

# Varying Mass Estimation and Force Ripple Compensation using Extended Kalman Filter for Linear Motor Systems

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**Abstract**—In many industrial fields, the mass information of a moving system is important and necessary to prevent undesired motion or failure and to control the system in its desired trajectory. One simple solution could be direct measurement of the mass using a sensor such as force sensor and accelerometer. However, it requires additional cost increase. In addition, it is not easy to measure the mass of a moving part in many cases. For those reasons, in this research, an online varying mass estimation algorithm is designed using an Extended Kalman Filter (EKF) without any additional sensors. Furthermore, the lumped disturbance compensating algorithm, which was designed by the authors in the previous research using EKF, is combined to obtain further position tracking performance. The effectiveness of the suggested method is validated through simulations. Additional verification with experiments is planned for future work.

## I. INTRODUCTION

With developments in industrial technology, automation and robotics have become widely used in many industries [1]. This phenomenon has led to improvement in productivity and uniform quality of products. However, when a process breaks down and the entire workflow stops, the loss in terms of time and cost is considerable because the working processes are organized with optimized schedules. Therefore, equipments are checked on a regular basis, fault tolerant design is established, and severe working conditions are avoided. As an example, the maximum inertial force at the robot arm is limited to certain amount to prevent fatigue fracture [2] due to repetitive bending moment at the lower part of the column. In this case, accurate mass information is absolutely necessary to calculate the exact inertial force.

Moreover, accurate mass information is required for system control such as position, velocity and acceleration control for the following reasons. First, the desired poles of closed loop control systems are usually chosen considering the mass of moving parts [3]. Secondly, the mass is included as a parameter in the inverse plant model which is usually used for feedforward control to obtain faster response.

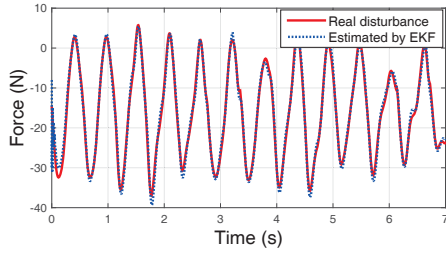
In some applications with an invariant mass system such as welding and screwing, just initial mass information could be sufficient for proper operation. However, the mass can change periodically in some transport works (i.e., pick and place work). Therefore, online estimation or measurement and adaptation for that varying mass are necessary. Attaching a sensor such as force sensor and acceleration sensor for direct measurement of the mass can be a solution. However, it is not easy to measure the mass of a moving part, and it also causes an increase of the hardware cost [4].

In this research, a software approach to estimate the varying mass during automated operation without any additional sensors is investigated. In a previous study [5], a lumped disturbance compensator was designed with an EKF of the 6th order. As explained in Section III, by expanding the EKF order from the 6th to the 7th, both the varying mass estimation and force ripple compensation for permanent magnet linear motor systems are achieved.

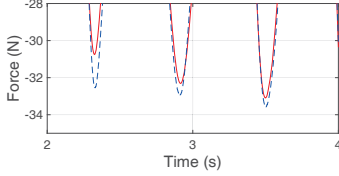
The rest of this paper is organized as follows. The necessity of mass information is presented with simulation results in Section II. The suggested EKF is designed in Section III. Then, the performance evaluation and analysis are shown in Section IV. Finally, conclusions are given in Section V.

## II. NECESSITY OF ACCURATE MASS INFORMATION

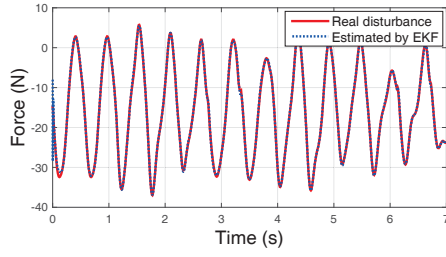
In this section, the necessity of accurate mass information is described using simulation results. With wrong mass information, the control performance of a mechanical system usually deteriorates. The performance according to the wrong mass information was analyzed for the case of the real mass of 6.7 kg, and the results are shown in Fig. 1, Fig. 2 and Table I. It should be noted that the desired velocity condition is given constant of 0.04 m/s to attenuate the influence by the different inertial force due to the mass error. As described in Fig. 1 and Fig. 2, the nominal mass is set to 3.4 kg, 6.7 kg and 9.9 kg, respectively. Fig. 1 shows the comparison between the real



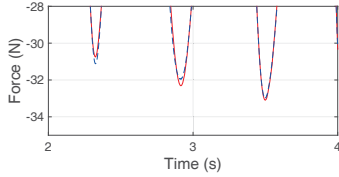
(a) When the nominal mass is 3.4 kg



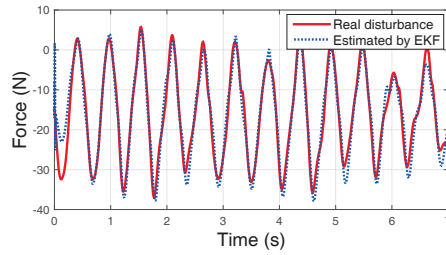
(b) Close-up of above subfigure (a)



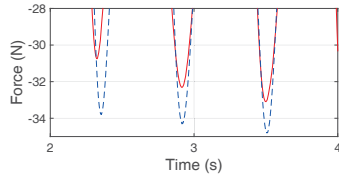
(c) When the nominal mass is 6.7 kg



(d) Close-up of above subfigure (c)

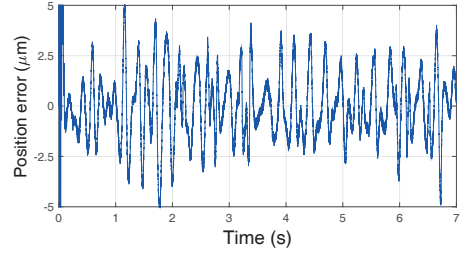


(e) When the nominal mass is 9.9 kg

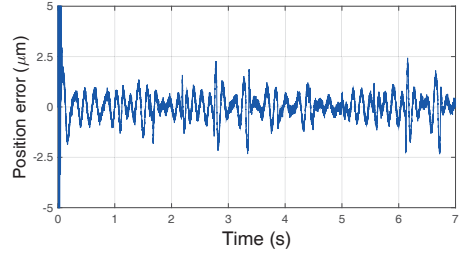


(f) Close-up of above subfigure (e)

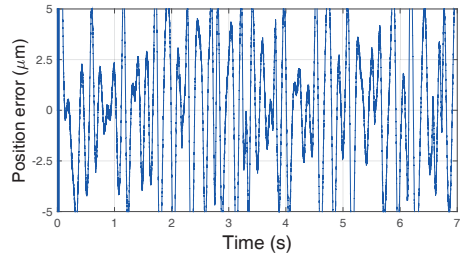
Fig. 1. Influence of mass information on lumped disturbance estimation. Real mass is 6.7 kg.



(a) When the nominal mass is 3.4 kg



(b) When the nominal mass is 6.7 kg



(c) When the nominal mass is 9.9 kg

Fig. 2. Influence of mass information on position tracking error. Real mass is 6.7 kg.

lumped disturbance and the estimated lumped disturbance by the 6th order EKF which does not consider the mass variation. Further detailed algorithmic description will be addressed in Section III. To clarify the differences among them, close-up figures are attached below each condition in Fig. 1. Moreover, Fig. 2 shows the position tracking error due to the wrong mass information. As clearly seen in Fig. 1 and Fig. 2, the control performance becomes better when the mass information is correct. In addition, the RMS position error is calculated for a quantitative evaluation and summarized in Table I. The results for the nominal mass of 17 kg are omitted in Fig. 1 and Fig. 2, but included in Table I.

### III. EXTENDED KALMAN FILTER DESIGN

As briefly mentioned in the previous sections, the 6th order EKF proposed by the authors in the previous research [5] was designed without the consideration of mass varying conditions. Therefore, poor performance is expected in varying mass condition as shown in Fig. 1, Fig. 2 and Table I. In this section, the order of the EKF is expanded from 6 to 7 to estimate the mass variation in real time and to utilize that information for the force ripple compensation as in the following procedures.

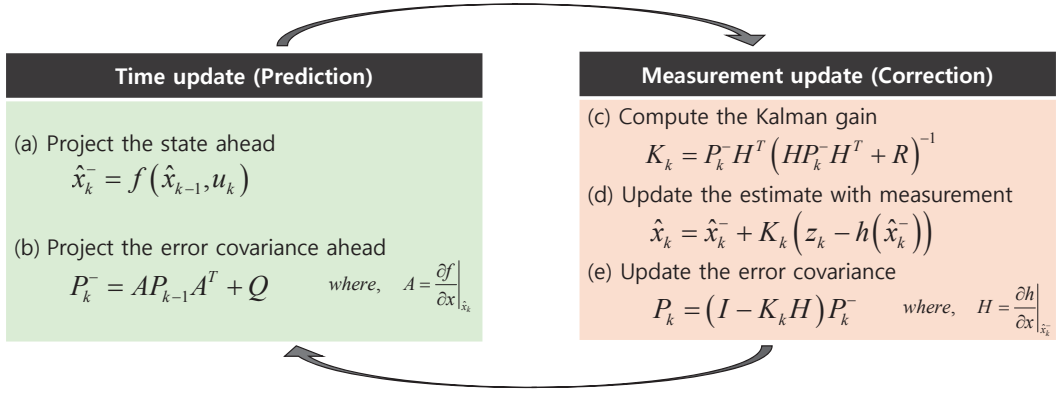


Fig. 3. Process of the EKF algorithm.

TABLE I  
RMS POSITION ERROR DUE TO WRONG MASS INFORMATION

Nominal mass	RMS position error
3.4 kg	1.63 $\mu\text{m}$
6.7 kg	0.55 $\mu\text{m}$
9.9 kg	3.42 $\mu\text{m}$
17 kg	4.32 $\mu\text{m}$

Reference trajectory: 0.04m/s\*t

First, the mechanical dynamics of a motor system can be written as

$$M\ddot{x}(t) + B\dot{x}(t) = u(t) + d(t), \quad (1)$$

where  $M$  is the mass of the mover,  $x(t)$  the position of the mover,  $B$  the viscous friction coefficient,  $u(t)$  the thrust force and  $d(t)$  the force ripple due to disturbances that are composed with constant and some harmonic components. With the assumption that the force ripple consists of up to the 4th order harmonic components, the force ripple  $d(t)$  in Eq. (1) can be decomposed as follows:

$$d(t) = \left[ (C_{A0} + C_0) + (C_{A1} + C_1) \cdot \cos\left(\frac{2\pi}{x_{pp}}(x + x_s)\right) + (C_{A2} + C_2) \cdot \sin\left(\frac{2\pi}{x_{pp}}(x + x_s)\right) + C_3 \cdot \cos\left(\frac{2\pi}{x_{pp}}2(x + x_s)\right) + C_4 \cdot \sin\left(\frac{2\pi}{x_{pp}}2(x + x_s)\right) + C_5 \cdot \cos\left(\frac{2\pi}{x_{pp}}3(x + x_s)\right) + C_6 \cdot \sin\left(\frac{2\pi}{x_{pp}}3(x + x_s)\right) + C_7 \cdot \cos\left(\frac{2\pi}{x_{pp}}4(x + x_s)\right) + C_8 \cdot \sin\left(\frac{2\pi}{x_{pp}}4(x + x_s)\right) \right], \quad (2)$$

where  $C_0 \sim C_8$  are the coefficients of the position dependent disturbance model,  $C_{A0} \sim C_{A2}$  are the coefficient offsets between the model and the real value, that are to be compensated by the EKF,  $x_{pp}$  is the pole pitch of the permanent magnets

and  $x_s$  is the initial position that is to be estimated by the EKF.

The state variable  $x$  for the EKF is designed as follows:

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} = \begin{bmatrix} x \\ \dot{x} \\ x + x_s \\ C_{A0} \\ C_{A1} \\ C_{A2} \\ 1/M \end{bmatrix}, \quad (3)$$

then, the state space representation is given as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -Bx_7x_2 + x_7u + \left[ x_7(x_4 + C_0) + x_7(x_5 + C_1) \cos\left(\frac{2\pi}{x_{pp}}(x_3)\right) + x_7(x_6 + C_2) \sin\left(\frac{2\pi}{x_{pp}}(x_3)\right) + x_7C_3 \cos\left(\frac{2\pi}{x_{pp}}(x_3) * 2\right) + x_7C_4 \sin\left(\frac{2\pi}{x_{pp}}(x_3) * 2\right) + x_7C_5 \cos\left(\frac{2\pi}{x_{pp}}(x_3) * 3\right) + x_7C_6 \sin\left(\frac{2\pi}{x_{pp}}(x_3) * 3\right) + x_7C_7 \cos\left(\frac{2\pi}{x_{pp}}(x_3) * 4\right) + x_7C_8 \sin\left(\frac{2\pi}{x_{pp}}(x_3) * 4\right) \right] \\ \dot{x}_3 &= x_2 \\ \dot{x}_4 &= 0 \\ \dot{x}_5 &= 0 \\ \dot{x}_6 &= 0 \\ \dot{x}_7 &= 0, \end{aligned} \quad (4)$$

where some assumptions as  $\dot{x}_s \simeq 0$ ,  $\dot{C}_{A0} \simeq 0$ ,  $\dot{C}_{A1} \simeq 0$ ,  $\dot{C}_{A2} \simeq 0$  and  $\dot{M} \simeq 0$  are applied. The first four assumptions are validated in [5]. The last assumption can be validated by

resetting this algorithm at every putting up and putting down operation.

Equation (4) is rewritten as follows through discretization using the forward rectangular approximation:

$$\begin{aligned}
x_1(k+1) &= x_1(k) + T_s x_2(k) \\
x_2(k+1) &= [1 - T_s B x_7(k)] x_2(k) + T_s x_7(k) u(k+1) \\
&\quad + T_s x_7(k) [x_4(k) + C_0] \\
&\quad + T_s x_7(k) [x_5(k) + C_1] \cos\left(\frac{2\pi}{x_{pp}} x_3(k)\right) \\
&\quad + T_s x_7(k) [x_6(k) + C_2] \sin\left(\frac{2\pi}{x_{pp}} x_3(k)\right) \\
&\quad + T_s x_7(k) C_3 \cos\left(\frac{2\pi}{x_{pp}} x_3(k) * 2\right) \\
&\quad + T_s x_7(k) C_4 \sin\left(\frac{2\pi}{x_{pp}} x_3(k) * 2\right) \\
&\quad + T_s x_7(k) C_5 \cos\left(\frac{2\pi}{x_{pp}} x_3(k) * 3\right) \\
&\quad + T_s x_7(k) C_6 \sin\left(\frac{2\pi}{x_{pp}} x_3(k) * 3\right) \\
&\quad + T_s x_7(k) C_7 \cos\left(\frac{2\pi}{x_{pp}} x_3(k) * 4\right) \\
&\quad + T_s x_7(k) C_8 \sin\left(\frac{2\pi}{x_{pp}} x_3(k) * 4\right) \\
x_3(k+1) &= x_3(k) + T_s x_2(k) \\
x_4(k+1) &= x_4(k) \\
x_5(k+1) &= x_5(k) \\
x_6(k+1) &= x_6(k) \\
x_7(k+1) &= x_7(k), \tag{5}
\end{aligned}$$

where  $T_s$  is the sampling time. The tuning parameters for the EKF are designed as follows:

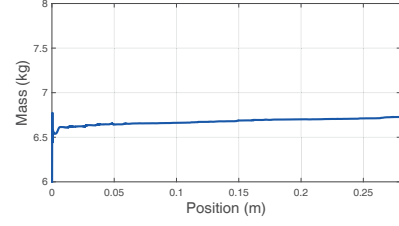
$$\begin{aligned}
\hat{x}_0 &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{M_n} \end{bmatrix}^T \\
\text{diag}(P_0) &= \begin{bmatrix} 10^{-4} & 10^{-4} & 10^{+3} & 10^{+8} & 10^{+6} & 10^{+3} & 10^{+6} \end{bmatrix} \\
\text{diag}(Q) &= \begin{bmatrix} 10^{-4} & 10^{+3} & 10^{-1} & 10^{+6} & 10^{+8} & 10^{+7} & 10^{-2} \end{bmatrix} \\
R &= \begin{bmatrix} 10^{-2} \end{bmatrix}, \tag{6}
\end{aligned}$$

where  $\hat{x}_0$  is the estimate of the initial state,  $P_0$  the initial error covariance matrix of the state,  $Q$  the system noise covariance matrix and  $R$  the measurement noise covariance matrix.

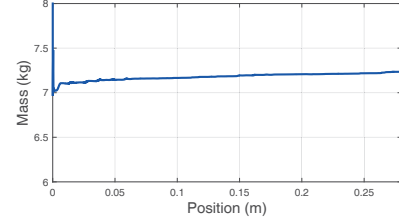
The process for the general EKF algorithm is shown in Fig. 3. As shown in Fig 3, the state variable  $x$  is recursively calculated through two processes: the 'Time update (Prediction)' and the 'Measurement update (Correction)'.

#### IV. PERFORMANCE VALIDATION

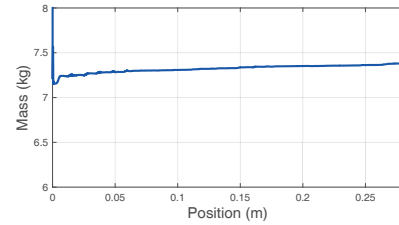
In this section, the performance of the EKF designed in Section III is validated through simulations. The simulation condition is similar to that described in Section II. Using the experimental data-set with 6.7 kg mass, the EKF algorithm is applied with different initial mass information, (i.e., different



(a) When the nominal mass is 3.4 kg



(b) When the nominal mass is 9.9 kg



(c) When the nominal mass is 20 kg

Fig. 4. Estimated mass with the proposed EKF algorithm using different initial mass information.

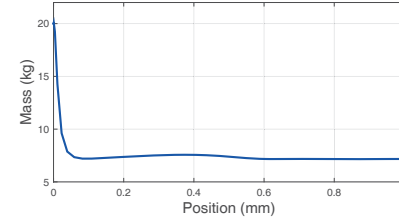
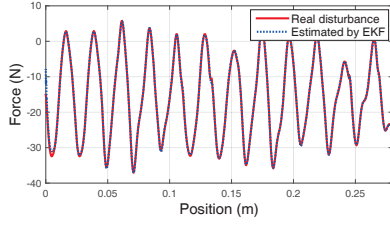


Fig. 5. Close-up figure of the starting region for the 20 kg nominal mass.

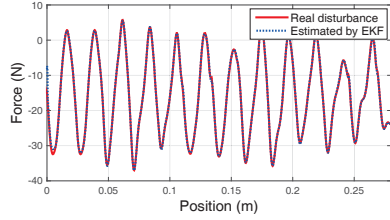
nominal mass information): 3.4 kg, 6.7 kg, 9.9 kg, 15 kg and 20 kg, respectively.

The estimated mass along the position (time) is shown in Fig. 4. For the sake of simplicity, only three cases among the five are shown. It is ascertained from Fig. 4 (c) that even though the initial mass information (i.e., the nominal mass) is given with about three times larger than the real value, the mass estimation works with a fast response. A close-up figure of the starting region for the 20 kg nominal mass case is shown in Fig. 5 to investigate the mass estimation rate. Notice that the unit of the x-axis is 'mm' in Fig. 5. The estimating rate, in other words, the convergence rate is considered fast enough for practical application.

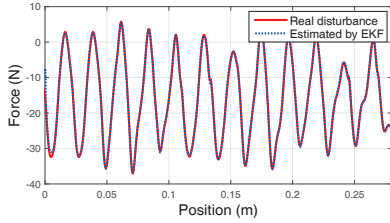
In addition, the estimated force ripple shape along the position that is to be compensated by the proposed EKF is



(a) When the nominal mass is 3.4 kg



(b) When the nominal mass is 9.9 kg



(c) When the nominal mass is 20 kg

Fig. 6. Estimated force ripple shape using the proposed EKF algorithm with different initial mass information.

TABLE II  
RMS POSITION ERROR BY THE PROPOSED EKF ALGORITHM  
WITH CONSTANT REFERENCE SPEED

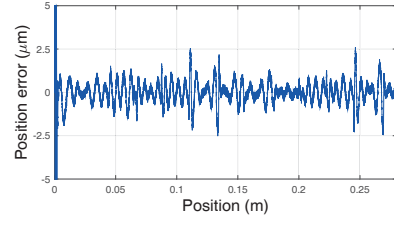
Nominal mass	Estimated mass	RMS position error
3.4 kg	6.70 kg	0.547 $\mu\text{m}$
6.7 kg	7.08 kg	0.557 $\mu\text{m}$
9.9 kg	7.21 kg	0.561 $\mu\text{m}$
15 kg	7.32 kg	0.568 $\mu\text{m}$
20 kg	7.35 kg	0.565 $\mu\text{m}$

Reference trajectory:  $0.04\text{m/s}\cdot t$

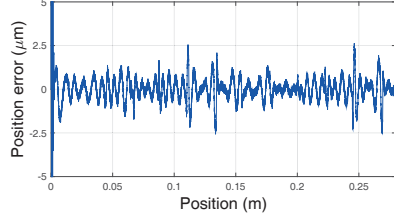
shown in Fig. 6. For the simplicity, the cases for the 6.7 kg and 15 kg nominal mass conditions are omitted in Fig. 6. Improved performance compared to Fig. 1 in Section II can easily be recognized. Regardless of the wrong mass information, almost identical force ripple shapes are obtained, and they also coincide well with the real disturbance shape. Moreover, the position tracking error is shown in Fig. 7, and the RMS position error is calculated and summarized in Table II as well.

Furthermore, additional simulations with sinusoidal reference trajectories are conducted. Equivalent results with the above conditions of constant speed are achieved. The RMS position tracking error is given in Table III.

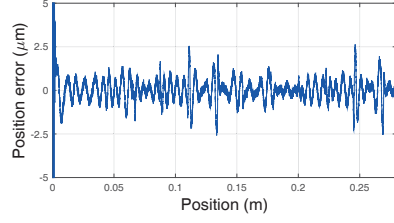
From those results, the effectiveness and necessity of sug-



(a) When the nominal mass is 3.4 kg



(b) When the nominal mass is 9.9 kg



(c) When the nominal mass is 20 kg

Fig. 7. Position tracking error using the proposed EKF algorithm with different initial mass information.

TABLE III  
RMS POSITION ERROR BY THE PROPOSED EKF ALGORITHM  
WITH SINUSOIDAL REFERENCE TRAJECTORY

Nominal mass	Estimated mass	RMS position error
3.4 kg	6.77 kg	0.639 $\mu\text{m}$
6.7 kg	6.92 kg	0.645 $\mu\text{m}$
9.9 kg	6.97 kg	0.647 $\mu\text{m}$
15 kg	7.01 kg	0.649 $\mu\text{m}$
20 kg	7.02 kg	0.649 $\mu\text{m}$

Reference trajectory:  $0.30\sin(2\pi/28\cdot t)$

gested method for the online mass estimation is verified.

## V. CONCLUSION

In this research, a 7th order EKF algorithm was designed to detect the varying mass during operation in real time and to compensate the force ripple for industrial automation systems. Through simulation results, it was validated that the proposed method can keep up with the mass variation with a fast response. This also leads to an increase in the force ripple estimation ability for the proposed EKF. Therefore, the position tracking performance can be maintained within a certain amount of position error regardless of the mass variation even though the mass increases to three times larger.

A follow up study with experimental validation is planned for future work.

#### ACKNOWLEDGMENT

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