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String tyre model for evaluating steering agility performance using tyre cornering force and lateral static characteristics

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ABSTRACT

The cornering force and lateral static characteristics of a tyre are fundamental factors that describe the steering feel for handling performance. However, it is difficult to justify the contribution of each factor when the tyre's cornering motion is evaluated through subjective assessment. Currently, the relaxation length of Pacejka's tyre model is close to describing these tyre motions. Therefore, this paper proposes a string tyre model based on the relaxation length in order to represent the steering performance. The proposed method provides a more accurate modelling of the steering agility performance. Therefore, it is possible to use this model to predict the steering response performance, and this is validated through comparison with a real relaxation length.

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KEYWORDS

Tyre model; steering response; steering agility; handling objective test; handling subjective test

Introduction

During the tyre development process, the tyre undergoes an indoor test, where only the tyre's performance is tested without being fitted to a vehicle. This test is undertaken in order to achieve the required steering performance. The three common test parameters that have been widely used in numerous tyre manufacturing companies for the indoor tests are cornering stiffness (C_α), lateral stiffness (K_L), and the distortion static characteristics (K_D). These three test procedures comprise the fundamental approach to determine the lateral motion of the tyre. In addition, each test provides a specific physical meaning of the tyre's lateral motion. When the three test parameters produce a high output value, it indicates that the steering response and effort become quicker and firmer, respectively.

However, when conducting the same three tyre test procedures on a moving vehicle, observing and analysing the performance results from the test procedures are very difficult because the initiation point of each test is unknown. Based on previous research, Pacejka string model with a relaxation parameter is the only tyre model that can closely characterise the three test parameters. However, there is very limited research regarding this tyre model, and there is potential for refining the model.

Through the contact patch length a, relaxation length σ, and carcass stiffness C_c parameter of the tyre, Pacejka tyre model can derive two equations: the lateral stiffness (K_L) and

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the cornering or lateral slip stiffness (C_α). The transfer function is derived from these two equations, and the dynamic response can be observed. Therefore, estimated values that closely resemble the actual parameters of the tyre are implemented in the model. Because the model is based on estimated arbitrary values, there are limitations to what the model can provide in actual vehicle testing environments.

In this paper, another equation is proposed that can be implemented in the tyre model: the equation for distortion static characteristics (K_D). This equation can closely replicate the characteristics of steering performance. With the implementation of the proposed third equation, the three components can be mathematically determined, and this provides more accuracy compared with the model using estimated arbitrary values.

The paper is organised as follows, in the second section, the standard model for Pacejka's string tyre model is reviewed, and its three components of the tyre test are described in detail. The third section describes the newly proposed tyre model, which implements the distortion static characteristics. In the fourth section, the tyre performance based on the proposed tyre model is evaluated and compared with a real relaxation length in order to validate the feasibility of the proposed tyre model. The last section concludes the paper.

Tyre modelling

A tyre's lateral stiffness is closely related to its steering agility. When the lateral stiffness and cornering stiffness are large, the steering agility becomes quicker. However, when the performance results are analysed directly from the test procedures and subjective evaluation, each parameter (lateral, distortion static stiffness, and cornering stiffness) cannot be correlated clearly. In order to overcome this issue, appropriate tyre modelling based on an evaluation method is proposed in this paper. In the current tyre models, the string tyre model and relaxation length are selected to describe the performance appropriately [1–3].

Review of previous research

Heydinger et al. [4] derived typical dynamic model that used lateral tyre force and lagged lateral force. This model expresses the relaxation length (L), which is represented as follows:

$$L = \frac{C_\alpha}{K_\mathrm{L}}, \tag{1}$$

where C_α (steady-state condition) and K_L represents the cornering stiffness and lateral stiffness, respectively. Each variable can be further expressed as follows:

$$C_\alpha = \left.\frac{\partial F_y}{\partial \alpha}\right|_{\alpha=0}, \tag{2}$$

$$K_\mathrm{L} = \left.\frac{\partial F_y}{\partial y}\right|_{y=0}, \tag{3}$$

where α and y are slip angle and the displacement of tyre elements from wheel centre, respectively.

From Equations (1) to (3), the typical first order dynamic model for lateral force based on the typical model can be derived and is represented as follows:

$$\tau_{\text{lag}} \dot{F}_{y\_\text{lag}} + F_{y\_\text{lag}} = F_y, \tag{4}$$

where F_y is the lateral force at a steady state and F_{y_lag} is the true lagged lateral force. In Equation (4), the relaxation time constant τ_{lag} is related to the relaxation length as follows:

$$\tau_{\text{lag}} = \frac{C_\alpha}{K_{\text{L}} V_x}. \tag{5}$$

Here, from Equations (1)–(5), the steering agility performance which is influenced by lagged lateral force can be proved by the relaxation length and typical first order dynamic model [1–3,5,6].

The string type tyre model was first studied by Von Schelippe in 1941 [6]. This specific tyre model consists of a set of endless strings, where each string provides a large number of tread elements. Figure 1 depicts the string tyre model proposed by Pacejka [1–3].

where x, v, X and Y are the longitudinal displacement and deflection of the string, lateral deflection, wheel plane axis and vertical axis of wheel plane axis on the horizontal plane, respectively. In order to determine the deflection factor v based on Figure 1, the boundary condition must be defined. The lateral force F_y, is expressed based on the following [1–3]

$$F_y = \int_{-\infty}^{\infty} C_c v(x)\,\text{d}x. \tag{6}$$

Here, C_c is the lateral carcass stiffness per unit length and it is a modulus of elasticity between lateral force and deflection of the string. In addition, as depicted in Figure 1, there are two sections in the Pacejka tyre model: the first part over the contact patch and the second part within the contact patch ‘a’.

In the first part ($x < -a$ or $x > a$), it is essential to determine the deflection factor v to obtain the lateral force of the tyre for side slip angle α, as described in Equation (6). Without friction between the road and tyre, only the shear force and tension of the string of each

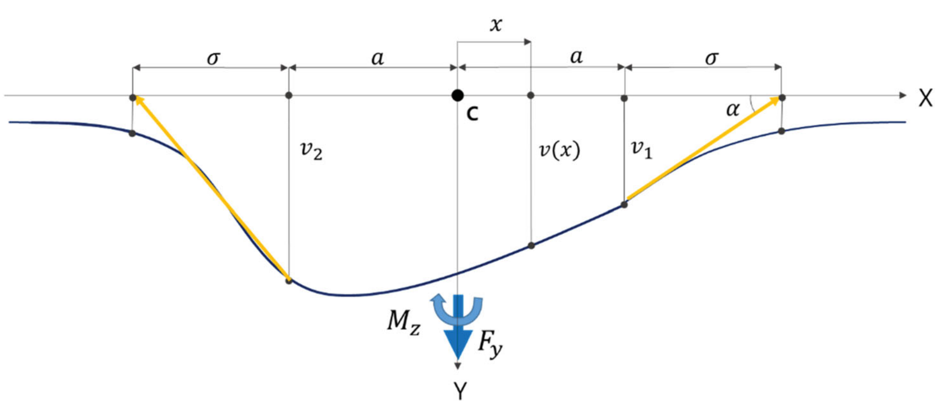

Figure 1. Top view of string tyre model.

string element is considered. Hence, the deflection factor v is represented as follows:

$$v = C_1 e^{-x/\sigma} \quad \text{if } x > a, \tag{7}$$

$$v = C_2 e^{+x/\sigma} \quad \text{if } x < -a, \tag{8}$$

$$\frac{\partial v}{\partial x} = -\frac{v_1}{\sigma} \quad \text{at } x = a, \tag{9}$$

$$\frac{\partial v}{\partial x} = \frac{v_2}{\sigma} \quad \text{at } x = -a, \tag{10}$$

where σ, v_i and C_i are the relaxation length of the string model, deflections of contact edges ($v(a) = v_1,\ v(-a) = v_2$), and integral coefficients, respectively. Then, Equation (6) can be calculated as follows [3]

$$F_y = \int_{-\infty}^{\infty} C_c v(x)\,\mathrm{d}x = C_c \int_{-a}^{a} v\,\mathrm{d}x + C_c\sigma(v_1 + v_2). \tag{11}$$

The lateral force F_y can be defined by Equation (11) through the definition of deflection of the string $v(x)$ within contact patch ($-a \le x \le a$) which is going to express with various concept.

Steady-state values of string model characteristic

The three common test parameters that have been widely used in numerous tyre manufacturing companies for the indoor test are cornering stiffness (C_α), lateral static characteristics (K_L), and the distortion static characteristics (K_D). These three test procedures comprise the fundamental approach to determine the lateral stiffness of the tyre. In addition, each procedure provides a specific physical meaning of the tyre's lateral stiffness from the previous study. These three factors measured through the indoor test are the characteristics of the tyre in the steady state.

Figure 2 describes the deformation to represent the lateral stiffness (K_L) from string tyre model. From the wheel plane axis X, a standing tyre is forced in the Y direction. In this process, it is assumed that the deflection of tyre within the contact patch ($-a \le x \le a$) is constant.

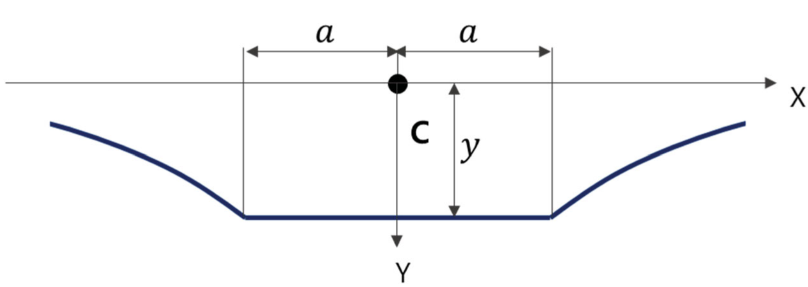

Figure 2. Steady-state deflection of the string tyre model for the lateral movement of the wheel centre.

From Equation (11), the lateral stiffness of the standing tyre can be derived as described in Equations (12)–(14):

$$v(x) = y, \quad -a \le x \le a, \tag{12}$$

$$F_y = 2C_c(\sigma + a)y, \tag{13}$$

$$K_L \triangleq \left.\frac{\partial F_y}{\partial y}\right|_{y=0} = 2C_c(\sigma + a). \tag{14}$$

Also when slip angle α is applied, in order to determine steady-state value of cornering stiffness, the slope of string is assumed to be close to a linear shape within contact patch as described in Figure 3.

Thus the cornering stiffness can be derived from Equation (11) as described in Equations (15)–(17):

$$v(x) = (-x + a + \sigma)\alpha, \quad -a \le x \le a, \tag{15}$$

$$F_y = 2C_c(\sigma + a)^2\alpha, \tag{16}$$

$$C_\alpha \triangleq \left.\frac{\partial F_y}{\partial \alpha}\right|_{\alpha=0} = 2C_c(\sigma + a)^2. \tag{17}$$

Lastly, the distortion stiffness K_D is the ratio of the moment M_z with respect to distortion angle θ, and it can be derived from the model as described in Figure 4.

The distortion moment equation can be defined as follows:

$$M_z = C_c \int_{-\infty}^{\infty} vx \, dx. \tag{18}$$

The distortion stiffness can be derived as described in Equations (19)–(21) for the arbitrary distortion angle :

$$v(x) = -x\theta, \quad -a \le x \le a, \tag{19}$$

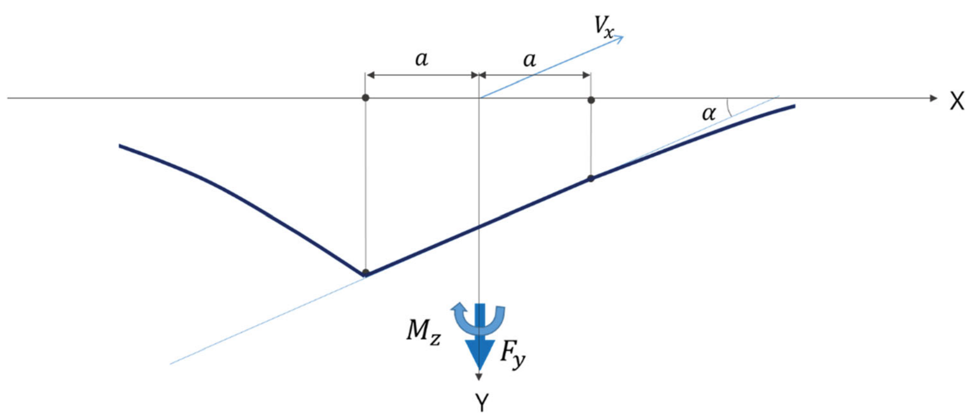

Figure 3. Steady-state deflection of the string tyre model for the side slip angle.

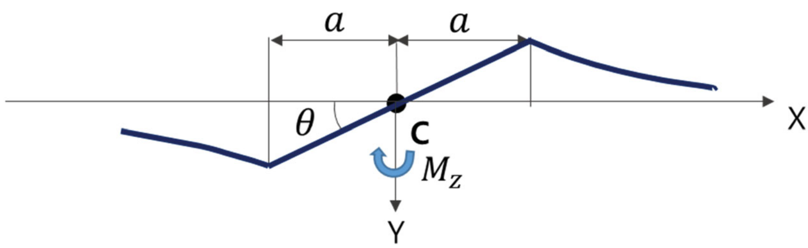

Figure 4. Steady-state deflection of the string tyre model for the torsion angle.

$$M_z = -K_D\theta = 2C_c a\left\{\sigma(\sigma + a) + \frac{1}{3}a^2\right\}\theta, \tag{20}$$

$$K_D \triangleq \left.\frac{\partial M_z}{\partial \theta}\right|_{\theta=0} = 2C_c a\left\{\sigma(\sigma + a) + \frac{1}{3}a^2\right\}. \tag{21}$$

Combining Equations (14) and (17), the relaxation length can be derived as follows:

$$\therefore \sigma = \frac{C_\alpha}{K_L} - a. \tag{22}$$

Equation (22) is different from relaxation length 'L' of Equation (1) defined by Heydinger et al. [4], and the relaxation length σ is more accurate value to express the dynamic tyre model. It will be proved in the model validation section.

Proposed tyre model

This paper proposes a novel mathematical expression to define the relaxation length through implementing distortion effect factors into the existing model.

To analysis the agility of tyre performance, transient state values should be discussed as well as steady state. In the previous studies [1–3,6], to describe the shape of string within contact patch, three different string models have been developed: single point string model, straight tangent model, and exact string model. Exact string model proposed by Segel [7] is developed with the assumption that velocity of each tread element with respect to road is zero. Single point string model assumes that tyre contact point is at one particular point. Straight tangent model is a simply linear approximation of the exact model. The contact patch line is defined as a linear extension of the deflection v_1 at the patch leading edge (Figure 5) [5].

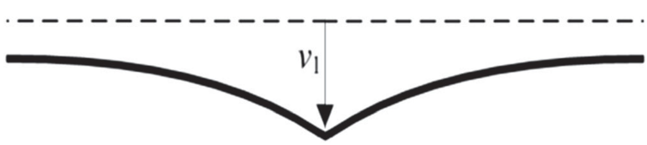

Figure 5. Single point string model.

The transfer function of the lateral force with respect to the side slip of single point model at transient state is expressed as follows [5,8]:

$$\bar{C}_{\alpha}(p) = 2C_{\mathrm{c}}(\sigma + a)^2 \frac{1}{1 + (\sigma + a)p}, \tag{23}$$

where p is Laplace variable with respect to travelled distance of the contact centre C. If the velocity of the contact centre C is constant, travelled distance s can be calculated in Equation (24)

$$s = V_x t, \tag{24}$$

where V_x and t are the velocity of the contact centre C and travelled time, respectively. Using Equations (17) and (24), the step response of Equation (23) can be described in the distance and time domain.

$$\bar{C}_{\alpha}(s) = 2C_{\mathrm{c}}(\sigma + a)^2(1 - \mathrm{e}^{-s/(\sigma+a)}) = C_{\alpha}(1 - \mathrm{e}^{-s/(\sigma+a)}), \tag{25}$$

$$\bar{C}_{\alpha}(t) = C_{\alpha}(1 - \mathrm{e}^{-t/((\sigma+a)/V_x)}). \tag{26}$$

Therefore, time constant of single point string model can be defined as follow.

$$\tau_{\mathrm{single}} = \frac{(\sigma + a)}{V_x}. \tag{27}$$

Through Equations (14) and (17), the time constant Equation (27) is same as the time constant of typical model described as Equation (5)The cornering stiffness of the straight tangent model at a transient state is can be expressed as follows [3]:

$$\bar{C}_{\alpha}(p) = 2C_{\mathrm{c}}(\sigma + a)^2 \frac{1}{1 + \sigma p}. \tag{28}$$

Transfer function of straight tangent model is also a first order function and the relaxation time constant τ_{straight} is suggested using sigma as follows with same method in Equations (25) and (26):

$$\tau_{\mathrm{straight}} = \frac{\sigma}{V_x}. \tag{29}$$

The cornering stiffness of the exact model described in Figure 6 at a transient state can be expressed as follows [3]:

$$\bar{C}_{\alpha}(p) = C_{\mathrm{c}} \frac{1}{p} \left\{ 2(\sigma + a) - \frac{1}{p}\left(1 + \frac{\sigma p - 1}{\sigma p + 1}\mathrm{e}^{-2pa}\right) \right\}. \tag{30}$$

It should be noted that the exact model described as Equation (30) is not in a simple first order form, and therefore time constant is not defined explicitly. However, comparing the three transfer functions, the exact model describes the real tyre best, and the others are the approximations of the exact model.

The accuracies of the approximated models are investigated in frequency responses. Figure 7 show that the straight tangent approximation is very closed to the exact model. Therefore, it is reasonable substituting exact model with the transfer function of straight

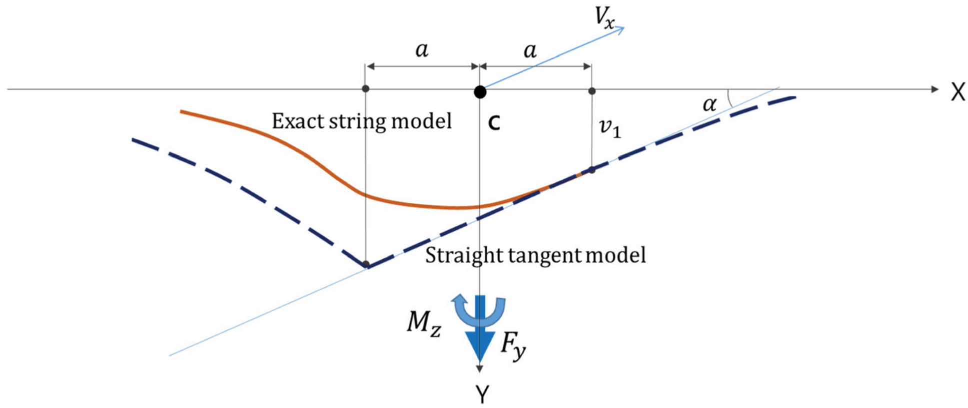

Figure 6. Exact and straight tangent model.

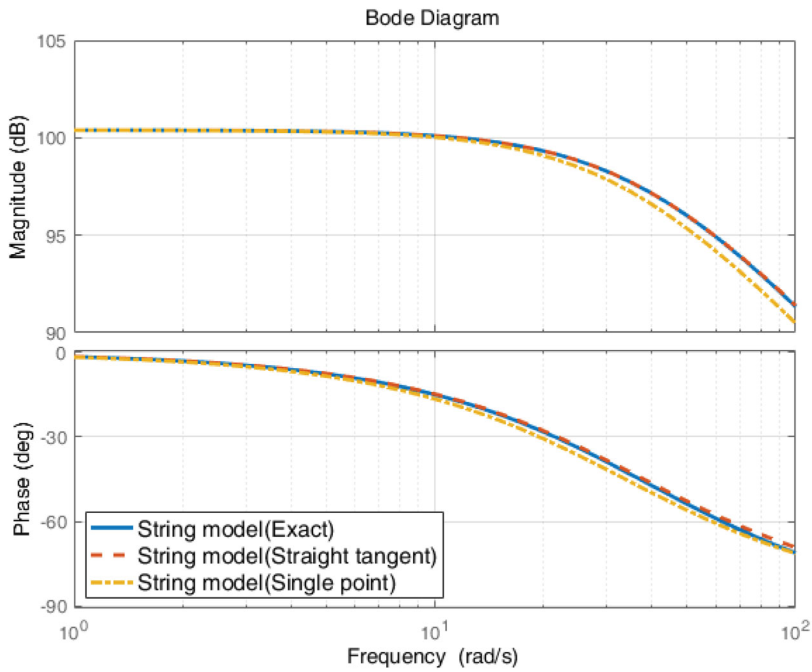

Figure 7. Bode plots of three models ($V_x = 120$ km/h).

tangent approximation, and the time constant of exact model can be represented by time constant of straight tangent model $\tau_{straight}$, rather than time constant of single point string model τ_{single} [3]. Thus it is very important to calculate the relaxation length σ using indoor test results and the time constant of straight tangent model can be a main factor to support the relation with steering response.

Model identification

To calculate relaxation length σ using Equation (22), the value of contact patch length 'a' is required. It is measured using the indoor test. As the dotted parts of Figure 8 show,

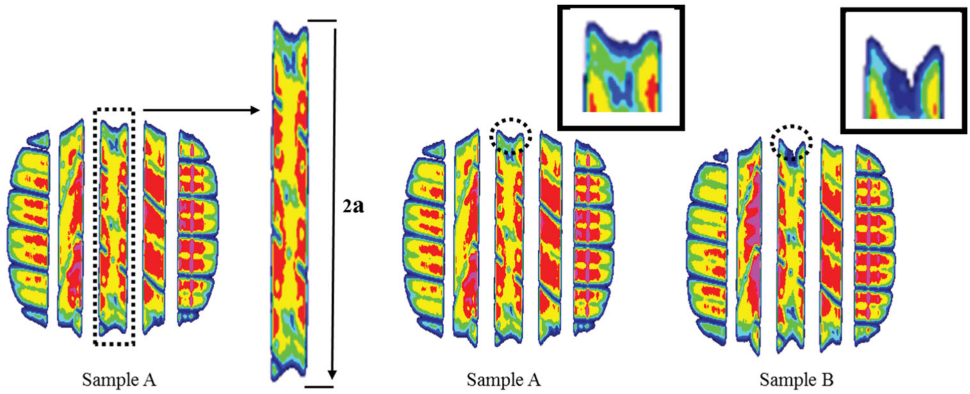

Figure 8. Foot shape of each test tyre.

determining the length of the contact patch is not a simple task. Even though the two samples have same pattern design, the blurred ends and sample-to-sample variations make the task quite delicate.

Furthermore, in the field, it is commonly known that the contact patch length has not a direct effect in determining the steering response as represented Equation (22): it rather contributes to determining the rolling resistance as well as the hydroplaning and wear performance. Therefore, in this paper, distortion stiffness K_D is considered to this mechanism instead of the contact patch length 'a'. The way to describe relaxation length σ without the measured value 'a' is considered

Therefore, a new relaxation length (σ) is proposed as expressed in Equation (31), which can be derived by combining Equations (14), (17), and (21) in addition

$$\therefore \sigma = \left(\frac{C_\alpha^3}{K_L^3} - \frac{3C_\alpha K_D}{K_L^2}\right)^{1/3}, \tag{31}$$

where C_α is the cornering stiffness, K_L the lateral static stiffness, and K_D the distortion static stiffness. This equation can closely replicate the characteristics of the steering performance by reflecting the effect of distortion static characteristics. Furthermore, the controversial factor 'a' is erased while determining the steering agility.

Also, this study investigates the connection between the proposed tyre model and the subjective evaluation method. A good starting point is acquiring the steering performance from a subjective test based on the driver's evaluation. The acquired steering performance from a subjective test based on the driver's evaluation is divided into two categories: steering response and torque. When the test driver feels the steering performance of the tyre, the most important factor is the visible reaction of the yaw motion and torque feedback felt by hands. After the subjective testing, the test driver expresses the distortion motion of the tyre as accurately as possible to the design engineer, and this can be expressed in numerous ways [3,4,6,9,10]. Therefore, it is reasonable to include the distortion static stiffness K_D to relaxation length σ.

Table 1. Effect of parameter variations on the proposed relaxation length (%).

Parameter	−20%	−10%	−5%	5%	10%	20%
C_α	−26.92	−13.08	−6.47	6.36	12.63	24.96
K_L	28.43	12.65	6.00	−5.44	−10.39	−19.08
K_D	2.74	1.39	0.70	−0.71	−1.43	−2.90

Effectiveness of each factor on relaxation length

It was found that the effect of each of the three factors can vary the results of the proposed relaxation length described in Equation (31). That means the steering agility performance can be predicted by using only indoor tests. Table 1 presents the effect of each factor on the relaxation length. When each parameter is changed artificially in the simulation with the variation ranging from −20% to 20%, the effectiveness of each factor to determine relaxation length can be confirmed. This shows the potential of tuning the steering agility performance of the tyre using the indoor static characteristics of steady-states deflection.

Performance assessment

In order to verify the validity of the modified tyre model with the proposed relaxation length defined in Equation (31) instead of Equation (1), it is compared with the experimental results based on indoor testing (Figure 9).

Comparison of relaxation length and experimental activity

In order to define a real relaxation length, all samples were tested in one day in order to reduce the effect of external noise due to the test conditions: temperature, humidity, etc.

Seven front tyre samples with the specifications in Table 2 were tested indoors without being fitted to a vehicle. Those tyres are made by a same manufacturer and summer tyre tread pattern design, and have with only minor design parameters are different. This is

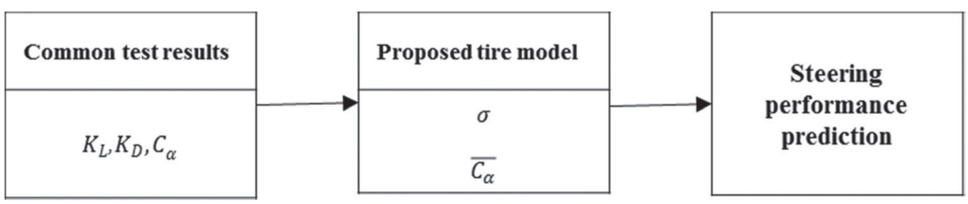

Figure 9. The flow of prediction of the steering performance section.

Table 2. Tyre indoor test and relaxation length results.

Sample	1	2	3	4	5	6	7
K_L ($\times 10^5$ N/m)	1.588	1.519	1.529	1.548	1.509	1.509	1.499
C_α ($\times 10^5$ N/rad)	1.046	1.027	1.027	1.022	1.027	1.035	1.035
K_D ($\times 10^3$ Nm/rad)	6.235	6.156	6.240	6.278	6.109	6.076	6.275
Relaxation length 'L'	0.659	0.676	0.672	0.660	0.680	0.686	0.691
Real relaxation length (m)	**0.600**	**0.615**	**0.610**	**0.600**	**0.616**	**0.625**	**0.630**
Proposed relaxation length (m)	**0.593**	**0.610**	**0.605**	**0.592**	**0.615**	**0.621**	**0.624**

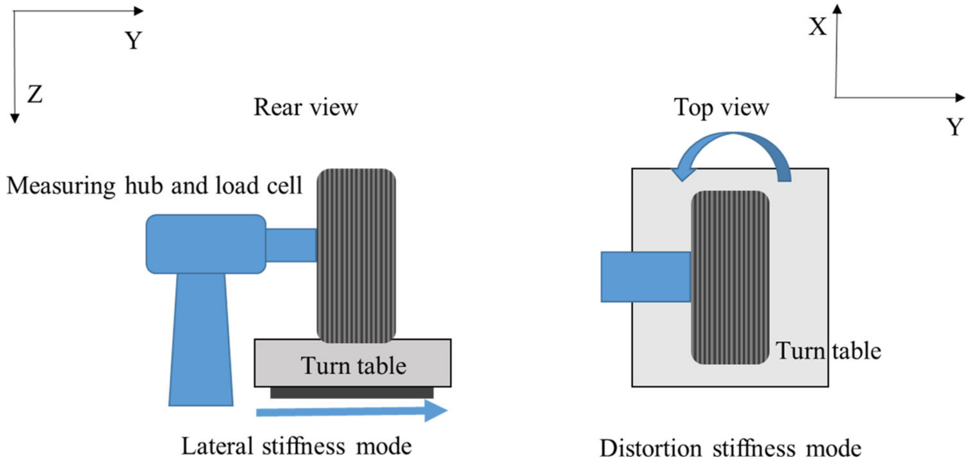

Figure 10. Static characteristic tester for lateral and distortion.

intended to evaluate the performance difference even with small changes. The cornering stiffness C_α and real relaxation length were determined by step response using MTS Flat-Trac® test machine, respectively. Here, when the side slip angle generated by the spindle torque of MTS Flat-Trac®, is applied to the test tyre, the output which is lagged lateral force with time delay, can be observed from the step response on the flat surface assuming the actual road surface. The test speed is 120 km/h under 3724N loads. Also, Static characteristic data is an important determining factor that influences the running situation of tyres and the steering performance, even though it is not running when measured. Figure 10 illustrates how the test machine is measuring the lateral stiffness (K_L) and distortion static stiffness characteristics(K_D). When force is applied to the tyre on a turntable, it is measured using a load cell. The turntable is shifted to the X axis in order to measure the lateral stiffness, and it is turned to the M_z axis for the distortion stiffness. The vertical load of each test mode can influence of the results. Therefore, the vertical load must be identified by real axle loading of test tyre and vehicle.

Model validation results

This section presents the experimental studies carried out validate the feasibility of the proposed tyre model. Table 2 presents the results of the indoor tyre tests. The proposed relaxation lengths were calculated based on the three parameters: lateral stiffness K_L, cornering stiffness C_α, and distortion stiffness K_D, and the real relaxation length results are presented for the seven different tyre samples by the results of experiments.

A comparison of the relaxation length and the real relaxation length demonstrated. The grey line of above Figure 11 represents the exact matching performance between experimental and calculated length (45° slope). The absolute value of relaxation length 'L' is longer than real and proposed values. As shown in Equation (22), this result is based on the consideration by subtracting the contact patch length of tyre. The proposed relaxation formula can be confirmed closer to the actual value than the relaxation length (L).

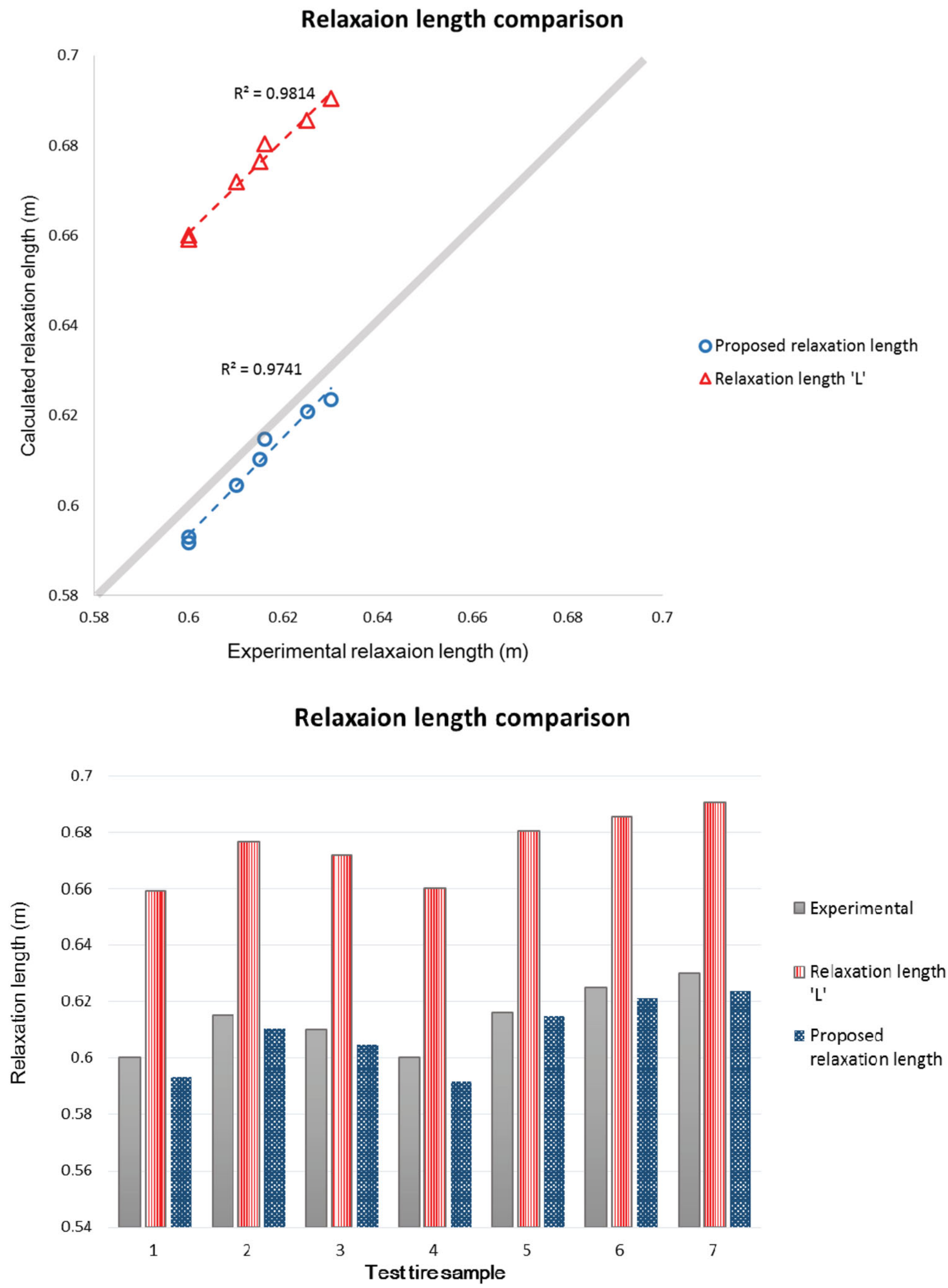

Figure 11. Validation results from comparing with real relaxation length.

This is the main contribution of this paper that steering response can be predicted by the proposed relaxation length which is closed to the real. Based on the comparison, this indicates that when the time constant of relaxation length proposed in Equation (31) is applied, it can replicate the characteristics of the steering performance much closely.

Conclusions

The concept of using the relaxation length to predict the steering response performance has been demonstrated to be appropriate in the string tyre model. This paper proposes a novel mathematical expression to define the relaxation length through implementing distortion effect factors into the existing model. Seven tyre samples were applied to the proposed tyre model; based on the string tyre model. The proposed relaxation length has more exact value to the real data than the relaxation length 'L'. This proved that determining the steering agility performance is possible through using only indoor test results. This can significantly reduce the evaluations costs and time associated with subjective tests, and tyre design engineers can use the results of the three parameters used in the indoor tests in order to tune the steering agility performance of a vehicle even before the vehicle is built. Further research will be conducted in order to investigate the steering feel performance with appropriate vehicle model, as well as the tyre.

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Disclosure statement

No potential conflict of interest was reported by the author(s).

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