## F2016-VDCF-004

# **GROUND VEHICLE ATTITUDE ESTIMATION THROUGH MAGNETIC-INERTIAL SENSOR FUSION**

## <sup>1</sup>Hwang, Yoonjin<sup>\*</sup>; <sup>1</sup>Seibum, Choi

<sup>1</sup>Korea Advanced Institute of Science and Technology, S. Korea

KEYWORDS – Vehicle Dynamics, Attitude Estimation, Sideslip, Euler Angle Estimation, Sensor Fusion

## ABSTRACT -

The main goal of this paper is to design accurate and model-independent vehicle attitudes estimator. The proposed estimator uses low cost 3 axis magnetometer and vehicle yaw rate and lateral acceleration sensors. The estimation skim can be divided into 2 parts: Euler angle estimation and sideslip estimation. An extended Kalman filter combined with a deterministic Euler angle estimator is utilized for robust and fast converging Euler angle estimation. Using the result of Euler angle estimator, a sideslip observer based on compensated planar kinematic model is proposed. The feasibility of proposed algorithm is verified with the commercial simulation tool, Carsim. Simulation includes possible noises of sensor based on manufacturer datasheets.

## TECHNICAL PAPER -

## 1. INTRODUCTION

An Electronic Stability System (ESC) is one of the successful control strategy to prevent unintended motion of the vehicle. According to the recent research[1], the ESC can reduce single-vehicle crashes up to 36% and rollover crashes up to 70%. The ESC system requires accurate information about the vehicle motion, which contains steering wheel angle, yaw rate, lateral acceleration and sideslip angle. Especially, the sideslip angle is an important factor to controllability. Tyre grip varies with the sideslip angle. The cornering stiffness increases along the sideslip, but once it hits its peak value, the tyre cannot make more force. That means when the sideslip exceed certain level, the driver cannot control it with steering. However, the sideslip angle cannot be measured directly while other parameters are given by sensors. Since the sideslip angle plays a key role to judge how much vehicle can react on driver's control, various estimation skims have been introduced for decades: a) estimation based on a dynamic model, which is called bicycle model, b) estimation by integrating a kinematic model for the plane motion, c) alternative sensors to measure the sideslip directly, and d) experiment-based methods.

First two model based approaches are most common ways to determine how much the vehicle slips. The estimation based on the bicycle model uses a reduced order model of a vehicle. It uses number of vehicle parameters, and only requires yaw rate and acceleration data. The accuracy of it is quite precise if the vehicle parameters are correct. However, the parameters are not easily determined and can be changed from their original value due to various reasons. To handle this problems, adaptive estimation methods have been introduced[2], but it still requires precise parameter adjustment and high computing power. The kinematic model observer doesn't need the accurate model parameters, but the Euler angle determination and the drifting issue are the main obstacles to design it[3]. Methods using GPS receivers to get the sideslip are introduced as an alternative[4; 5]. Multiple GPS receivers can provide the sideslip, but most of production vehicles are not

equipped with multiple GPS receivers. Moreover, the sampling rate of low cost GPS receivers is 1Hz to 10Hz, which is too low to use as a reference.

Yoon proposed a method with a magnetometer, IMU, and GPS for robust sideslip estimation. This method can stand on a wide range of road condition such as frictions, bank angle or inclination. The Kalman filter integrates Euler angles with IMU data, and checks the angles by comparing the rotation of the geomagnetic vector. This effectively eliminate a drift issue when using an IMU integration. With those three Euler angles and GPS velocity, the compensated kinematic model is used to get the sideslip angle of the vehicle. However, in this research, 6-DOF IMU and very high performance GPS receiver are used to estimate the vehicle state. Both those type of high performance sensors are not common equipment in production vehicles due to the cost of them.

This paper proposes a new method to obtain the sideslip angle with factory-equipped and low cost sensors. The proposed method only requires yaw rate and lateral acceleration, standard equipment of ESC systems, and three-axis magnetometer (TAM), used for E-compass. The estimation consists of 2 steps: Euler angle estimator and sideslip angle estimator. The Euler angle estimator is based on a sensor fusion through Kalman filtering of magnetic sensor and yaw rate sensor. Then, using the estimated Euler angle, kinematic model based sideslip estimation is performed. This proposed estimator doesn't need any vehicle model parameter, so it can be adopted various vehicle without individualization. Since this method also gives the data of roll, pitch, and yaw angle, it can be used for the roll over protection, suspension control, or even navigation systems.

This paper is organized as follows. Section 2 describes how measurement vectors react with arbitrary rotation and a methodology to find corresponding rotational matrix with vector measurements. Section 3 deals with the design of the Euler angle estimator by using a Kalman filter through a magnetic-inertial sensor fusion and the sideslip estimator with a compensated planar kinematic model. Section 4 verify the performance of proposed estimators on simulations and experiments.

## 2. THEORETICAL BASIS

## 2.1 3-Axis Magnetometer

A magnetometer measures the geomagnetic field which can be assumed as a constant vector in local area. The geomagnetic field has its direction to the magnetic north pole and its magnitude about 500mG. Since the geomagnetic field is fixed in Earth-fixed reference frame (R-frame), the measurement in body-fixed frame (m-frame) is varying with the rotation of the vehicle. There are several ways to represent a body rotation with Euler angles; this paper use ZYX successive rotation with yaw angle  $\psi$ , pitch angle  $\theta$ , and roll angle  $\varphi$ .



Figure 1: Successive ZYX rotation

The measured geomagnetic vector in m-frame can be expressed as (2.1).

$$\vec{B}^{m} = \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta\\ -\cos\phi\sin\psi + \sin\phi\sin\theta\cos\psi & \cos\phi\cos\psi + \sin\phi\sin\theta\sin\psi & \sin\phi\cos\theta\\ \sin\phi\sin\psi + \cos\phi\sin\theta\cos\psi & -\sin\phi\cos\psi + \cos\phi\sin\theta\sin\psi & \cos\phi\cos\theta \end{bmatrix} \vec{H}$$
(2.1)

 $\vec{H}$  is the geomagnetic field measured in the R-frame, and  $\vec{B}^m$  is the geomagnetic field measured in m-frame.

Since the rotation model has components with Euler angles, the differentiation of (2.1) with time can provide an equation with angular rates.

$$\dot{\vec{B}} = \vec{\omega} \times \vec{B} = \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix} \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix}$$
(2.2)

However, the unique solution of (2.2) cannot be obtained by itself. There are 3 equations and 3 unknown variables, p, q and r, but considering the constraint of the magnitude of  $\vec{B}$ , one of the equations can be represented by others. Therefore, to obtain the unique solution, there should be at least one more condition. Then, yaw rate r can plays a role. By measuring r from a sensor, (2.2) becomes solvable, and the result is as follows.

$$p = (-\dot{B}_y + rB_x) / B_z$$

$$q = (\dot{B}_x - rB_y) / B_z$$
(2.3)

### 2.2 Tri-axial Attitude Determination

Tri-axial attitude determination (TRIAD)[6-8] is a method to determine the attitude of body with multiple vector measurements. As mentioned in previous section, the attitude cannot be obtained as unique solution with one vector measurement.



Figure 2: (a) Any vector on shaded plane can be the rotation axis from V to V' (b) The rotation axis can be uniquely defined with two vector set (V, W)

However, with multiple measurement, so called Wahba's problem[9], there are more constraints than required, so the system is overdetermined. TRIAD algorithm provides the

optimal solution that minimize certain weight for each vector.

First, the triad vectors from measurement and reference vectors V and W are set as follows.

$$\vec{V}^{m} = R_{R}^{m} \vec{V}^{R}$$

$$\vec{W}^{m} = R_{R}^{m} \vec{W}^{R}$$

$$\vec{v}_{1} = \vec{V}_{m}, \quad \vec{v}_{2} = \vec{V}_{m} \times \vec{W}_{m}, \quad \vec{v}_{3} = \vec{v}_{1} \times \vec{v}_{2}$$

$$\vec{w}_{1} = \vec{V}_{R}, \quad \vec{w}_{2} = \vec{V}_{R} \times \vec{W}_{R}, \quad \vec{w}_{3} = \vec{w}_{1} \times \vec{w}_{2}$$
(2.4)

From that orthogonal triad basis, the rotational relationship from R-frame to m-frame can be obtained.

$$R_{R,Triad}^{m} = [\hat{\vec{v}}_{1} | \hat{\vec{v}}_{2} | \hat{\vec{v}}_{3}][\hat{\vec{w}}_{1} | \hat{\vec{w}}_{2} | \hat{\vec{w}}_{3}]^{T}$$
(2.5)

 $R_{R,Triad}^{m}$  is the rotation matrix calculated from R-frame to m-frame with same weight for each measurement, where  $\hat{\vec{v}}_i$  and  $\hat{\vec{w}}_i$  are the normalized vectors of  $\vec{v}_i$  and  $\vec{w}_i$ .

## 3. GROUND VEHICLE ATTITUDE ESTIMATON

### 3.1 EKF with Pseudo Angular Velocity

To combining the measurement model given in (2.1) and the transient model with the angular velocity obtained by (2.3), the extended Kalman filter[10] is chosen to solve severe nonlinearity on measurement model.

The transient model is[11]

$$\dot{\phi} = p + (q\sin\phi + r\cos\phi)\tan\theta$$
  

$$\dot{\theta} = q\cos\phi - r\sin\phi \qquad (3.1)$$
  

$$\dot{\psi} = p\cos\phi + q\sin\phi\sin\theta + r\cos\phi\sin\theta$$

and the measurement model is exactly same as (2.1).

With the transient model and measurement model, the Kalman filter is applied to estimate the orientation of the vehicle.

$$\hat{x}_{k}^{-} = F_{k-1}\hat{x}_{k-1}^{+}$$

$$P_{k}^{-} = F_{k-1}P_{k-1}^{+}F_{k-1}^{T} + Q_{k-1}$$

$$K_{k} = P_{k}^{-}H_{k}^{T}[H_{k}P_{k}^{-}H_{k}^{T} + R]^{-1}$$

$$\hat{x}_{k}^{-} = \hat{x}_{k}^{-} + K_{k}(z_{k} - H_{k}\hat{x}_{k}^{-})$$

$$P_{k}^{+} = (I - K_{k}H_{k})P_{k}^{-}$$
(3.2)

where, 
$$F = \frac{\partial \hat{x}}{\partial \hat{x}}, H = \frac{\partial z}{\partial \hat{x}}, x = \begin{bmatrix} \phi & \theta & \psi \end{bmatrix}^T$$

The covariance Q and R are assumed that they only have diagonal components, and treated

.

as tuning parameters since their distribution is unknown.

### 3.2 TRIAD with Pseudo Gravity Vector

EKF introduced in previous section is quite accurate without the measurement noise or misalignment. However, in the real world, the measurement noise is not avoidable, and the small error in measurement can be stacked and make the system diverge, called drifting. To prevent the drifting issue, a deterministic estimation can be an effective alternative. While TRIAD method requires least 2 vector measurements, the supposed system measures only 1 vector, which is the geomagnetic field. To deal with this problem, a virtual vector measurement can be introduced.

$$\vec{g}^{m} = [0 \quad a_{y} \quad \sqrt{1 - a_{y}^{2}}]$$
 (3.3)

The vector  $\vec{g}^m$  is a pseudo gravity vector, assuming the vehicle motion doesn't make inertial force and the road doesn't have any inclination angle.

To restrict the effect of TRIAD estimation when (2.8) is not valid, a time varying gain, TF (transient factor) is introduced. TF has its value from 0 to 1. A low TF means the vehicle motion is quiet steady longitudinal motion, whereas a high TF indicates the dynamic motion of vehicle. TF is defined as

EKF and TRIAD is combined through TF as follows.

$$\hat{x} = \hat{x}_{EKF} + (1 - TF)L(\hat{x}_{EKF} - \hat{x}_{TRIAD})$$
(3.4)

where  $\hat{x} = \begin{bmatrix} \phi & \theta & \psi \end{bmatrix}^T$  and *L* is a tunable gain.

### 3.3 Sideslip Estimation

The velocity kinematics[12] of the vehicle on the plane is used widely to estimate the longitudinal and lateral velocity of vehicle.

$$\begin{bmatrix} \dot{v}_x \\ \dot{v}_y \end{bmatrix} = \begin{bmatrix} 0 & r \\ -r & 0 \end{bmatrix} \begin{bmatrix} v_x \\ v_y \end{bmatrix} + \begin{bmatrix} a_{xw} \\ a_y \end{bmatrix} + g \begin{bmatrix} \sin \hat{\theta} \\ -\sin \hat{\phi} \cos \hat{\theta} \end{bmatrix}$$
(3.5)

The corresponding Luenberger form observer is as follows.

$$\dot{\hat{v}} = \begin{bmatrix} \dot{\hat{v}}_x \\ \dot{\hat{v}}_y \end{bmatrix} = \begin{bmatrix} 0 & r(t) \\ -r(t) & 0 \end{bmatrix} \begin{bmatrix} \hat{v}_x \\ \hat{v}_y \end{bmatrix} + \begin{bmatrix} a_{xw} \\ a_y \end{bmatrix} + g \begin{bmatrix} \sin \hat{\theta} \\ -\sin \hat{\phi} \cos \hat{\theta} \end{bmatrix} + L(t)y$$

$$\hat{y} = \begin{bmatrix} \hat{v}_x \\ \hat{v}_y \end{bmatrix}$$
(3.6)

Since the transient matrix varies with time, observer gain L(t) is selected by using frozentime pole-placement to stand for the time varying system.[13]

$$\dot{\tilde{x}} = \begin{bmatrix} -L_1 & r \\ -L_2 - r & 0 \end{bmatrix} \tilde{x}$$

$$\det(\lambda I - (A - LC)) = \det \begin{bmatrix} \lambda + L_1 & -r \\ L_2 + r & \lambda \end{bmatrix} = \lambda^2 + L_1 \lambda + r(r + L_2)$$
(3.7)

Then, choose  $L_1$  and  $L_2$  as

$$L_{1} = 2\kappa |r|$$

$$L_{2} = (\kappa^{2} - 1)r$$
(3.8)

The system meets the stability condition by LaSalle's theorem, with the Lyapunov function candidate as follows.

$$V = \frac{1}{2} (\kappa^2 \tilde{v}_x^2 + \tilde{v}_y^2) \ge 0$$
  

$$\dot{V} = -2\kappa^3 |r| \tilde{v}_x^2 \le 0$$
(3.9)

### **4. SIMULATION RESULTS**

#### 4.1 Simulation Results

The verification of proposed Euler angle estimator is performed using a commercial vehicle dynamics simulation tool, Carsim. The main focus of the test is to verify the robustness of the estimator with angled road which is hard to deal with the conventional methods, and with various states consist of quasi-static, steady and dynamic condition.

Case	Driver control	Bank	Incline
Case I	Double Lane Change	Flat	Flat
Case II	Straight	-20°	Flat
Case III	Straight	Flat	Hill
Case IV	Double Lane Change	$0^{\circ} \rightarrow 10^{\circ} \rightarrow 0^{\circ}$	Hill

Table 1: Test scenarios

Case I is a standard double lane change test. As shown is figure 3 the single TRIAD methods cannot follow the angles during the turn, whereas EKF effectively estimates the Euler angles including the yaw angle during the severe turning manoeuver. The combined estimator shows no difference with standard EKF, since right after the turn ended TRIAD is stabilized rapidly and it has no effect at dynamic region due to the conditional gain SF.

Case II is about constant bank angle. There is some peaks at the very first time of manoeuver due to the limitation of road creating on simulation tool. For roll and pitch angle 3 of them all show quiet good estimation performance, but in the case of yaw, the combined estimator can hold the steady state error much less than the standard EKF.

Case III, the vehicle goes through the hill. Since the inclination is not stiff, TRIAD also has

low disturbance. The combined estimator and the standard EKF shows similar levels of error.

Case IV is the complicate manoeuver, which covers all previous cases at once. As shown in figure 6, each estimators has predictable performance until 7 sec. However, after 7 second, the drift issue occurs with the standard EKF and the combined estimator both. The error of EKF is getting larger slowly with time, but the combined estimator effectively restrict the divergence of error with TRIAD method.



Figure 5: Case III, Straight, Hill

Figure 6: Case IV, Double Lane Change, Combined

The results of sideslip estimation is compared with simple integration of uncompensated planar kinematic model. There is unknown bias in the yaw rate and acceleration measurement, those leads the integration to diverge. The proposed sideslip observer with the model compensated with the effect of gravity and time varying gain eliminate the divergence issue from the integration, and at the same time it keeps the estimation error under

## acceptable level.



Figure 7 Straight, Flat

Figure 8 DLC, Combined

## 4. CONCLUSION

Without using the vehicle model parameter, such as moment of inertia, mass or cornering stiffness, it has been proven the ideal performance and feasibility of the low cost vehicle attitude estimator. The proposed estimator follows the reference values whether the vehicle is under dynamic condition or not. Since it does not contain any vehicle model parameters, the proposed estimator can be applied to any vehicle directly only with the initial calibration process.

The originalities distinguished from the previous researches are the following: improvement of the converging time and reducing the steady state error with a combination of the deterministic estimation(TRIAD) and the integrational estimation(EKF), and robustness of the vehicle attitude estimation against to the road angles.

However, according to the previously reported papers, since the geomagnetic field is very weak compared with the magnetic field generated by electric devices, using the magnetometer on the ground can be affected by the environment nearby. The hard and soft iron effect also could be a problem. Therefore, handling the magnetic noise and distinguish it from the geomagnetic field, and on-board calibration method is required. A real-car-based experiment is on preparation.

## REFERENCES

- [1] NHTSA. "Statistical Analysis of the Effectiveness of Electronic Stability Control (ESC) Systems Final Report" [J]. 2007,
- [2] Oh, J. J., & Choi, S. B. "Vehicle velocity observer design using 6-d imu and multipleobserver approach" [J]. 2012, 13(4): 1865-1879.
- [3] Kim, H., & Ryu, J. "Sideslip angle estimation considering short-duration longitudinal velocity variation" [J]. 2011, 12(4): 545-553.
- [4] Bevly, D. M., Ryu, J., & Gerde, J. C. "Integrating INS sensors with GPS measurements for continuous estimation of vehicle sideslip, roll, and tire cornering stiffness" [J]. 2006, 7(4): 483-493.
- [5] Yoon, J.-H., & Peng, H. "Vehicle sideslip angle estimation using two single-antenna GPS receivers" [J]. American Society of Mechanical Engineers, 2010, 863-870.
- [6] Bar-Itzhack, I. Y., & Harman, R. R. "Optimized TRIAD algorithm for attitude determination" [J]. 1997, 20(1): 208-211.
- [7] Markley, F. L. "Attitude determination using vector observations: a fast optimal matrix algorithm" [J]. 1993,
- [8] Shuster, M. D. "The TRIAD algorithm as maximum likelihood estimation" [J]. 2006, 54(1): 113-123.
- [9] Wahba, G. "A least squares estimate of satellite attitude" [J]. 1965, 7(3): 409-409.
- [10] Haykin, S. S., Haykin, S. S., & Haykin, S. S. 2001. *Kalman filtering and neural networks*: Wiley Online Library.
- [11] Oh, J., & Choi, S. B. "Vehicle roll and pitch angle estimation using a cost-effective six-dimensional inertial measurement unit" [J]. 2013, 227(4): 577-590.
- [12] Farrelly, J., & Wellstead, P. 1996. *Estimation of vehicle lateral velocity*. Paper presented at the Control Applications, 1996., Proceedings of the 1996 IEEE International Conference on.
- [13] Son, H., & Choi, S. 2008. A study on design of a robust vehicle side-slip angle observer using an integration of kinematic and bicycle model. Paper presented at the KSAE 2008 Annual Conf., Daejeon, Korea.