time extended Kalman filter (EKF), interacting multiple model (IMM) filter, parameter adaptation, vehicle dynamics model.

## I. INTRODUCTION

I N CONTRADISTINCTION to two-wheel drive (2WD) vehicles, all-wheel drive (AWD) vehicles allow the distribution of the driving torque generated by the engine to both the front and rear wheels by attaching a transfer case at the transmission output shaft. Recently, the number of vehicles with AWD systems installed has increased, even for on-road vehicles, because 2WD can allow excessive slip during the generation of tire-road traction force due to increases in engine power and drivetrain efficiency. The electronically actuated AWD system discussed in this paper differs from a passive-type AWD, because it controls the amount of driving torque distributed to the sub-driveshaft by changing the engagement force of the wet clutch in the transfer case.

Manuscript received April 19, 2017; revised August 24, 2017 and November 3, 2017; accepted November 24, 2017. Date of publication December 7, 2017; date of current version April 16, 2018. This work was supported in parts by the National Research Foundation of Korea grant funded by the Korea government (MSIP) (No. 2017R1A2B4004116), the Ministry of Science and ICT, Korea, under the Information Technology Research Center support program (IITP-2017-2012-0-00628) supervised by the Institute for Information and communications Technology Promotion, and the BK21 Plus Program. The review of this paper was coordinated by Dr. A. Chatterjee. (*Corresponding author: Seibum B. Choi*)

The authors are with the Department of Mechanical Engineering, Korea Advanced Institute of Science and Technology, Daejeon 34141, Korea (e-mail: bourne@kaist.ac.kr; sbchoi@kaist.ac.kr).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TVT.2017.2779155

The control algorithms that govern the operation of the vehicles active safety technologies, e.g., the anti-lock braking system, electronic stability control, active front steering, and roll stability control, use data on the vehicles states, primarily of longitudinal velocity, sideslip angle, and tire force [1]–[3]. In particular, the tire force can be a critical state for the performance enhancement of vehicle control systems [4]–[9]. Although there have been numerous approaches to estimating vehicle states and tire forces, most of them are applicable only to 2WD vehicle dynamics. Because 2WD and AWD vehicles have different drivetrain structures, it is necessary to design an idiosyncratic vehicle state estimator that is applicable to AWD vehicles.

Previous studies of tire force estimation can be classified into two approaches. One estimated the tire force only through invehicle sensors but not independently at each wheel. The other involved the estimation of tire force using additional sensors. In [10], an estimator for vehicle planar motion states and tire force was suggested using the extended Kalman filter (EKF) by expressing the force state in second-order random walk Kalman filter form. However, this estimator was limited because it did not estimate the individual tire force and had a computational burden due to its use of a high-order model. In [11], an algorithm for estimating tire-road forces and friction coefficient was proposed using a single-track model and the Burckhardt/Kiencke adaptive tire model, but the tire forces could not be estimated independently due to its simple structure. Moreover, the braking condition was not included in this suggested estimator. In [12], a random walk Kalman filter that considered the combined tire force was proposed. Unfortunately, this estimator is vulnerable to sensor noise, because it requires the derivative value of the wheel angular velocity signal. In [13], a tire force estimator for road friction coefficient identification was suggested using the discrete-time EKF. However, this method could not estimate the individual lateral tire force, because it separated the tire model for the road friction estimation from the vehicle dynamics model for the vehicle states estimation. In [14], a sliding-mode observer scheme for tire force estimation was proposed. Although this study showed better performance than the other filters, the algorithm assumed no braking torque to satisfy the asymptotic stability criteria.

Regarding tire force estimation using in-vehicle sensors, various studies have attempted to enhance understanding about vehicle states. However, information about the vehicle states was limited, because only in-vehicle sensors were used. This approach had limitations on enhancing the accuracy of the estimator and on estimating more detailed vehicle states than those

<sup>0018-9545 © 2017</sup> IEEE. Personal use is permitted, but republication/redistribution requires IEEE permission.

See http://www.ieee.org/publications\_standards/publications/rights/index.html for more information.

of the previously discussed studies. For these reasons, another type of study, which estimated vehicle states in detail by installing additional sensors, was conducted. In [15], an estimator of vehicle motion states and tire force was suggested using the vehicle model combined with the Dugoff tire model; estimator validation was conducted in real time. Although it is meaningful that a method for estimating the independent tire force at each wheel was suggested, the estimation accuracy relied on a complex tire model that included trigonometric terms with predetermined tire parameters. Furthermore, the ability to mount in the electronic control unit (ECU) was not sufficiently considered due to the integration of all equations in a single state-space form. In [16], an estimator for vehicle planar motion states, roll angle, tire force, and tire parameters using the EKF was suggested. As this algorithm did not consider estimating the vehicle motion states without additional measurements, it was inadequate to apply to a vehicle control system in real time.

A second approach expanded the understanding of tire force but suffered from the limitation that it requires additional sensors for estimation. Requiring additional sensors implies that it could not be applied to production vehicles. In [17], a realtime independent tire force estimation strategy at each wheel was suggested and validated using both the EKF and the unscented Kalman filter (UKF). Although the method suggested was practical, because it did not require additional sensors nor the compensation for the uncertainty of the tire parameters, the transient estimation response for the lateral tire forces at each wheel was unacceptable due to the omission of the tire model. Moreover, the estimator was validated using only one scenario. Therefore, this paper suggests a novel method of real-time longitudinal and lateral tire force estimation at each wheel for an AWD vehicle based on the planar full car model. Two distinctive aspects of this study are that it applies a preliminary estimator of the vehicle sideslip angle while calculating the vehicles lateral velocity state and that it simplifies the steady tire model that can not only represent nonlinear tire characteristics based on lateral load transfer, but that is also suitable for on-board application. In addition, several effective algorithms were applied in this study to consider the parameter uncertainty and intrinsic hardware characteristic of the transfer case. First, this study suggests an adaptive method for reducing the uncertainty of the clutch friction coefficient, which should be considered, because it changes depending on the inner conditions of the clutch, such as oil temperature and engagement force. Secondly, a novel method for AWD tire force estimation, which reflects the clutch lock-up and slipping states simultaneously, is suggested, using the interacting multiple model (IMM) method. For the studies of clutch systems with one output shaft, the current state of the clutch has been determined using the relative angular velocity and transmitted torque [18], [19]. By contrast, it is difficult to determine whether an AWD clutch is in a lock-up or slipping state, as it has a two-output shaft, i.e., the allowable transferred torque can change depending on external factors, e.g., the engine torque, the difference between the front and rear final reduction gear ratios, and the front and rear wheel vertical loads. To deal with the aforementioned problem, this study adopted the IMM



Fig. 1. Model for AWD vehicle states estimation. (a) Full car dynamic model.(b) Drivetrain structure of the test vehicle.

filter, which calculates the estimated value by using the weighted probability of two different dynamic models [24]–[26].

The organization of this paper is as follows. The vehicle dynamic equations for the estimator design are described in Section II. Section III suggests sideslip angle observer and clutch friction coefficient adaptation algorithm working separately at the upper part of the tire force estimator. Section IV describes the method of IMM filter based tire force estimation for AWD vehicle. Section V explains experimental setup of the test vehicle. Then, Section VI provides experimental results to validate the advantages of the developed estimator.

## **II. VEHICLE MODELS**

Although an AWD system cannot distribute the driving torque independently to each wheel, information on the individual tire forces is important not only for advanced AWD controllers but also for integrated chassis controller design. Fig. 1(a) illustrates the planar vehicle model that includes several vehicle motion states and tire forces. Following the information in Fig. 1(a), the governing equations of motion are as follows:

$$m\dot{v}_x = F_{x1}\cos(\delta_1) + F_{x2}\cos(\delta_2) - F_{y1}\sin(\delta_1) - F_{y2}\sin(\delta_2) + F_{x3} + F_{x4} - \frac{\rho_{air}}{2}C_dAv_x^2 + mv_y\gamma(1)$$

 $m\dot{v}_y = F_{x1}\sin(\delta_1) + F_{x2}\sin(\delta_2) + F_{y1}\cos(\delta_1)$ 

$$+F_{y2}\cos(\delta_2) + F_{y3} + F_{y4} - mv_x\gamma$$
(2)

$$I_{z}\dot{\gamma} = l_{f} \{F_{x1}\sin(\delta_{1}) + F_{x2}\sin(\delta_{2}) + F_{y1}\cos(\delta_{1}) + F_{y2}\cos(\delta_{2})\} + t_{w} \{-F_{x1}\cos(\delta_{1}) + F_{x2}\cos(\delta_{2}) + F_{y1}\sin(\delta_{1}) - F_{y2}\sin(\delta_{2}) - F_{x3} + F_{x4}\} - l_{r}(F_{y3} + F_{y4})$$
(3)

where  $v_x$ ,  $v_y$ , and  $\gamma$  are the longitudinal velocity, lateral velocity, and yaw rate, respectively, at the vehicle's center of gravity (CG).  $F_x$ ,  $F_y$ ,  $\delta$ ,  $\rho_{air}$ ,  $C_d$ , A,  $t_w$ ,  $l_f$ ,  $l_r$ , m, and  $I_z$  are the longitudinal tire force, lateral tire force, wheel steering angle, density of air, aerodynamic drag coefficient, front cross-sectional area, half of track width, distance from front axle to the vehicle's CG, distance from rear axle to the vehicle's CG, total vehicle mass, and vehicle yaw inertia, respectively.

#### A. AWD Wheel Dynamics Model Set

1) Slipping State: When the transfer case clutch is in the slipping state, the AWD system can actively control the amount of torque transferred to the front shaft through adjusting the engagement force. Referring to the wheel dynamic model presented in [21], the dynamic equation of motion for the wheel can be modified as follows:

$$I_{w}\dot{\omega}_{i} = r_{c}F_{c}\mu_{c}\frac{i_{f}}{2} - T_{bi} - R_{e}F_{xi} - R_{e}R_{r}F_{zi}, i = 1, 2 \quad (4)$$
$$I_{w}\dot{\omega}_{i} = (T_{t} - r_{c}F_{c}\mu_{c})\frac{i_{r}}{2} - T_{bi} - R_{e}F_{xi} - R_{e}R_{r}F_{zi}$$
$$i = 3, 4 \quad (5)$$

where  $\omega_i, T_t, i_f, i_r, T_{bi}, R_{ri}$ , and  $F_{zi}$  are the wheel angular velocity at each wheel; transmission output torque, which can be obtained easily from the CAN signal of the engine torque and gear ratio; front final reduction gear ratio; rear final reduction gear ratio; braking torque; rolling resistance; and vertical load at each wheel, respectively.  $I_w$  and  $R_e$  are the wheel moment of inertia and effective wheel radius, respectively.

2) Lock-Up State: When the transfer case clutch is in the lock-up state, the AWD system cannot actively control the amount of torque transferred to the front shaft. This means that the front to rear torque distribution ratio is determined not by the clutch engagement force but by the external conditions. Then, the dynamic equation of motion for the wheel is expressed as follows:

$$I_w \dot{\omega}_i = \frac{T_f}{2} i_f - T_{bi} - R_e F_{xi} - R_e R_r F_{zi}, i = 1, 2 \quad (6)$$

$$I_w \dot{\omega}_i = \frac{T_r}{2} i_r - T_{bi} - R_e F_{xi} - R_e R_r F_{zi}, i = 3, 4$$
(7)

where  $T_f$  and  $T_r$  are the front shaft torque and rear shaft torque respectively. The sum of both is equal to  $T_t$ .

## B. Tire-Road Relation Model to Integrate Vehicle Dynamics

1) Steady Tire Model: Among several tire-road relation models in previous studies, the Dugoff tire model has been mostly adopted to estimate individual tire lateral force. Although the Dugoff model is quite simple compared to other tire models, it is still not appropriate for tire force estimation in real time due to a trigonometric function term in the model. Also, this model includes both cornering stiffness and road friction coefficient parameters that must be identified in advance. Generally, the main reason to include the tire model in the vehicle dynamics is to consider the nonlinear tire effect caused by both the vertical load transfer and vehicle sideslip angle. However, the augmentation of cornering stiffness as a state can cause a computational burden in the estimator [14]. This paper suggests the application of a simple tire model that addresses the nonlinear effect in the vehicle dynamics to avoid all of the above drawbacks. The



Fig. 2. Bicycle model.

modified lateral tire force model is described as follows:

$$\bar{F}_{y} = C_{1} \left( 1 + k_{1} \frac{F_{zi} - F_{zi,n}}{F_{zi,n}} \right) \alpha + C_{2} \left( 1 + k_{2} \frac{F_{zi} - F_{zi,n}}{F_{zi,n}} \right) \alpha^{2}$$
(8)

where  $F_y$  is the lateral tire force in the steady-state,  $C_1$  is the cornering stiffness and  $C_2$  is the auxiliary cornering stiffness.  $k_1$  and  $k_2$  are adjustment factors.  $F_{zi,n}$  is the nominal vertical load without including both the longitudinal and lateral acceleration effects. In this study,  $F_{zi}$  is calculated using the equation presented in [16].

2) *Dynamic Tire Model:* A first-order dynamic model can be applied as follows, to address the lagging behavior of the tire force generation:

$$\frac{\sigma}{v_x}\dot{F}_{yi} + F_{yi} = \bar{F}_{yi}, i = 1, 2, 3, 4.$$
(9)

Here,  $\sigma$  is the relaxation length, which consists of two tire parameters. It can be expressed as follows:

$$\sigma = \frac{C_{\alpha}}{K_L} \tag{10}$$

where  $C_{\alpha}$  is the same as  $C_1$  in (8) and  $K_L$  is the lateral stiffness.

## **III. PRELIMINARY ESTIMATOR**

#### A. Vehicle Sideslip Angle Estimation Based on Bicycle Model

The vehicle sideslip angle value must be obtained to integrate the tire-road relation model in vehicle dynamics. Previous studies of tire force estimation assumed that the sideslip angle could be estimated by adding a vehicle velocity sensor in the heading direction [11], [15] or measured by adding a lateral velocity sensor [16]. However, to apply the developed estimator in the real-time chassis controller of a production vehicle, the sideslip angle should be estimated using only in-vehicle sensors. To deal with the aforementioned problem, a bicyclemodel-based sideslip angle observer, that demonstrates robust performance toward sensor noise was applied independently as a measurement of the vehicle dynamic model for the tire force estimation.

Based on the bicycle model given in Fig. 2, the following state-space form is obtained:

$$\dot{x} = Ax + Bu, \quad y = Cx + Du \tag{11}$$



Fig. 3. Variation of clutch static friction coefficient depending on engagement force and temperature.

where

$$\begin{aligned} x &= \begin{bmatrix} \beta \\ \gamma \end{bmatrix}, y = \begin{bmatrix} \gamma \\ a_y \end{bmatrix}, u = \delta_f \\ A &= \begin{bmatrix} -\frac{2(C_f + C_r)}{mv_x} & \frac{-2(C_f l_f - C_r l_r)}{mv_x^2} - 1 \\ \frac{-2(C_f l_f - C_r l_r)}{I_z} & \frac{-2(C_f l_f^2 + C_r l_r^2)}{I_z v_x} \end{bmatrix}, B = \begin{bmatrix} \frac{2C_f}{mv_x} \\ \frac{2C_f l_f}{I_z} \end{bmatrix} \\ C &= \begin{bmatrix} 0 & 1 \\ \frac{-2(C_f + C_r)}{m} & \frac{-2(C_f l_f - C_r l_r)}{mv_x} \end{bmatrix}, D = \begin{bmatrix} 0 \\ \frac{2C_f}{m} \end{bmatrix} \end{aligned}$$

Here,  $C_f$  and  $C_r$  denote the front and rear cornering stiffness, respectively.  $\delta_f$  is the front wheel steering angle. Then, the bicycle-model-based observer is designed as follows:

$$\dot{\hat{x}} = A\hat{x} + Bu + K(y - \hat{y}).$$
 (12)

By using the pole-placement method, i.e., allowing the characteristic equation of observation error dynamics to have the multiple root  $p_o$ , the observer gain matrix K is determined as follows [22]:

$$K = \begin{bmatrix} \frac{I_z(C_f l_f - C_r l_r)}{2C_f C_r(l_f + l_r)} p_o^2 - 1 & \frac{1}{v_x} \\ 2p_o & \frac{m(C_f l_f^2 + C_r l_r^2)}{I_z(C_f l_f - C_r l_r)} \end{bmatrix}$$
(13)

where  $p_o$  is a positive constant. The elements of matrix K that have variable denominators are set to switch zero to avoid an undefined condition.

## B. Transfer Case Clutch Friction Coefficient Adaptation

Compared with a dry clutch, the wet clutch friction coefficient changes significantly depending on the operating conditions, i.e., temperature and engagement force. Fig. 3 shows the variation in the clutch static friction coefficient of a commercial transfer case; these data were obtained from an SAE J1646 test [20]. However, the exact value of the clutch friction coefficient cannot be identified in the ECU because a temperature sensor is not built into the transfer case. Therefore, real-time estimation of the clutch friction coefficient is necessary to improve the estimation performance. Among the several approaches to estimating this model parameter in real time, the adaptation method was adopted in this paper because it guarantees asymptotical stability when certain conditions are met. If the transfer case clutch is not fully engaged, the front shaft wheel dynamic equation is described as follows:

$$I_w(\dot{\omega}_1 + \dot{\omega}_2) = (\mu_{c,n} + \xi_\alpha) r_c F_c i_f$$
  
-  $R_e (F_{x1} + F_{x2}) - R_e R_r (F_{z1} + F_{z2})$   
(14)

where  $\mu_{c,n}$  is the nominal friction coefficient of the transfer case wet clutch,  $i_f$  is the front final reduction gear ratio,  $r_c$  is the effective radius of the clutch pad,  $F_c$  is the clutch engagement force, and  $\xi_{\alpha}$  is the uncertainty of the clutch friction coefficient which is compensated through adaptation.

From the intuition that the longitudinal acceleration force is determined by the ratio of the front to rear vertical load, (14) can be edited as follows:

$$r_{c}F_{c}i_{f}\xi_{\alpha} = I_{w}(\dot{\omega}_{1} + \dot{\omega}_{2}) - \mu_{c,n}r_{c}F_{c}i_{f}$$
$$+ R_{e}ma_{x}\frac{(F_{z1} + F_{z2})}{F_{z}} + R_{e}R_{r}(F_{z1} + F_{z2}) = z. \quad (15)$$

Using the gradient method [23], the adaptation form to estimate parameter uncertainty is designed as follows:

$$\dot{\hat{\xi}}_{\alpha} = \frac{\gamma_{\alpha}(z - \phi \hat{\xi}_{\alpha})}{1 + n_{\alpha} \phi^2} \phi$$
(16)

where  $\gamma_{\alpha}$  is the adaptation gain,  $n_{\alpha}$  is the normalizing gain, and  $\phi$  is the regressor defined as follows:

$$\phi = r_c F_c i_f. \tag{17}$$

#### **IV. ESTIMATOR DESIGN**

In this section, the estimator of the vehicle states and the tire force is proposed and validated experimentally. When a vehicle is on a homogenous surface while the transfer case is in a lock-up state, the torque is distributed depending only on the ratio of vertical load. However, when the transfer case is in a slipping state, maximum allowable torque is upper-bounded by transmission output torque. The IMM filter presented in this section deals with these situations. Under a situation that is governed by several dynamic models, the performance of the estimator using an IMM filter can be improved compared with that using a single model (SM). Applying the model probability which is obtained based on a stochastic in filtering process as a weighting factor, the IMM filter integrates the estimation results of each model. Also, the IMM filter can contribute to improving estimation performance in various road conditions, driving scenarios, and varying clutch engagement conditions because it can decrease the instability caused by the immediate switchover between the estimator's dynamic models. The nonlinear state-space equation of each model and the output equation can be expressed as follows:

$$\dot{\mathbf{x}}(t) = f^{i} \left( \mathbf{x}(t), \mathbf{u}(t) \right) + \mathbf{w}(t)$$
$$\mathbf{y}(t) = h \left( \mathbf{x}(t), \mathbf{u}(t) \right) + \mathbf{n}(t).$$
(18)

The state vector  $\mathbf{x}(t)$  consists of the longitudinal velocity, the lateral velocity, the yaw rate, the four wheel angular velocities,

1

and the tire forces:

$$\mathbf{x}(t) = \begin{bmatrix} v_x, v_y, \gamma, \boldsymbol{\omega}_{[1 \times 4]}, \mathbf{F}_{[1 \times 8]} \end{bmatrix}^T$$
$$= \begin{bmatrix} x_1, x_2, \dots, x_{15} \end{bmatrix}^T.$$
(19)

Here,  $\omega_{[1\times4]}$  and  $\mathbf{F}_{[1\times8]}$  are the wheel angular velocity vector and the tire force vector, respectively:

$$\boldsymbol{\omega}_{[1\times 4]} = [\omega_1, \omega_2, \omega_3, \omega_4] \tag{20}$$

$$\mathbf{F}_{[1\times 8]} = [F_{x1}, F_{x2}, F_{x3}, F_{x4}, F_{y1}, F_{y2}, F_{y3}, F_{y4}].$$
(21)

The process and measurement noise vectors,  $\mathbf{w}(t)$  and  $\mathbf{n}(t)$ , are assumed to be zero mean, white, and uncorrelated.

The input vector  $\mathbf{u}(t)$  consists of the front wheel steering angle and the transfer case engagement force, i.e.,

$$\mathbf{u}(t) = [\delta_f, F_c]^T = [u_1, u_2]^T$$
. (22)

The measurements are:

$$\mathbf{y}(t) = \begin{bmatrix} v_x, v_y, a_x, a_y, \gamma, \boldsymbol{\omega}_{[1 \times 4]} \end{bmatrix}^T.$$
 (23)

Front wheel angular velocities were used as the measured values of  $v_x$ , and the sideslip angle obtained from the bicycle-modelbased observer (see Section III-A) was used as measured value of  $v_y$ . The particular function  $f^i$  of each model and observation function h can be expressed as follows:

$$\begin{cases} f_1^1 = \frac{1}{m} \{ (x_8 + x_9) \cos(u_1) - (x_{12} + x_{13}) \sin(u_1) \\ + x_{10} + x_{11} - \frac{1}{2} \rho_{air} C_d A x_1^2 \} + x_2 x_3 \\ f_2^1 = \frac{1}{m} \{ (x_8 + x_9) \sin(u_1) + (x_{12} + x_{13}) \cos(u_1) \\ + x_{14} + x_{15} \} - x_1 x_3 \\ f_3^1 = \frac{1}{I_z} [l_f \{ (x_8 + x_9) \sin(u_1) + (x_{12} + x_{13}) \cos(u_1) \} \\ + t_w \{ (-x_8 + x_9) \cos(u_1) + (x_{12} - x_{13}) \sin(u_1) \\ - x_{10} + x_{11} \} - l_r (x_{14} + x_{15}) ] \\ f_4^1 = \frac{1}{I_w} \left\{ \left( \frac{l_r}{L} - \frac{a_x h_{cg}}{gL} \right) \frac{T_i i_f}{2} - T_{b1} - R_e (x_8 - R_r F_{z1}) \right\} \\ f_5^1 = \frac{1}{I_w} \left\{ \left( \frac{l_f}{L} - \frac{a_x h_{cg}}{gL} \right) \frac{T_i i_f}{2} - T_{b2} - R_e (x_9 - R_r F_{z2}) \right\} \\ f_6^1 = \frac{1}{I_w} \left\{ \left( \frac{l_f}{L} + \frac{a_x h_{cg}}{gL} \right) \frac{T_i i_r}{2} - T_{b3} - R_e (x_{10} - R_r F_{z3}) \right\} \\ f_7^1 = \frac{1}{I_w} \left\{ \left( \frac{l_f}{L} + \frac{a_x h_{cg}}{gL} \right) \frac{T_i i_r}{2} - T_{b4} - R_e (x_{11} - R_r F_{z4}) \right\} \\ f_8^1 = 0, f_9^1 = 0, f_{10}^1 = 0, f_{11}^1 = 0 \\ f_{12}^1 = \frac{x_1}{\sigma} \left( -x_{12} + \bar{F}_{y1} \right), f_{13}^1 = \frac{x_1}{\sigma} \left( -x_{13} + \bar{F}_{y2} \right) \\ f_{14}^1 = \frac{x_1}{\sigma} \left( -x_{14} + \bar{F}_{y3} \right), f_{15}^1 = \frac{x_1}{\sigma} \left( -x_{15} + \bar{F}_{y4} \right) \end{cases}$$

$$(24)$$

$$\begin{aligned} f_1^2 &= \frac{1}{m} \left\{ (x_8 + x_9) \cos(u_1) - (x_{12} + x_{13}) \sin(u_1) \\ &+ x_{10} + x_{11} - \frac{1}{2} \rho_{air} C_d A x_1^2 \right\} + x_2 x_3 \\ f_2^2 &= \frac{1}{m} \left\{ (x_8 + x_9) \sin(u_1) + (x_{12} + x_{13}) \cos(u_1) \\ &+ x_{14} + x_{15} \right\} - x_1 x_3 \\ f_3^1 &= \frac{1}{I_z} \left[ l_f \left\{ (x_8 + x_9) \sin(u_1) + (x_{12} + x_{13}) \cos(u_1) \right\} \\ &+ t_w \left\{ (-x_8 + x_9) \cos(u_1) + (x_{12} - x_{13}) \sin(u_1) \\ &- x_{10} + x_{11} \right\} - l_r \left( x_{14} + x_{15} \right) \right] \\ f_4^2 &= \frac{1}{I_w} \left( \mu_c r_c u_2 \frac{i_f}{2} - T_{b1} - R_e x_8 - R_e R_r F_{z1} \right) \\ f_5^2 &= \frac{1}{I_w} \left( \mu_c r_c u_2 \frac{i_f}{2} - T_{b2} - R_e x_9 - R_e R_r F_{z2} \right) \\ f_6^2 &= \frac{1}{I_w} \left\{ (T_t - \mu_c r_c u_2) \frac{i_r}{2} - T_{b3} - R_e x_{10} - R_e R_r F_{z3} \right\} \\ f_7^2 &= \frac{1}{I_w} \left\{ (T_t - \mu_c r_c u_2) \frac{i_r}{2} - T_{b4} - R_e x_{11} - R_e R_r F_{z4} \right\} \\ f_8^1 &= 0, \ f_9^1 &= 0, \ f_{10}^1 &= 0, \ f_{11}^1 &= 0 \\ f_{12}^1 &= \frac{x_1}{\sigma} \left( -x_{12} + \bar{F}_{y1} \right), \ f_{13}^1 &= \frac{x_1}{\sigma} \left( -x_{13} + \bar{F}_{y2} \right) \\ f_{14}^1 &= \frac{x_1}{\sigma} \left( -x_{14} + \bar{F}_{y3} \right), \ f_{15}^1 &= \frac{x_1}{\sigma} \left( -x_{15} + \bar{F}_{y4} \right) \end{aligned}$$
(25)

Superscript 1 implies the lock-up model and 2, the slipping model. The front wheel steering angles are assumed to be equal  $(\delta_1 = \delta_2 = \delta_f)$ . The IMM filter is divided into four parts: interacting, filtering, model probability updating and estimation fusion [24]–[26], which are respectively discussed in the following subsections:

## A. Interacting

With the values of the associated state vector, probability, and covariance of each model from the previous step, the IMM filter computes the mixed values for the filter. First, the mixed probability is obtained as follows:

$$\bar{p}_k^i = \sum_{j=1}^2 H_{ij} \hat{p}_{k-1}^j, i = 1, 2$$
 (27)

where  $H_{ij}$  is the probability transition matrix indicating the probability that the vehicle dynamics model will transition from model *j* to *i*. The index number 1 indicates the lock-up model and 2 the slipping model. Considering the amount of clutch engagement force and relative slip mathematically, the equation of the probability transition matrix can be expressed as:

$$\begin{cases}
H_{11} = \max\left(0.91 + 0.09r_l - \max\left(\frac{d\omega - 2}{2}, 0\right), 0\right) \\
H_{12} = 0.01 + 0.05r_l \\
H_{21} = \min\left(0.09 - 0.09r_l + \max\left(\frac{d\omega - 2}{2}, 0\right), 1\right) \\
H_{22} = 0.99 - 0.05r_l
\end{cases}$$
(28)

where  $r_l$  is the lock-up ratio and  $d\omega$  is the relative angular velocity between the front and rear shafts. These are defined as:

$$r_l = \min\left[1, \frac{\mu_c r_c F_c}{\left(\frac{l_r}{L} - \frac{a_x h_{cg}}{gL}\right) T_t}\right]$$
(29)

$$d\omega = \frac{(\omega_3 + \omega_4)}{2i_r} - \frac{(\omega_1 + \omega_2)}{2i_f}$$
(30)

where L and  $h_{cg}$  are wheelbase length and height from the ground to the vehicle's CG, respectively.  $a_x$  and g are the longitudinal and gravitational acceleration, respectively. With the value of the mixed probability, the initial mixed state vector and the covariance of each model are:

$$\bar{\mathbf{x}}_{k-1}^{i} = \sum_{j=1}^{2} H_{ij} \hat{p}_{k-1}^{j} \hat{\mathbf{x}}_{k-1}^{j} / \bar{p}_{k}^{i}$$
(31)  
$$\bar{P}_{k-1}^{i} = \sum_{j=1}^{2} H_{ij} \hat{p}_{k-1}^{j} [\hat{P}_{k-1}^{j} + (\hat{\mathbf{x}}_{k-1}^{j} - \bar{\mathbf{x}}_{k-1}^{i})$$
$$\times (\hat{\mathbf{x}}_{k-1}^{j} - \bar{\mathbf{x}}_{k-1}^{i})^{T}] / \bar{p}_{k}^{i}.$$
(32)

#### B. Filtering

With the state vector and the covariance of each model resulting from interacting stage, Bayesian filtering of each model is performed. Among several methods, the discrete-time EKF algorithm, which is applicable to nonlinear systems with discontinuous measurements, was adopted in this study [27].

1) Process Update: Based on the process models of lock-up and slipping state, *a priori* estimates of the state and covariance of the current step can be obtained. The process update procedures are as follows:

$$\hat{\mathbf{x}}_{k-}^{i} = f_{k}^{i} \left( \bar{\mathbf{x}}_{k-1}^{i}, \mathbf{u}_{k} \right)$$
(33)

$$\hat{P}_{k-}^{i} = F_{k}^{i} \bar{P}_{k-1}^{i} \left(F_{k}^{i}\right)^{T} + Q$$
(34)

where  $f_k^i$  is the discretized result from each continuous process model, which can be obtained using the Euler approximation method.  $F_k^i$  is the Jacobian matrix of  $f_k^i$  with respect to the state vector, **x**. 2) Measurement Update: Applying the measurement  $y_k$  in a feedback correction, *a posteriori* estimates of the state and the covariance of the current step can be obtained. The measurement update procedures are as follows:

$$K_{k}^{i} = \hat{P}_{k-}^{i} H^{T} \left( H \hat{P}_{k-}^{i} H^{T} + R \right)^{-1}$$
(35)

$$\hat{\mathbf{x}}_{k+}^{i} = \hat{\mathbf{x}}_{k-}^{i} + K_{k}^{i} \left( \mathbf{y}_{k} - H \hat{\mathbf{x}}_{k-}^{i} \right)$$
(36)

$$\hat{P}_{k+}^{i} = \left(I - K_{k}^{i}H\right)\hat{P}_{k-}^{i}$$
(37)

where H is the Jacobian matrix of h with respect to the state vector,  $\mathbf{x}$ .

Here, even a subtle change in the noise covariance matrices of Q and R greatly affects the estimation performance, especially in the transient state. For properly determining Q and R, it should be noted that a relatively small value of covariance is used in reliable measurements, while high covariance is used in uncertain dynamics, e.g., random walk model. In this study, the values of the covariance matrices were selected as:

$$Q = \text{diag}[1, 1, 0.01, 1, 1, 1, 1, \mathbf{Q}_{F[1 \times 8]}]$$
(38)  

$$R = \text{diag}[0.001, 0.1, 0.0001, 0.01, 0.01, \cdots$$
  

$$0.00028, 0.00028, 0.00028, 0.00028]$$
(39)

where  $\mathbf{Q}_{F[1\times 8]}$  are the covariance vectors of tire forces. To improve the estimation performance in various experimental cases, a variable value of  $\mathbf{Q}_{F[1\times 8]}$  was used, depending on the conditions as follows:

$$\mathbf{Q}_{F[1\times8]} = \begin{cases} \operatorname{diag}[1000_{[1\times4]}, 20000_{[1\times4]}], \ if \ \delta_f > 0.005 \\ \operatorname{diag}[20000_{[1\times8]}], \ if \ \delta_f > 0.005 \ \& \ T_t > 100 \\ \operatorname{diag}[20000_{[1\times4]}, 1000_{[1\times4]}], \ else. \end{cases}$$

$$(40)$$

## C. Model Probability Updating

Based on the *a posteriori* estimates of each model resulting from the filtering stage, a probability update is then conducted. Assuming that the process noise and the measurement noise of each model are Gaussian, the likelihood function is as follows:

$$\Gamma_{k}^{i} = \frac{\exp\left(-\frac{1}{2}\left(v_{k}^{i}\right)^{T}\left(S_{k}^{i}\right)^{-1}v_{k}^{i}\right)}{\sqrt{\left|2\pi S_{k}^{i}\right|}}$$
(41)

m

where  $v_k^i$  and  $S_k^i$  are the measurement innovation and covariance terms respectively, defined as:

$$v_k^i = \mathbf{y}_k - H\hat{\mathbf{x}}_{k-}^i \tag{42}$$

$$S_k^i = H\hat{P}_{k-}^i H^T + R.$$
 (43)

Then, the probability of model i at time k is obtained from:

$$\hat{p}_{k}^{i} = \frac{\bar{p}_{k}^{i} \Gamma_{k}^{i}}{\sum_{i=1}^{2} \bar{p}_{k}^{i} \Gamma_{k}^{i}}.$$
(44)

### D. Estimation Fusion

By applying the associated probability of each model as weights, the final mixed state vector and the covariance can



Fig. 4. Block diagram of real-time AWD vehicle states estimation algorithm.

be obtained as follows:

$$\hat{\mathbf{x}}_k = \sum_{i=1}^2 \hat{p}_k^i \hat{\mathbf{x}}_k^i \tag{45}$$

$$\hat{P}_{k} = \sum_{i=1}^{2} \hat{p}_{k}^{i} \left[ \hat{P}_{k}^{i} + \left( \hat{\mathbf{x}}_{k}^{i} - \hat{\mathbf{x}}_{k} \right) \left( \hat{\mathbf{x}}_{k}^{i} - \hat{\mathbf{x}}_{k} \right)^{T} \right].$$
(46)

# V. EXPERIMENTAL SETUP

To validate the performance of the suggested estimator, an experiment was conducted using a full-size AWD vehicle. Fig. 1(b) shows the type of drivetrain used in this experiment. Fig. 4 is a block diagram of the proposed estimator. The elements of the CAN signals, i.e.,  $a_x, a_y, \gamma, T_e, \delta_f$ , and  $\omega_{[4\times 1]}$ , which are used in the estimator were obtained by bypassing the vehicle's onboard diagnostics (OBD)-II channel. The engagement force of the transfer case was obtained from the pressure sensor, which is planned to be embedded in the vehicle CAN signal for commercial transfer case product. Wheel force transducer (WFT) equipment was installed at each wheel to measure the tire force. Also, telemetric front and rear shaft torque sensors and a vehicle motion sensor (RT3000) were attached and used only for data validation. A Dewetron device was used as the data acquisition system. All data were sampled at 500 Hz. To verify the estimator under various driving scenarios, the vehicle experiment was performed at the proving ground of the Korea Automotive Technology Institute. Here, the AWD operation was governed by the on-board logic of the commercial transfer case. Considering the ECU-mountability of the proposed algorithm, the estimator sampling time was set at 5 ms. Table I shows the principal parameter values and vehicle specifications that were used in the estimator.

#### VI. EXPERIMENTAL RESULTS

## A. Transfer Case Clutch Friction Coefficient Adaptation

The first test of the experiment involved longitudinal acceleration without steering, and the effect of adaptation was verified before the tire force estimator was employed. The vehicle started to accelerate from the stopped state with constant throttle. The adaptation algorithm was coded to run by checking the

 TABLE I

 Vehicle Specifications and Tire Modeling Parameters

Parameter	Value	Parameter	Value
l <sub>f</sub>	1.471 m	$l_r$	1.539 m
$h_{cq}$	0.61m	$2t_w$	1.63 m
m	2050 kg	$I_z$	$4200  \text{kg} \cdot \text{m}^2$
$I_w$	$0.9  \text{kg} \cdot \text{m}^2$	$R_e$	0.368 m
$\tilde{R_r}$	0.015	i f	3.916
$i_r$	3.909	$\vec{C}_1$	60913 N/rad
$\dot{C}_2$	-81456 N/rad <sup>2</sup>	$K_L$	108422 N/m
$\tilde{k_1}$	0.85	$k_2^L$	0.15



Fig. 5. Experimental setup and test environment of AWD vehicle. (a) Exterior of the test vehicle. (b) Data acquisition devices and sensors. (c) Vehicle test in a proving ground. (d) Beam roller climbing test.

persistent excitation condition [23] in real-time, which guaranteed convergence of the parameter to the real value. Fig. 6 shows the validation results of the adaptation. Fig. 6(a) shows the adaptation results on dry asphalt. With several clutch uncertainties of different magnitudes, the results approached the real values. Fig. 6(b) shows the adaptation results on wet asphalt, which also approached the real values. The adaptation time was longer on dry asphalt than in the wet asphalt case. Regardless of the



Fig. 6. Experimental result of clutch friction coefficient adaptation. (a) Dry asphalt. (b) Wet asphalt.

 TABLE II

 DESIGN PARAMETERS OF ADAPTATION ALGORITHM

Parameter	Value	Parameter	Value
$\gamma_{lpha}$	$2\cdot 10^{-8}$	$n_{lpha}$	$10^{-10}$

road condition, the adaptation algorithm notably improved the accuracy of the clutch friction coefficient. However, the adaptation value did not perfectly converge to the real value because (15) can be intermittently invalid in experimental situations. Table II shows the design parameter values used in the adaptation algorithm.

## B. Acceleration in a Straight Line

Fig. 7 shows the experimental results of tire force estimation on dry asphalt. The test scenario was the same described in Section VI-A. Here, the proposed estimator was validated with the clutch friction coefficient adaptation algorithm embedded. Fig. 7(g) shows the vehicle inputs. Here, the longitudinal tire forces between the front and rear wheels were distributed in proportion to the front and rear vertical loads. Thus, the clutch was close to the lock-up state, and the probability of the lock-up model was almost 1. Then, the probability of lock-up model decreased slightly as the transmission output torque and the clutch engagement force dropped; the probability of lock-up model finally approached 0 as the clutch force diminished (see Fig. 7(e)). Using the clutch friction adaptation algorithm, the proposed estimator provided results highly correlated with the measurements (see red dashed lines of Fig. 7(a)-(d)). However, without the clutch friction coefficient adaptation algorithm, the lock-up ratio,  $r_l$ , could not be calculated accurately because the uncertainty of the clutch friction coefficient was not adequately compensated for. This led to an improper model probability of  $\hat{p}_k^i$  (see Fig. 7(f)). However, the error in the estimated longitudinal tire forces without adaptation was not much different than that with adaptation because the engagement force was strong enough to cause the probability of the lock-up model to persist near 1, regardless of the application of the adapted parameter value (see black dotted lines of Fig. 7(a)–(d)). Fig. 8 shows the experimental results on wet asphalt (see Fig. 8(a)-(d)). Fig. 8(g) shows the vehicle inputs. Similar to the dry asphalt case, the probability of the lock-up model remained at almost 1 during the initial engagement, then gradually decreased to 0. However, the difference in the model probability with and without



Fig. 7. Acceleration on dry asphalt. (a) Front left longitudinal tire force. (b) Front right longitudinal tire force. (c) Rear left longitudinal tire force. (d) Rear right longitudinal tire force. (e) Model probability (w/ adaptation). (f) Model probability (w/o adaptation). (g) Vehicle inputs.

adaptation was greater than for dry asphalt case (see Fig. 8(e) and (f)). Without the adaptation, the proposed estimator not only could not properly calculate the probability of each model, but also yielded inaccurate estimation results, especially when the AWD was in the slipping state. Therefore, the estimator with adaptation showed better results than that without adaptation (see t = 12.5 s to 20 s of Fig. 8(a)–(d)). In the wet asphalt case, there was a range in which the model probability changed rapidly from lock-up to the slipping state during the engagement of the clutch (see t = 12.5 s to 14 s of Fig. 8(e)), which was due to the increase in the relative slip in those ranges (see Fig. 8(h)).

# C. Slalom Test

The second part of the experiment was the slalom test. While the vehicle velocity was maintained at 80 km/h, the transient steering maneuver was initiated (see Fig. 9(f)). Using the simple tire model, the proposed estimator showed satisfactory performance during the lateral transient response (see Fig. 9(a)–(d)) but minor discrepancies, i.e., underestimation on one side and overestimation on the other side, stood out at each peak point of the left and right wheels. Since there was almost no intervention





Fig. 8. Acceleration on wet asphalt. (a) Front left longitudinal tire force. (b) Front right longitudinal tire force. (c) Rear left longitudinal tire force. (d) Rear right longitudinal tire force. (e) Model probability (w/ adaptation). (f) Model probability (w/o adaptation). (g) Vehicle inputs. (h) Angular velocity difference between front and rear shaft.

Fig. 9. Slalom test @ 80 km/h. (a) Front left lateral tire force. (b) Front right lateral tire force. (c) Rear left lateral tire force. (d) Rear right lateral tire force. (e) Model probability. (f) Vehicle inputs. (g) Angular velocity difference between front and rear shaft.

of the transfer case clutch before the steering, the probability of the slipping model persisted close to 1. However, with the engagement of the transfer case clutch from t = 3 s, the probability of the slipping model declined intermittently (see Fig. 9(e)). The fluctuating response of the model probability is due to both variation of engagement force and the relative slip in the clutch during the slalom test (see Fig. 9(f) and (g)). However, the model probability is not dominant here because the dynamic equation for lateral tire force is exactly the same in both models.

# D. Slow Ramp Steer With Acceleration

The third test of the experiment was the slow ramp steer with acceleration. This scenario was selected for the validation of the proposed estimator in a case of combined tire force generation. Here, the vehicle started to accelerate with an initial velocity of 40 km/h and steadily increased the steering angle at the same time (see Fig. 10(j)). The probability of the lock-up model was near to 0 before steering and slightly increased due to the weak

engagement of the clutch (see Fig. 10(i)). The estimated values of the lateral tire force at each wheel were slightly different from the measurements at the initial transient state and at steady state. The estimated values of longitudinal tire forces had less error than those for the lateral tire force. Considering the combined effect of the tire force generation and vertical load transfer, the proposed estimator showed highly accurate results for both longitudinal and lateral tire forces (see Fig. 10(a)–(h)).

# E. Beam Roller Climbing

The fourth part of the experiment was beam roller climbing, in which the role of the AWD is greatly anticipated. The vehicle started to accelerate while the rear wheels were driven on a roller and the front wheels were hung on a beam (see Fig. 5(d)). Before the vehicle climbed over the beam, the AWD clutch was in the lock-up state (see Fig. 11(f)). Although the AWD clutch was fully engaged and controlled, the occurrence of relative slip was inevitable (see t = 3.3 s of Fig. 11(g)) and most of the transmission output torque was transferred to the front wheels because the vehicle was being driven on nonhomogeneous





Fig. 11. Climbing beam roller. (a) Front left longitudinal tire force. (b) Front right longitudinal tire force. (c) Rear left longitudinal tire force. (d) Rear right longitudinal tire force. (e) Model probability. (f) Vehicle inputs. (g) Angular velocity difference between front and rear shaft.

Fig. 10. Slow ramp steer with acceleration. (a) Front left longitudinal tire force. (b) Front right longitudinal tire force. (c) Rear left longitudinal tire force. (d) Rear right longitudinal tire force. (e) Front left lateral tire force. (f) Front right lateral tire force. (g) Rear left lateral tire force. (h) Rear right lateral tire force. (i) Model probability. (j) Vehicle inputs.

surface. Thus, the lock-up model, which was defined to distribute the transmission output torque according to the proportion of vertical load, was not valid here. Nevertheless, the proposed IMM filter detected this situation. Then, the model probability of the slipping states started to increase (see Fig. 11(e)), eventually reaching 1. As for the longitudinal tire forces at the front wheels, the IMM filter provided estimation results showing that most of the traction force was generated in the front wheels, which matched well with the measurement data (see Fig. 11(a) and (b)). As for the longitudinal tire forces at rear wheels, the IMM filter showed estimation results of almost no traction force because the rear wheels were being driven on a slippery road; this result also agreed well with the measurement data (see Fig. 11(c) and (d)). However, at the early stage of traction force generation i.e., t = 3 s, the IMM filter could not estimate the traction force well because the probability change occurred at approximately t = 4.3 s. The black dotted line (see Fig. 11(a)–(d)) shows the tire force estimation results for the SM filter with the lock-up constraint; this SM could not be used to estimate the tire force accurately because it detected the current situation as the lock-up state.

# VII. CONCLUSION

This paper proposed a method for the estimation individual tire force, especially for the AWD vehicle. To improve the estimation performance, the AWD clutch friction coefficient was adapted by using a wheel dynamics model. Also, a vehicle sideslip angle observer that is robust to sensor noise was adopted independently in consideration of the ECU-mountability. Then, the IMM filter algorithm, which simultaneously considers both the clutch slipping and lock-up states based on the stochastic process in the filtering was adopted by integrating the information on the preliminary estimator and the vehicle CAN data. The experimental results validated that the proposed method performed better than that did the SM Kalman filter, under various driving conditions. Specifically, the adaptation algorithm improved the accuracy of individual tire force, especially on wet asphalt. Also, the proposed estimator was able to support the accurate estimation of tire force on a road with a nonhomogeneous friction coefficient without causing estimator instability or severe chattering by the switching of model dynamics. Having the strength to operate using only in-vehicle sensors, the proposed estimation algorithm is expected to be applied to a real-time AWD system controller, and ultimately to the vehicle dynamics control area for the logic development of an integrated chassis controller.

#### ACKNOWLEDGMENT

The authors would like to thank C. W. Moon and C. H. Jeong of the Korea Automotive Technology Institute and J. H. Jeon of Hyundai Dymos Company for their efforts in carrying out experiments. This paper has its origin in the project titled "Development of 1,300 Nm Electro-Hydraulic AWD Modularization System and Control Technology to Improve Driving Force and Safety," without which this paper could not have been completed.

#### REFERENCES

- S. B. Choi, "Antilock brake system with a continuous wheel slip control to maximize the braking performance and ride quality," *IEEE Trans. Control Syst. Technol.*, vol. 16, no. 5, pp. 996–1003, Sep. 2008.
- [2] M. Choi and S. B. Choi, "Model predictive control for vehicle yaw stability with practical concerns," *IEEE Trans. Veh. Technol.*, vol. 63, no. 8, pp. 3539–3548, Oct. 2014.
- [3] J. Yoon, W. Cho, B. Koo, and K. Yi, "Unified chassis control for rollover prevention and lateral stability," *IEEE Trans. Veh. Technol.*, vol. 58, no. 2, pp. 596–609, Feb. 2009.
- [4] H. Kim, S. Lee, and J. K. Hedrick, "Active yaw control for handling performance improvement by using traction force," *Int. J. Automot. Technol.*, vol. 16, no. 3, pp. 457–464, Jun. 2015.
  [5] Y. F. Lian, Y. Zhao, L. L. Hu, and Y. T. Tian, "Cornering stiffness and
- [5] Y. F. Lian, Y. Zhao, L. L. Hu, and Y. T. Tian, "Cornering stiffness and sideslip angle estimation based on simplified lateral dynamic models for four-in-wheel-motor-driven electric vehicles with lateral tire force information," *Int. J. Automot. Technol.*, vol. 16, no. 4, pp. 669–683, Aug. 2015.
- [6] B. Li, A. Goodarzi, A. Khajepour, S. Chen, and B. Litkouhi, "An optimal torque distribution control strategy for four-independent wheel drive electric vehicles," *Veh. Syst. Dyn.*, vol. 53, no. 8, pp. 1172–1189, Aug. 2015.
- [7] H. Hamann, J. K. Hedrick, S. Rhode, and F. Gauterin, "Tire force estimation for a passenger vehicle with the unscented Kalman filter," in *Proc. IEEE Intell. Veh. Symp.*, Dearborn, MI, USA, Jun. 8–11, 2014. pp. 814–819.
- [8] K. Nam, S. Oh, H. Fujimoto, and Y. Hori, "Estimation of sideslip and roll angles of electric vehicles using lateral tire force sensors through RLS and Kalman filter approaches," *IEEE Trans. Ind. Electron.*, vol. 60, no. 3, pp. 988–1000, Mar. 2013.
- [9] K. Nam, H. Fujimoto, and Y. Hori, "Lateral stability control of in-wheelmotor-driven electric vehicles based on sideslip angle estimation using lateral tire force sensors," *IEEE Trans. Veh. Technol.*, vol. 61, no. 5, pp. 1972–1985, Jun. 2012.
- [10] L. R. Ray, "Nonlinear state and tire force estimation for advanced vehicle control," *IEEE Trans. Control Syst. Technol.*, vol. 3, no. 1, pp. 117–124, Mar. 1995.

- [11] G. Baffet, A. Charara, and G. Dherbomez, "An observer of tire-road forces and friction for active security vehicle systems," *IEEE/ASME Trans. Mechatronics*, vol. 12, no 6, pp. 651–661, Dec. 2007.
- [12] W. Cho, J. Yoon, S. Yim, B. Koo, and K. Yi, "Estimation of tire forces for application to vehicle stability control," *IEEE Trans. Veh. Technol.*, vol. 59, no. 2, pp. 638–649, Feb. 2010.
- [13] M. Choi, J. J. Oh, and S. B. Choi, "Linearized recursive least square methods for real-time identification of tire-road friction coefficient," *IEEE Trans. Veh. Technol.*, vol. 62, no. 7, pp. 2906–2918, Sep. 2013.
- [14] G. Baffet, A. Charara, and D. Lechner, "Estimation of vehicle sideslip, tire force and wheel cornering stiffness," *Control Eng. Practice*, vol. 17, no. 11, pp. 1255–1264, Nov. 2009.
- [15] M. Doumiati, A. C. Victorino, A. Charara, and D. Lechner, "Onboard real-time estimation of vehicle lateral tire-road forces and sideslip angle," *IEEE/ASME Trans. Mechatronics*, vol. 16, no. 4, pp. 601–614, Aug. 2011.
- [16] J. Kim, "Identification of lateral tyre force dynamics using an extended Kalman filter from experimental road test data," *Control Eng. Practice*, vol. 17, no. 3, pp. 357–367, Mar. 2009.
- [17] A. Rezaeian *et al.*, "Novel tire force estimation strategy for real-time implementation on vehicle applications," *IEEE Trans. Veh. Technol.*, vol. 64, no. 6, pp. 2231–2241, Jun. 2015.
- [18] J. J. Oh, S. B. Choi, and J. Kim, "Driveline modeling and estimation of individual clutch torque during gear shifts for dual clutch transmission," *Mechatronics*, vol. 24, no. 5, pp. 449–463, Aug. 2014.
- [19] M. Bachinger, M. Stolz, and M. Horn, "A novel drivetrain modelling approach for real-time simulation," *Mechatronics*, vol. 32, pp. 67–78, Dec. 2015.
- [20] Glossary of Terms Lubricated Friction Systems, USA: SAE International, Mar. 2012.
- [21] R. Rajamani, Vehicle Dynamics and Control, 2nd ed. New York, NY, USA: Springer-Verlag, 2006.
- [22] J. J. Oh and S. B. Choi, "Vehicle velocity observer design using 6-D IMU and multiple-observer approach," *IEEE. Trans. Intell. Transp. Syst.*, vol. 13, no. 4, pp. 1865–1879, Dec. 2012.
- [23] P. A. Ioannou and J. Sun, *Robust Adaptive Control*. 2nd ed., Upper Saddle River, NJ, USA: Prentice-Hall, 1996.
- [24] K. Jo, K. Chu, and M. Sunwoo, "Interacting multiple model filter-based sensor fusion of gps with in-vehicle sensors for real-time vehicle positioning," *IEEE Trans. Intell. Transp. Syst.*, vol. 13, no. 1, pp. 329–343, Mar. 2012.
- [25] H. A. P. Blom and Y. Bar-Shalom, "The interacting multiple model algorithm for systems with Markovian switching coefficients," *IEEE Trans. Autom. Control*, vol. 33, no. 8, pp. 780–783, Aug. 1988.
- [26] H. Tsunashima, M. Murakami, and J. Miyataa, "Vehicle and road state estimation using interacting multiple model approach," *Veh. Syst. Dyn.*, vol. 44, no. 1, pp. 750–758, 2006.
- [27] D. Simon, Optimal State Estimation. Hoboken, NJ, USA: Wiley, 2006.



Hojin Jung received the B.S. and M.S. degrees in mechanical engineering from the Korea Advanced Institute of Science and Technology (KAIST), Daejeon, Korea, in 2012 and 2014, respectively, where he is currently working toward the Ph.D. degree in mechanical engineering. His research interests include vehicle dynamics and control, vehicle system modeling and simulation, and control theory.



Seibum B. Choi (M'09) received the B.S. degree in mechanical engineering from Seoul National University, Seoul, South Korea, in 1985, the M.S. degree in mechanical engineering from the Korea Advanced Institute of Science and Technology (KAIST), Daejeon, South Korea, in 1987, and the Ph.D. degree in controls from the University of California, Berkeley, CA, USA, in 1993. From 1993 to 1997, he was with the Institute of Transportation Studies, University of California, working on the development of automated vehicle control systems. Through 2006, he was with

TRW, Livonia, MI, USA, where he worked on the development of advanced vehicle control systems. Since 2006, he has been with the Faculty of the Mechanical Engineering Department, KAIST. His research interests include fuel-saving technology, vehicle dynamics and control, and active safety systems. He is a Member of the American Society of Mechanical Engineers, the Society of Automotive Engineers, and the Korean Society of Automotive Engineers.