

# Model-based prediction of steering response

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## Abstract

To predict steering response, this paper proposes a string tire model based on relaxation length and a vehicle model designed by using effective cornering force. The cornering force, lateral static, and distortion characteristics of a tire are significant factors that describe steering response. Nevertheless, it is difficult to define the contribution of each factor when a tire's cornering motion is evaluated through subjective assessment. A new concept of tire model, which adopts distortion stiffness ( $K_D$ ), is defined. A modified vehicle dynamics model is adapted to represent tire force lag and converted cornering stiffness ( $C_\alpha$ ) based on effective cornering force. This integration scheme is proposed to provide a more accurate modeling for steering response performance. The integration model is confirmed by dynamic response and validated through comparison with a steering response assessment by a subjective test engineer. The test results demonstrate the ability of this approach to predict steering response.

## Keywords

Tire model, steering response, steering agility, handling objective test, handling subjective test, tire dynamics

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## Introduction

Contrary to common opinion, a strong coupling of a firm tire and chassis can generate fast steering response. To verify steering response, a huge variety of design parameters and testing methods have been developed. However, most of the existing solutions are too expensive to be practical due to cost of real test facilities and simulation software.

To predict steering response, this work integrated tire and vehicle modeling aims to achieve the following objectives:

- to predict steering response with a simplified model;
- to utilize existing test machines and parameters globally;
- to verify the validity in an easy way.

The three common parameters that have been widely used by numerous tire manufacturing companies for indoor tests are cornering stiffness ( $C_\alpha$ ), lateral stiffness ( $K_L$ ), and distortion stiffness ( $K_D$ ). These three test parameters comprise the fundamental approach to determining the lateral motion of a tire. In addition, each parameter provides a specific physical meaning of the tire's lateral motion.<sup>1,2</sup> However, when these three

tire test procedures are conducted on a moving vehicle, observing and analyzing the performance results from each test procedure is very difficult because the exact initiation point of each test is unknown.

Based on previous research, the typical dynamic model and string model with a relaxation length is an appropriate tire model that can closely characterize a tire's lateral dynamic motion.<sup>1,2</sup>

In this study, another equation is adopted that can be implemented in the tire model: an equation for distortion stiffness ( $K_D$ ), which can closely replicate the characteristics of steering response. With the implementation of the proposed additional equation, the three unknown property values can be mathematically determined, and this provides greater accuracy than the

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previous model, which uses the controversial estimated value.

In the vehicle modeling part of this study, a single-track vehicle model was designed, which mainly modified the tire force lag and cornering stiffness ( $C_\alpha$ ). From the tire modeling, a dynamic tire model is adopted in the equation of motion to represent tire force lag. To represent the steering compliance and roll behavior effect, the cornering stiffness ( $C_\alpha$ ) is converted to the effective cornering force measured by a suspension parameter measuring machine. Therefore, it is expected that this integration can be proposed to predict the steering response.

The remainder of this paper is organized as follows. In the next section, the standard string tire model is reviewed, and its three tire test properties are described in detail. The third section describes the newly proposed tire model, which implements distortion stiffness ( $K_D$ ). The fourth section presents the single-track vehicle model with effective cornering force. In the fifth section, the dynamic response based on the proposed tire and vehicle models are evaluated and compared with actual tire performance subjective assessment results to validate the feasibility of the proposed model. The last section concludes this paper.

## Review of previous research

A tire's lateral stiffness is closely related to its steering response. When the lateral stiffness and cornering stiffness are high, the steering response becomes quicker. On the other hand, when the performance results are analyzed directly by subjective evaluation, each parameter (lateral, distortion static stiffness, and cornering stiffness) cannot be correlated clearly. To overcome this issue, appropriate tire modeling based on an evaluation method is proposed in this paper. In previous tire models, the magic formula tire model and the LuGre tire model were initially introduced, improved, and corrected.<sup>1,3,4</sup> These are good representatives to obtain the cornering stiffness from experimental friction curves. However, these models are not suitable to predict steering response with steady-sated values of lateral,

distortion static stiffness, and cornering stiffness. In this work, only the cornering stiffnesses are obtained by the magic formula equation and an experimental indoor test. The details will be discussed in the next section.

The string tire model and relaxation length were selected to describe the performance appropriately.<sup>1</sup>

Guenther (1990) and Heydinger (1991) derived the typical dynamic model used for lateral tire force lag. This model expresses the relaxation length ( $L$ ) as<sup>5</sup>

$$L = \frac{C_\alpha}{K_L} \quad (1)$$

where  $C_\alpha$  and  $K_L$  represent the cornering stiffness at a steady state and lateral stiffness, respectively. Each variable can be further expressed as<sup>5</sup>

$$C_\alpha = \left. \frac{\partial F_y}{\partial \alpha} \right|_{\alpha=0} \quad (2)$$

$$K_L = \left. \frac{\partial F_y}{\partial y} \right|_{y=0} \quad (3)$$

where  $\alpha$  and  $y$  are the slip angle and the lateral displacement of tire elements from the wheel plane center, respectively.<sup>5</sup>

From equations (1) to (3), the relaxation time constant with respect to tire force lag ( $\tau_{lag}$ ) is related to the relaxation length as follows:

$$\tau_{lag} = \frac{C_\alpha}{K_L V_x} \quad (4)$$

The string tire model was first studied by Von Schelippe in 1941.<sup>2</sup> This specific tire model assumes that a tire is a set of endless strings, where each string provides a large number of tread elements. Figure 1 depicts the string tire model proposed by Von Schelippe and Pacejka.<sup>1</sup>

In Figure 1,  $a$ ,  $x$ ,  $v$ ,  $\alpha$ ,  $\sigma$ ,  $X$ , and  $Y$  are the contact patch length, the longitudinal displacement and deflection of the string, the slip angle, the relaxation length, as well as the wheel plane axis and vertical axis of the wheel plane axis on the horizontal plane, respectively.<sup>1,2</sup>

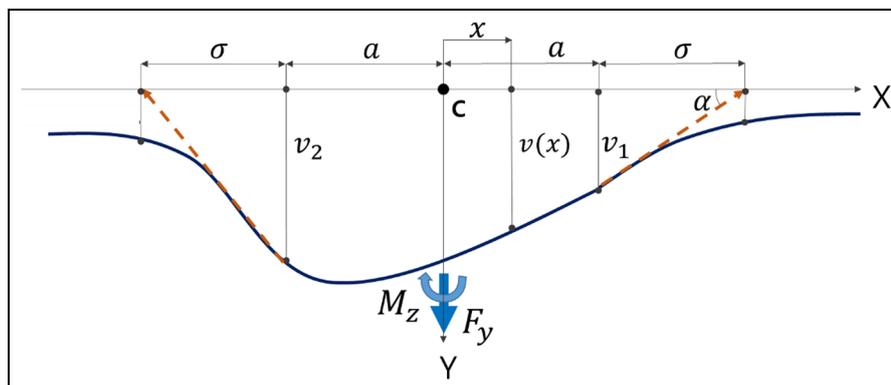


Figure 1. Top view of string tire model.

### Proposed tire model

The three test parameters that have been commonly used by most tire companies for indoor testing are cornering stiffness ( $C_\alpha$ ), lateral stiffness ( $K_L$ ), and distortion stiffness ( $K_D$ ). These three test procedures comprise the fundamental approach to determine the lateral motion of a tire. In addition, each parameter provides a specific physical meaning of the tire's lateral motion from the previous study. These three factors measured through indoor testing reveal the characteristics of a tire at a steady state, and they are represented by the following equations from a previous study:<sup>1</sup>

$$K_L \triangleq \left. \frac{\partial F_y}{\partial y} \right|_{y=0} = 2C_c(\sigma + a) \quad (5)$$

$$C_\alpha \triangleq \left. \frac{\partial F_y}{\partial \alpha} \right|_{\alpha=0} = 2C_c(\sigma + a)^2 \quad (6)$$

$$K_D \triangleq \left. \frac{\partial M_z}{\partial \theta} \right|_{\theta=0} = 2C_c a \left\{ \sigma(\sigma + a) + \frac{1}{3}a^2 \right\} \quad (7)$$

Here,  $C_c$  was used to denote the carcass tension in the previous study. However, it is the total modulus of elasticity, between lateral force and deflection of the string, represented by not only carcass tension but also the sum of nylon, steel belt, and other reinforcement materials. Combining equations (5) and (6), the relaxation length can be derived as

$$\therefore \sigma = \frac{C_\alpha}{K_L} - a \quad (8)$$

The formula in equation (8) newly expresses the relaxation length which is comparable to that defined as  $L$  in equation (1). The next subsection will discuss the accuracy of the newly expressed relaxation length  $\sigma$ .

#### Transient state values of string model characteristic

To analyze the response of tire performance, transient state values should be discussed as well as steady state values. In previous studies, three different string models

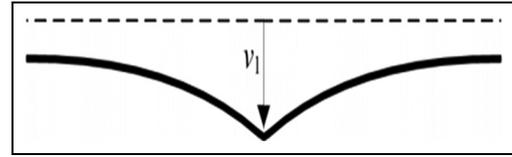


Figure 2. Single point string model.

have been developed to describe string shape within a contact patch:<sup>1,2</sup> the single-point string model, the straight tangent model, and the exact string model, as shown Figures 2 and 3.

The exact string model proposed by Segel in 1966 assumes that the velocity of each tread element with respect to the road is zero. The single-point string model assumes that the tire contact point is at one particular point. The straight tangent model is a simple linear approximation of the exact model. The contact patch line is defined as a linear extension of the deflection  $v_1$  at the patch leading edge.<sup>1</sup>

The time constant of the single-point string model can be defined as<sup>1</sup>

$$\tau_{single} = \frac{(\sigma + a)}{V_x} \quad (9)$$

The transfer function of the straight tangent model is also a first-order function, and the relaxation time constant  $\tau_{straight}$  is defined as<sup>1</sup>

$$\tau_{straight} = \frac{\sigma}{V_x} \quad (10)$$

The cornering stiffness of the exact model at a transient state cannot be described in a simple first-order form; therefore, the time constant is not defined explicitly.<sup>1,2,5</sup> However, comparing the three transfer functions, the exact model describes a real tire best, and the others are approximations of the exact model.

The accuracies of the approximated models are investigated in terms of frequency response. Figure 4 can be expressed by the values of time constants in

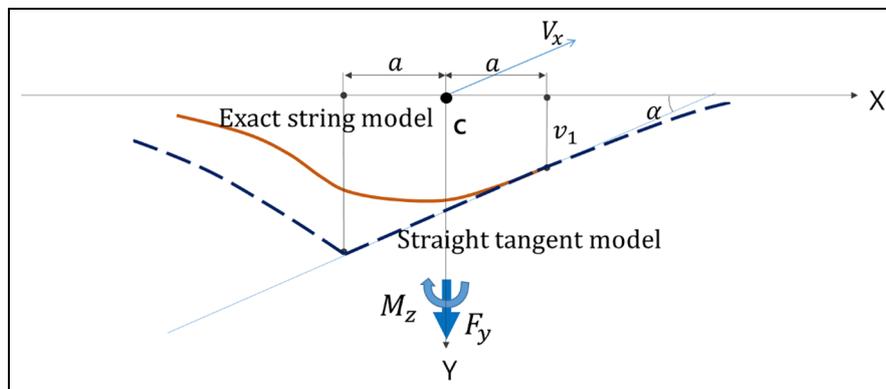


Figure 3. Exact and straight tangent model.

equations (9) and (10) and it shows that the straight tangent approximation is very close to the exact model.

Therefore, it is reasonable to substitute the exact model with the transfer function of straight tangent approximation, and the time constant of the exact model can be represented by the time constant of the straight tangent model  $\tau_{straight}$ . Therefore, it makes sense to calculate the relaxation length  $\sigma$  with proper expressions.

**Model identification**

To calculate relaxation length  $\sigma$  using equation (8), the value of contact patch length ‘a’ is required. It is measured by indoor testing. Practically, determining the length of the contact patch is not a simple task. Even if each sample has the same pattern design, the sample-to-sample variations make the task quite delicate since the end of the contact patch has a highly nonlinear shape.

Therefore, in this work, a new method of describing relaxation length  $\sigma$  using distortion stiffness  $K_D$  was investigated instead of the controversial contact patch length ‘a’.

A new relaxation length ( $\sigma$ ), which can be derived by combining equations (5), (6), and (7), is proposed:

$$\therefore \sigma = \left( \frac{C_\alpha^3}{K_L^3} - \frac{3C_\alpha K_D}{K_L^2} \right)^{\frac{1}{3}} \tag{11}$$

where  $C_\alpha$  is the cornering stiffness,  $K_L$  is the lateral stiffness, and  $K_D$  is the distortion stiffness. This equation can closely replicate the characteristics of steering response by reflecting the effect of distortion static characteristics on the existing models. Furthermore, it can be formulated without the controversial factor ‘a’.

Also, this work investigated the connection between the proposed tire model and the subjective evaluation method. A good starting point is acquiring the steering performance from a subjective test based on a test engineer’s evaluation. The acquired steering feels from a subjective assessment based on the test engineer’s evaluation is divided into two categories: steering response and torque. When the test engineer feels the steering

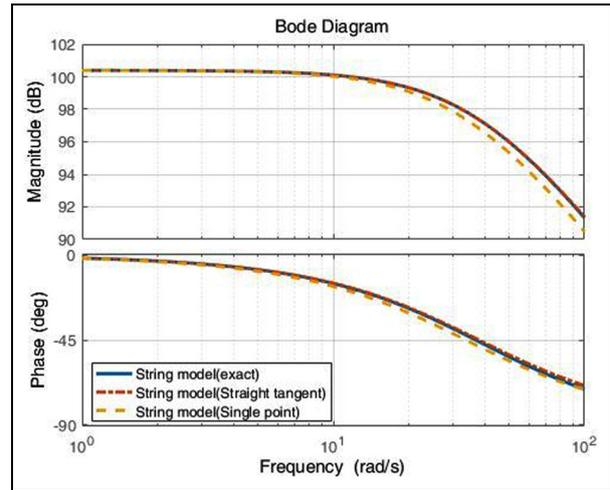


Figure 4. Bode plots of three models ( $V_x = 100$  km/h).

performance of the tire, the most important factor is the visible reaction of the yaw motion and torque feedback felt by hands. After subjective testing, engineers express the distortion motion of a tire as accurately as possible to the design engineers as well as the yaw response of a vehicle, and this can be expressed in numerous ways.<sup>1,6-9</sup> Therefore, it makes a sense to include the distortion static stiffness  $K_D$  to the relaxation length  $\sigma$ .

**Vehicle modeling**

The single-track model depicted in Figure 5 is used to describe the vehicle lateral dynamics. The three-degree-of-freedom (3DOF) linear model, which is a proper representation of lateral dynamics in the steering response assessment region, is employed. Roll and pitch motions are assumed to be not excited and are ignored.

**Single track model equation of motion**

The equations of motion for the model are given as

$$\dot{V}_y = \frac{2F_{yf}}{m} + \frac{2F_{yr}}{m} - V_r \tag{12}$$

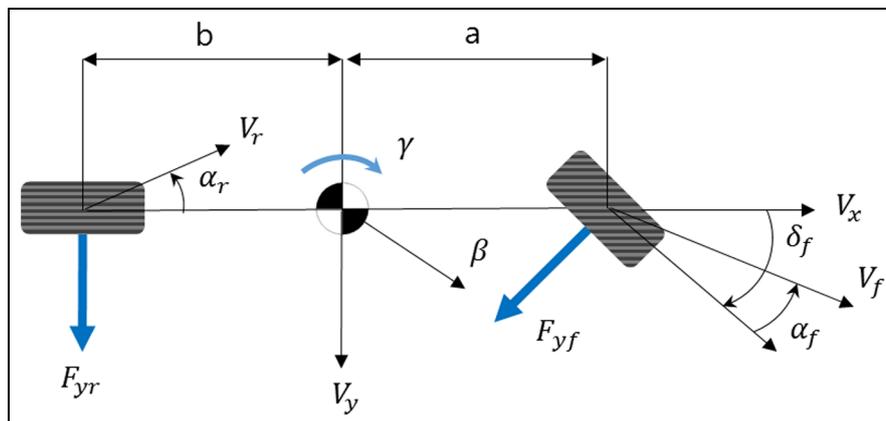


Figure 5. Single-track model for vehicle lateral dynamics.

$$\dot{r} = \frac{2aF_{yf}}{I_z} + \frac{2bF_{yr}}{I_z} \quad (13)$$

$$\dot{F}_{yf} = -\frac{F_{yf}}{\tau_f} + \frac{C_f}{\tau_f} \left( \delta_f - \frac{V_y}{V_x} - \frac{ar}{V_x} \right) \quad (14)$$

$$\dot{F}_{yr} = -\frac{F_{yr}}{\tau_r} + \frac{C_r}{\tau_r} \left( -\frac{V_y}{V_x} - \frac{br}{V_x} \right) \quad (15)$$

Here, the relaxation time constant ( $\tau_f$ ) from equations (10) and (11) is reflected in equations (14) and (15).

### State space realization of the linear time invariant (LTI) system

The system modeled using a set of ordinary differential equations can be represented in the following state equations:

$$\begin{Bmatrix} \dot{V}_y \\ \dot{r} \\ \dot{F}_{yf} \\ \dot{F}_{yr} \end{Bmatrix} = \begin{bmatrix} 0 & -V_x & \frac{2}{m} & \frac{2}{m} \\ 0 & 0 & \frac{2a}{I_z} & -\frac{2b}{I_z} \\ -\frac{C_f}{\tau_f V_x} & -\frac{aC_f}{\tau_f V_x} & -\frac{1}{\tau_f} & 0 \\ -\frac{C_r}{\tau_r V_x} & -\frac{bC_r}{\tau_r V_x} & 0 & -\frac{1}{\tau_r} \end{bmatrix} \begin{Bmatrix} V_y \\ r \\ F_{yf} \\ F_{yr} \end{Bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{C_f}{\tau_f R} \\ 0 \end{bmatrix} \delta_f \quad (16)$$

$$Y(S) = \begin{Bmatrix} r \\ a_y \end{Bmatrix} = \begin{Bmatrix} r \\ \dot{V}_y + V_x r \end{Bmatrix} \quad (17)$$

Here, the transfer function ( $Y(S)/\delta_f$ ) can be obtained through the steering response using the front steering angle input  $\delta_f$  for every vehicle dynamic response frequency, where the cornering stiffnesses  $C_f$  and  $C_r$  are changed to effective cornering stiffnesses, which are explained in detail in the next section.

### Effective of suspension compliance effect on cornering force

Since the single-track model does not consider the vertical load transfer, it cannot express the understeering tendency of a vehicle. Therefore, the suspension compliance effect is also implemented as depicted in Figure 6. Suspension compliance expresses the input force variation in the suspension components and bushings resulting from changes in the road surface received through the tire.

Suspension compliance provides the benefit of extracting the cornering stiffness variable output. The effective cornering stiffness affected by the steering compliance can be calculated as follows:<sup>1</sup>

$$\alpha^e = \alpha + \frac{\partial \alpha}{\partial F_y} F_y + \frac{\partial \alpha}{\partial M_z} M_z \quad (18)$$

$$F_{yi} = C_i \alpha^e_i \quad i = (1, 2) \quad (19)$$

$$M_{zi} = (C_i \alpha^e_i) n_i \quad (20)$$

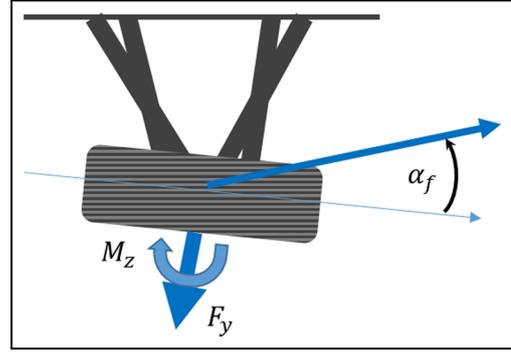


Figure 6. Steering due to wheel suspension compliance effect.

Table 1. Measured cornering stiffness and that converted to effective axle cornering stiffness.

Axle	Measured by Flat-Trac®	Effective cornering force
Front	1860 N/deg	1075 N/deg
Rear	1700 N/deg	1477 N/deg

Then, equations (19) and (20) are substituted into equation (18) as follows:<sup>1</sup>

$$\alpha^e_i = \alpha_i + \frac{\partial \alpha}{\partial F_y} C_i \alpha^e_i + \frac{\partial \alpha}{\partial M_z} (C_i \alpha^e_i) n_i \quad (21)$$

Next, equation (21) is substituted into equation (19) as follows:<sup>1</sup>

$$F_{yi} = C_i \alpha^e_i = \frac{C_i}{1 - \frac{\partial \alpha}{\partial F_y} C_i - \frac{\partial \alpha}{\partial M_z} C_i n_i} \alpha_i = C_i^* \alpha_i \quad (22)$$

where  $C_i$  is the cornering stiffness of each axle,  $n_i$  is the pneumatic trail of the tire, and  $C_i^*$  is the effective cornering stiffness. The cornering stiffness of each axle and the pneumatic trail of a tire were measured using an MTS Flat-Trac® test machine. Therefore, the effective cornering stiffness can be calculated by equation (22).

Equation (22) shows that the cornering stiffness of the front axle tends to be decreased, and that of the rear axle tends to be relatively increased. The results presented in Table 1 show how much force can be changed through the suspension compliance effect. The compliance effects on the vehicle dynamics using a full vehicle model is presented in Figure 7. These obtained results are quite similar to actual evaluation results. To obtain the compliance coefficients  $\partial \alpha / \partial F_y$  and  $\partial \alpha / \partial M_z$ , in equation (22), a compliance suspension parameter test machine was used for each test vehicle, as shown Figure 7.

A simulation was conducted using a B segment sedan with 195/55R16 size tires. While the tire model is adopted in the vehicle model, the relaxation time constant and cornering stiffness values are implemented as described in equations (14) and (15). Figure 8 shows the dynamic response of yaw and lateral acceleration



Figure 7. Suspension parameter test machine.<sup>11</sup>

obtained using the measured cornering stiffness values and the steady-state gain at a low frequency region. Some of these values do not fully correspond with the results obtained from the field objective test. The magnitude of yaw rate and lateral acceleration without compliance effect have similar characteristics that appear in the sports car class as shown in Table 2.

Therefore, it is crucial to use the effective cornering stiffness ( $C_{\alpha}^*$ ) for the proposed model. It demonstrates that the newly configured value corresponds more accurately with the results from the field objective test.

The detailed description of the objective test method and the results are beyond the scope of this paper. To improve the accuracy of the proposed tire model, the effective axle cornering stiffness is used for the target value, and the adjustment factors are applied to the cornering stiffness transfer function.

Table 2. Practical magnitude of yaw rate and lateral acceleration of each vehicle class.

	Normal range of SUV class	Normal range of Sports class
Magnitude of yaw rate @ 0.2 Hz (absolute value)	0.2–0.3	0.3–0.45

### Model validation

The validity of the proposed tire and vehicle model was verified by dynamic response tests and subjective assessment. There are two phases to identify the steering response. The initial phase is the front axle response time represented by the yaw rate. Afterwards, the second phase is the rear axle response expressed by the lateral acceleration. Therefore, the total response delay can be represented by the phase lag of lateral acceleration as shown Figure 8 and it has high correlation with subjective assessment according to field experiments.<sup>12,13</sup>

The transfer function of the vehicle model based on the relaxation time constant and effective cornering force effect was implemented for each test tire. The relaxation time constant ( $\tau_f, \tau_r$ ) in equation (10) is defined using equation (11). In the vehicle model, the dynamic response is represented by lateral acceleration, where input  $\delta_f$  is applied to observe the dynamic response.<sup>12,13</sup> Then it is evaluated at each frequency and the performance ranking is confirmed, as shown in Figures 3 and 4.

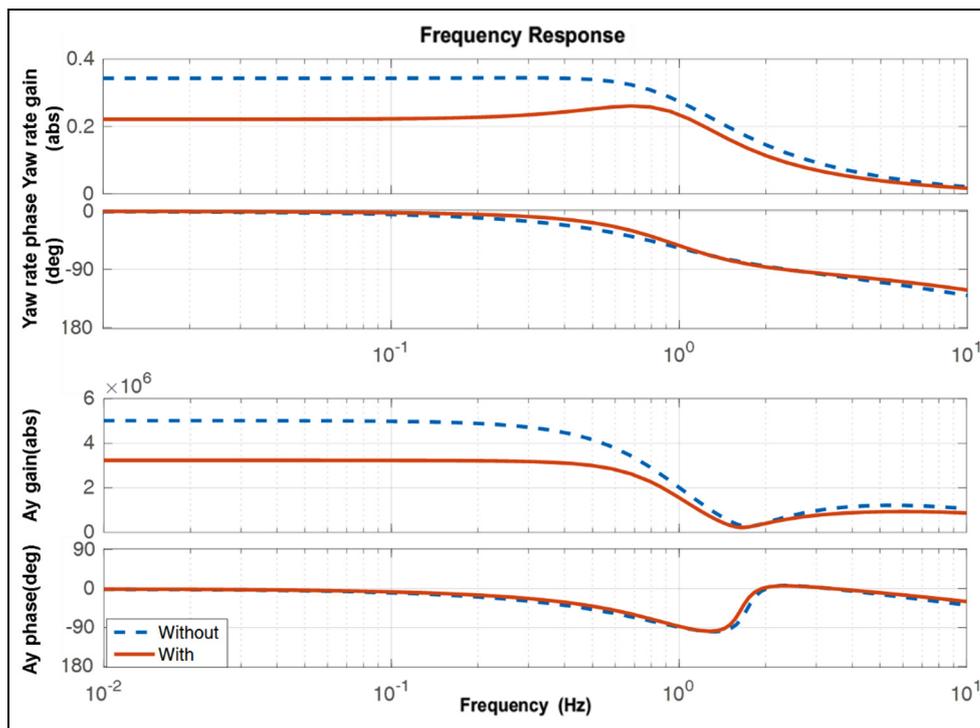


Figure 8. Dynamic response results of yaw and lateral acceleration by the effective cornering force.

**Table 3.** Tire indoor test results.

Sample	Case 1				Case 2				
	A	B	C	D	E	F	G	H	I
$K_L$ ( $\times 10^5 \text{ N/m}$ )	1.184	1.202	1.258	1.221	1.152	1.189	1.198	1.221	1.234
$C_\alpha$ ( $\times 10^5 \text{ N/rad}$ )	1.250	1.256	1.242	1.235	1.226	1.219	1.224	1.220	1.260
$K_D$ ( $\times 10^3 \text{ Nm/rad}$ )	4.080	4.570	4.130	4.414	3.812	4.865	4.695	4.767	4.767
Relaxation length (m)	0.588	0.576	0.549	0.563	0.593	0.569	0.568	0.555	0.565

**Table 4.** Tire subjective test results.

Sample	A	B	C	D	E	F	G	H	I
Subjective rating	6.500	6.625	6.750	6.625	6.500	6.750	7.000	7.250	7.500

Each case sample was tested on one day to reduce the effect of external noise, such as different test engineers as well as vehicle and environmental conditions.

A total of nine tire samples of two cases with the specifications in Table 3 were tested indoors under the same load condition (580 kgf) without being fitted to a vehicle. Those tires were made by the same manufacturer with a summer tire tread pattern design, and only minor design parameters were different. This was intended to evaluate performance differences in relation to very minor changes.

Tables 3 and 4 present the results of the indoor tire tests and subjective tests, respectively. In Table 3, the proposed relaxation lengths were calculated based on three parameters: lateral stiffness  $K_L$ , cornering stiffness  $C_\alpha$ , and distortion stiffness  $K_D$ , which were obtained through the average of five iterative tests. The cornering stiffness  $C_\alpha$  was measured using an MTS Flat-Trac® test machine. In Table 4, the subjective rating results are presented for the nine tire samples based on SAE evaluation method.<sup>12</sup>

In general, each subjective test engineer has a unique frequency of steering maneuvers when assessing steering response performance. To analyze the correlation between the dynamic response and subjective test results, it is necessary to investigate the major frequency input used for the subject since results are greatly affected by the frequency contents.<sup>6</sup>

To minimize this issue, the specific steering maneuver frequencies performed by each test engineer need to be determined. Figure 9 shows the spectrum of steering input and vehicle yaw rate response for a steering response test.

After each test engineer's major steering maneuver frequency is identified, more accurate analysis of the dynamic response of yaw rate and lateral acceleration can be achieved. In this work, the dynamic response was evaluated within the steering maneuver frequency near 1 Hz reflecting each test engineer's driving style.

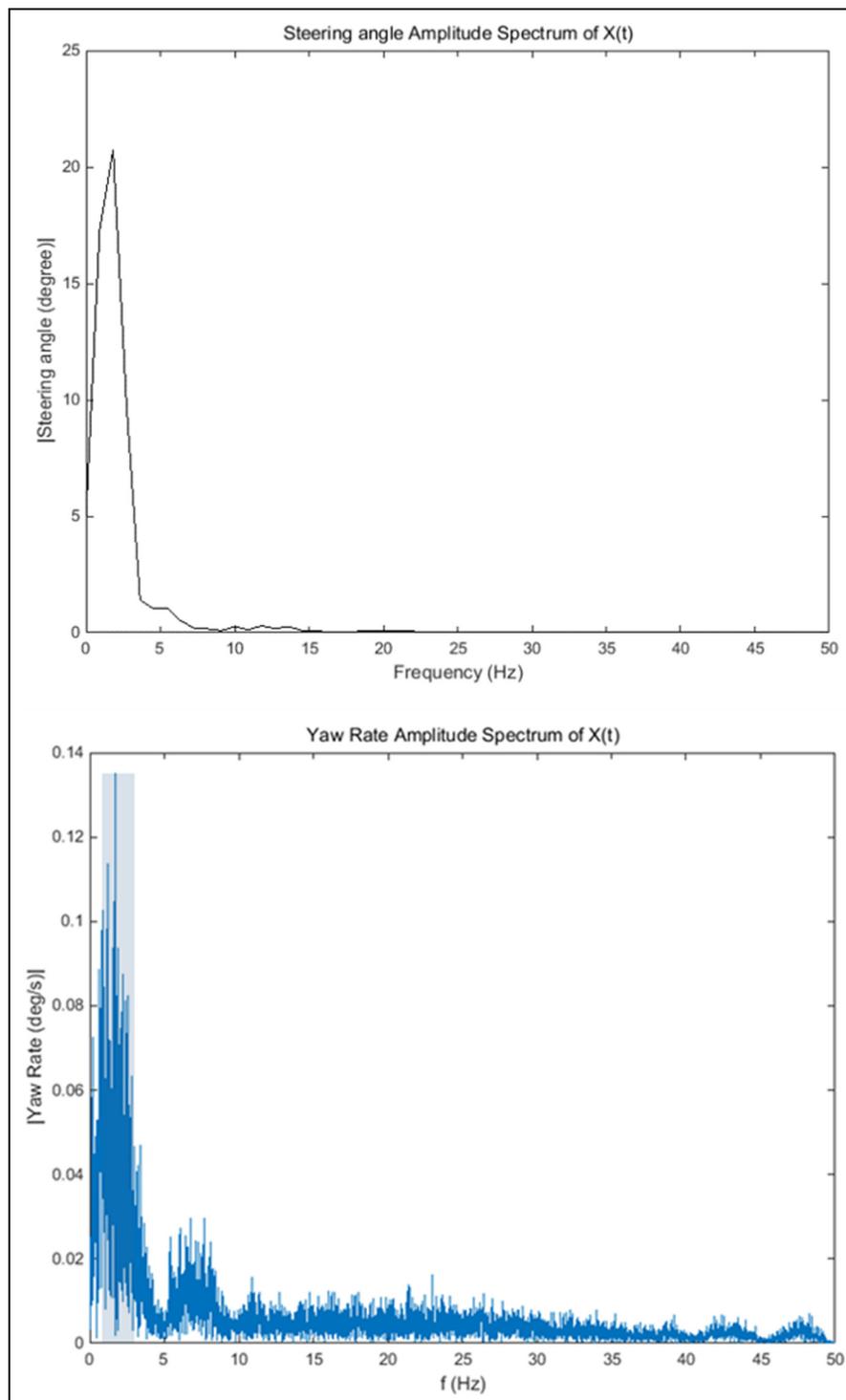
### Model prediction results

In the vehicle model, the dynamic response is represented by lateral acceleration where input  $\delta_f$  is applied to observe the dynamic response.<sup>5,10,14</sup> The lateral acceleration response of each sample A-I was evaluated in the frequency domain by using the parameters in Table 3 and equations (16) and (17). Also, the performance ranking is confirmed as shown in Figures 10. The variation of the dynamic response test was negligible. However, there are clear differences between subjective ratings in the scale of 0–1.<sup>12</sup>

The integration model was confirmed by dynamic responses and validated through comparison with a steering response assessment by the subjective test engineer. Figures 11 and 12 show the correlation obtained using a linear least square regression method (Draper and Smith, 1981). The results demonstrate that the proposed tire dynamic model, described in equations (10) and (11) correlates with the subjective rating results much better than the single-point model, expressed in equation (1), does. Using the new model,  $r^2$  values were improved from 0.84 and 0.86 to as much as 0.95. High  $r^2$  value means that the model reflects the actual movement.<sup>15</sup> Based on these results, it can be concluded that this integration model is suitable to predict tire steering responses.

### Conclusion

The concept of using the relaxation length to predict the steering response performance has been applied to the string tire model. Three common parameters, stiffness ( $C_\alpha$ ), lateral stiffness ( $K_L$ ), and the distortion stiffness ( $K_D$ ), are considered at the same time while relaxation length is defined to enhance the accuracy of the proposed tire steering response model, which can be used even before a vehicle is built. The modified



**Figure 9.** Steering maneuver and yaw rate spectrum in a subjective test.

single-track vehicle model is adapted to represent the relaxation time constant ( $\tau_f, \tau_r$ ) and converted cornering stiffness ( $C_\alpha$ ) based on effective cornering force. This integration is based on the fact that tires influence the lateral dynamics. The modified vehicle dynamics model was applied to nine tire samples with two different classes. This integration model yielded good estimation results correlating with the subjective assessment very well.

- The relaxation length in equation (11) is proposed.
- The modified single-track vehicle model is adapted.
- This integration model was applied to the nine tire samples.
- Good estimation results were obtained correlating with the subjective assessment.

The proposed model has the potential to significantly reduce the evaluation efforts associated with field

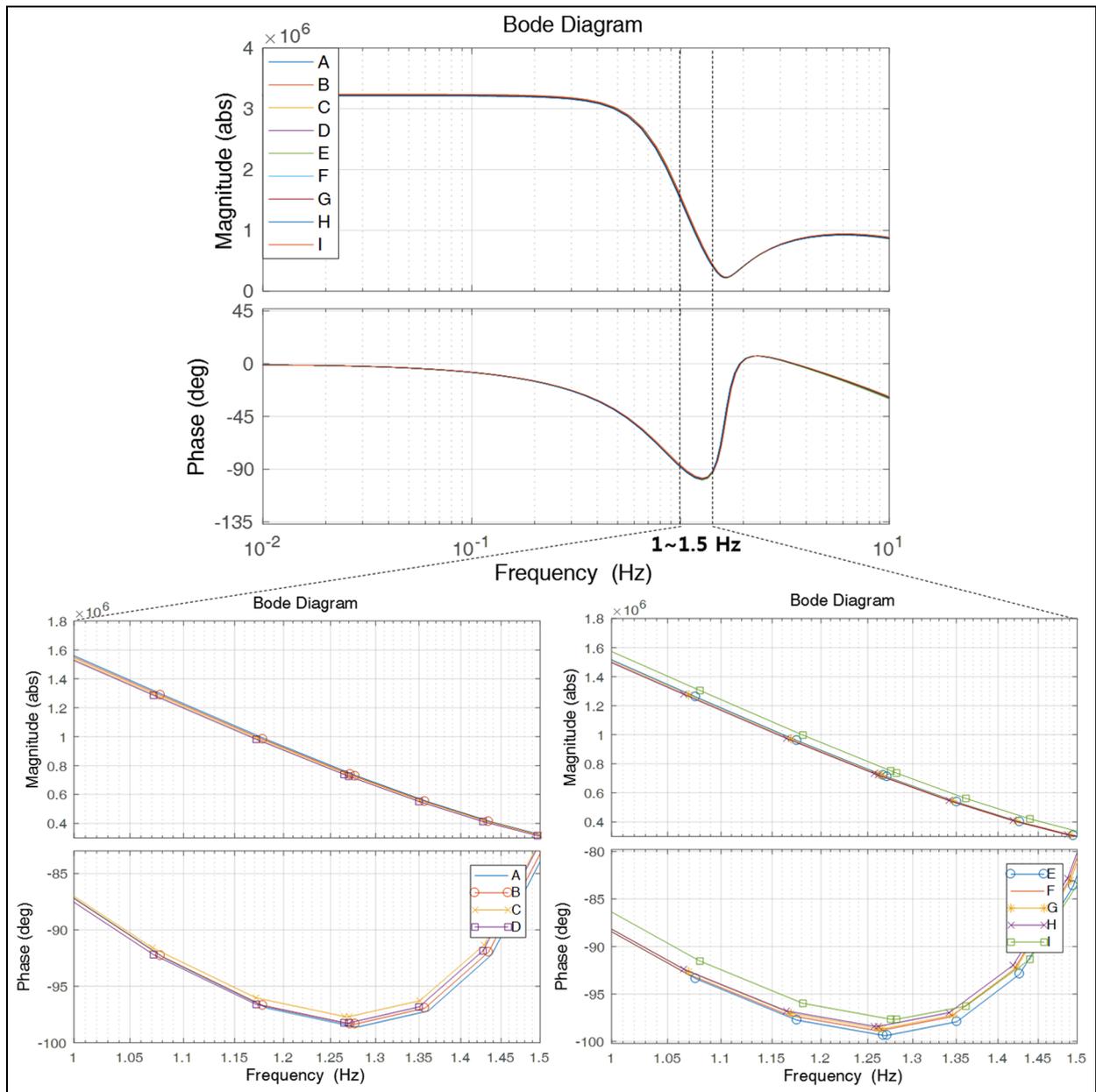


Figure 10. Dynamic response analysis of lateral acceleration at 1.2 Hz.

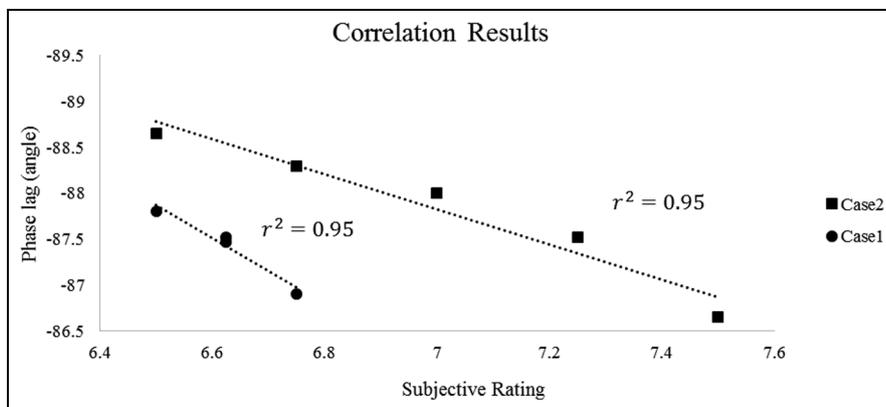
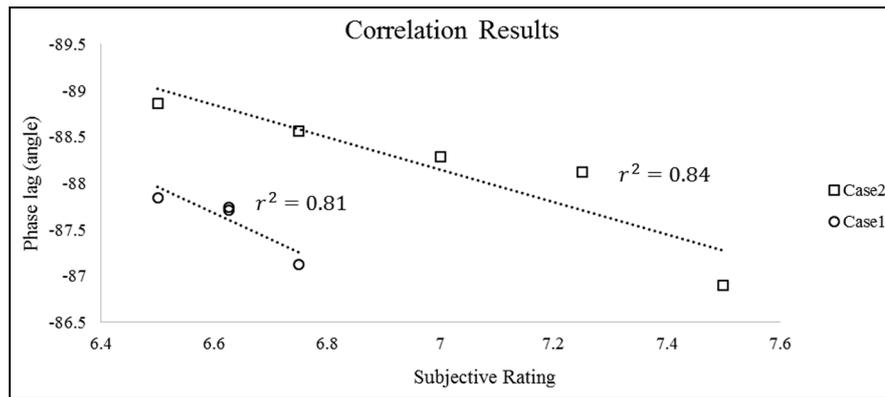


Figure 11. Correlation results of subjective rating and phase lag of lateral acceleration (with proposed straight tangent model).



**Figure 12.** Correlation results of subjective rating and phase lag of lateral acceleration (with typical time constant).

testing, and tire design engineers can conjugate the results of the tire and vehicle characteristics used in indoor tests. Therefore, the model-based prediction of steering response may play a significant role in replacing tire field tests on a vehicle with just indoor tire tests.

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#### References

1. Pacejka H *Tire and vehicle dynamics*. Amsterdam, Netherlands: Elsevier, 2012.
2. von Schilippe B and Dietrich R. *Das Flattern eines bepneuten Rades*. Bericht 140 der Lilienthal-Gesellschaft. NACA TM 1365, pp. 125–147, 1954.
3. Deur J, Asgari J and Hrovat D. A dynamic tire friction model for combined longitudinal and lateral motion. In: *2001 ASME international mechanical engineering congress and exposition*, New York, NY, 11–16 November 2001, pp. 1–8. New York: ASME.
4. Zhu JJ, Khajepour A and Esmailzadeh E. Total dynamic response of a PSS vehicle negotiating asymmetric road excitations. *Veh Syst Dyn* 2012; 50: 1835–1859.
5. Rajamani R. *Vehicle dynamics and control*. Berlin, Germany: Springer Science Business Media, 2011.
6. Gil Gómez GL, Nybacka M, Bakker E et al. Findings from subjective evaluations and driver ratings of vehicle dynamics: steering and handling. *Veh Syst Dyn* 2015; 53: 1416–1438.
7. Krüger HP, Neukum A and Schuller J. A workload approach to the evaluation of vehicle handling characteristics. SAE technical paper 0148-7191, 2000.
8. Winkler CB. Simplified analysis of the steady-state turning of complex vehicles. *Veh Syst Dyn* 1998; 29: 141–180.
9. Wolff K, Kraaijeveld R and Hoppermans J. Objective evaluation of subjective driving impressions. In: *2008 JSAE annual congress*, Yokohama, Japan, SAE paper 2008-08-0222, 2008.
10. Morris KA. *Introduction to feedback control*. San Diego: Academic Press Inc., 2000.
11. Gil G. Hybrid numerical integration scheme for highly oscillatory dynamical systems. Dissertation, University of Arizona, 2013.
12. SAE. *Subjective rating scale for vehicle handling*. SAE standard J 1441\_201609. SAE international, Warrendale, PA, USA, 2016.
13. Zschocke AK and Albers A. Links between subjective and objective evaluations regarding the steering character of automobiles. *Int J Automot Technol* 2008; 9: 473–481.
14. Heydinger GJ, Garrott WR and Chrstos JP. The importance of tire lag on simulated transient vehicle response. SAE technical paper 0148-7191, 1991.
15. Data S and Frigerio F. Objective evaluation of handling quality. *Proc IMechE Part D: J Automobile Eng* 2002; 216: 297–305.