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Abstract

The purpose of this paper is to estimate accurately the vehicle attitudes, i.e. the vehicle roll and pitch angles. It is assumed that a set of data obtained from a low-price six-dimensional inertial measurement unit is available. This includes the linear acceleration of the vehicle and the angular rates of all axes. In addition, the observer exploits the data from the wheel speed sensors, and the steering-wheel angle, which are already available for recent production cars. Using the above, based on the combination of the velocity kinematics and pseudointegration of the angle kinematics, a novel scheme for reference angle selection dependent on the cornering-stiffness adaptation is adopted to observe the angles. The stability of each component of the proposed observer is investigated, and a set of assessments to confirm the performance of the entire system is arranged via experiments using a real production sport utility vehicle.

Keywords

Adaptive algorithm, inertial measurement unit, observers, pitch angle, roll angle, state estimation, vehicle dynamics

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Introduction

As a result of the heavy dependence on the use of vehicles by humans, the cost of the associated driving hazards always accompanies the convenience. In order to reduce the risk factors as much as possible, numerous engineers have been inspired to work on advancing the electronic safety control technology of vehicles; these various types of computerized electronic safety control technology facilitate securing the comfort and safety of passengers. For instance, knowing the vehicle pitch, and thus the road gradient, contributes to generating the desired clutch pressure values in controlling an automated manual transmission¹ and a dual-clutch transmission,^{2,3} or in processing the laser sensor signals for longitudinal vehicle control schemes⁴ such as collision damage mitigation, idle stop-and-go system, obstacle avoidance or adaptive cruise control. Also, the availability of information on the roll angle contributes to the safety and convenience of passengers through implementation of roll control,⁵⁻⁷ vehicle yaw stability $control^{8-12}$ and roll-over mitigation.¹³⁻¹⁵ Also, more fundamentally, identification of the vehicle attitudes facilitates estimation of other vehicle states by increasing the vehicle model accuracy for the studies in which the roll and pitch effects are neglected.¹⁶⁻¹⁸

The problem related to this field, however, is the need for a high-precision sensor to measure the required vehicle states. This is a critical hindrance for vehicles with a wide range of costs to have such technologies mounted and, without a solution, vehicle safety may only remain reachable for the affluent minority. In order to raise the passenger comfort and safety level, accurate information on the vehicle states must be available even with an affordable sensor. Also, this assumption must be satisfied under all conditions, regardless of the severity of vehicle motion, since the driving environment varies widely between different drivers and various driving conditions. Such requirements have been the major limitation in the previous efforts to develop wholly satisfactory vehicle attitude observers.

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Schiffman¹⁴ attempted to identify the roll dynamics via sensor integration, but this involves drift error and is effective for only a short duration. Some studies attempted to use inertial sensors to form the observer which estimates information on the vehicle attitude, but they required a Global Positioning System (GPS).¹⁹⁻²¹ Here, the option of GPS use is unfavourable, since using the GPS signals for purposes other than a navigation service requires additional hardware and processors. Furthermore, heavy dependence on the GPS signal frequently causes the system robustness to deteriorate when the vehicle enters areas with weak signals (i.e. driving near a tall building or through a tunnel). Tseng and co-workers^{22,23} designed observers that use both a vehicle model and sensor kinematics to obtain roll and pitch angle information, but they involved an unqualified assumption regarding the vehicle yaw rate and ignored the lateral and vertical velocity components. Park et al.²⁴ and Kim et al.²⁵ proposed roll dynamics estimators using modified bicycle models, but they assumed fixed cornering stiffnesses and omitted experiments performed on slippery surfaces (i.e. surfaces with a high-slip condition). Hence, the scope of this research is to maximize the estimation performance of vehicle attitude with only a low-cost six-dimensional (6D) inertial measurement unit (IMU), regardless of how severely a vehicle is manoeuvred, and without the aid of a GPS.

This research introduces a novel scheme to estimate the vehicle roll and pitch angles through combining the velocity observer and the sensor kinematics. While the bicycle-model-based observer estimation and pseudointegration of the angle kinematics are combined in accordance with the estimated front and rear cornering stiffnesses obtained by an adaptation scheme to generate the reference angle used in the attitude observer, the observer itself revises the estimation using the gyroscope sensor measurements. This process of multiple estimations induces a high estimation accuracy for both the low-frequency components and the high-frequency components of the changing vehicle attitudes.

The basic organization of this paper is as follows. The second section describes the way in which the flow of logic can be facilitated as follows: first, the general layout of the observer with the block diagram is displayed; second, the principle behind the primary longitudinal velocity approximation based on the angular velocities of the wheels is briefly dealt with; third, the cornering-stiffness adaptation is described; fourth, the details of the bicycle-model-based observer which is used to generate the lateral velocity used for pseudointegration are focused on; fifth and finally, the formation of the two reference angles obtained by the velocity kinematics and pseudointegration, and the attitude observer design obtained by the fusion of the two references, are described. The third section displays the results of the real-car-based experiments performed under different scenarios that verify a robust vehicle attitude estimation performance.



Figure I. General flow chart of the subcomponents of the proposed vehicle attitude observer. 6D: six-dimensional: IMU: inertial measurement unit.

Observer design

General observer flow chart

The general structure of the entire observer is given in Figure 1.

Longitudinal velocity estimation

The vehicle controller area network mounted on modern mass production cars provides the speeds of each individual wheel. When these speed data are transformed so that they correspond to the speed at the centre of gravity (CG) using the vehicle yaw rate and wheel track compensation, a rough estimation of the longitudinal velocity of the vehicle can be obtained. However, this value is equivalent to the real velocity of the vehicle, only when there is no longitudinal or lateral slip involved in the tyres. Taking an extreme case, a vehicle without an anti-lock braking system may still have a significant amount of inertia to maintain the longitudinal velocity even when the brakes have locked all the wheels. In this case, the vehicle velocity approximated from the wheel speed sensors is unusable.

To maximize the range of reliable wheel speed use, only the speeds of the undriven wheels (rear wheels, in the case of front-drive vehicles) are taken into account when the vehicle is accelerating to minimize the wheel spin effect. The maximum value of the four-wheel speed data is taken when the brake is applied, since wheels cannot rotate more rapidly than the velocity of the car but only become slower or even locked during braking.



Figure 2. Bicycle model representation of a vehicle.

The result is then filtered with a rate limiter, which takes the physical limitation and a_x as the references to restrict the change in the vehicle velocity within the realistically allowable boundary. This refinement gives v_{car} , a rough estimation of the vehicle velocity, which is used throughout the rest of this paper.

Cornering-stiffness adaptation

Figure 2 shows the bicycle model representation of the vehicle which deals with the lateral dynamics. With the assumptions that the left- and the right-hand sides of the vehicle experience identical dynamics with equivalent tyre cornering stiffnesses and that the longitudinal velocity of the vehicle is slowly varying, the bicycle model considerably reduces the amount of required computational effort and has been proven to be effective for identification of the vehicle states. Also, the fact that there is no requirement for a high IMU sensor input makes the bicycle model robust against sensor error.

On the other hand, however, a weakness involved in the use of the bicycle model is that the unmodelled disturbance including the road bank angle and the discrepancy between the nominal cornering stiffness and the actual cornering stiffness may cause a significant amount of deviation from the real lateral dynamics of the vehicle. To alleviate this problem, lateral dynamics compensation with knowledge of the road disturbance through the estimation of the total vehicle roll and pitch angles with respect to the earth axes is required. Also, model error due to parameter uncertainty is reduced through the front- and rear-cornering-stiffness adaptation.

On the basis of the bicycle model, the front and rear cornering stiffnesses can be updated using an adaptive scheme. Simple moment balance equations lead to the expression for the front-tyre lateral force given by

$$F_{yf} = \frac{ml_r a_y + I_z \dot{r}}{l_f + l_r} \tag{1a}$$

and the expression for the rear-tyre lateral force given by

$$F_{yr} = \frac{ml_f a_y - I_z \dot{r}}{l_f + l_r} \tag{1b}$$

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where m, I_z , a_y and r are the vehicle mass, the yaw inertia, the lateral acceleration and the yaw rate respectively. Here, it must be noted that each tyre force corresponds to the total lateral force on each axle of the actual vehicle. Also, structural investigation of the bicycle model leads to the expression for the front-tyre slip angle given by

$$\alpha_f = \beta + \frac{l_f}{v_x} r - \delta_f \tag{2a}$$

and the expression for the rear-tyre slip angle given by

$$\alpha_r = \beta - \frac{l_r}{v_x} r \tag{2b}$$

where v_x , β and δ_f are the longitudinal velocity, the side-slip angle and the front-tyre steering angle respectively. In order to connect equations (1) and (2) regarding the vehicle dynamics, a tyre model is essential. For simplicity, the linear tyre model is chosen to give a simple relationship between the tyre lateral force and the tyre slip angle, according to

$$F_{yf} = -C_f \alpha_f \tag{3a}$$

$$F_{yr} = -C_r \alpha_r \tag{3b}$$

The linear tyre model shows increasing inaccuracy with increasing tyre slip, because the cornering stiffnesses are left as constants. Hence, the actual implementation is compensated by the tyre cornering-stiffness adaptation, and so C_f and C_r in the linear tyre model are not necessarily constants.²⁶

After equation (2) is substituted into equation (3) and some algebraic manipulation is performed, we can obtain

$$\beta = -\frac{F_{yf}}{C_f} + \delta_f - \frac{l_f}{v_x}r$$
(4a)

$$\beta = -\frac{F_{yr}}{C_r} + \frac{l_r}{v_x}r$$
(4b)

In order to eliminate the unknown variable β (the sideslip angle), equations (4a) and (4b) are equated to give

$$\frac{F_{yf}}{C_f} - \frac{F_{yr}}{C_r} = \delta_f - \frac{l_f + l_r}{v_x}r$$
(5)

Here, $1/C_f$ and $1/C_r$ are modelled as

$$\frac{1}{C_f} = \left(\frac{1}{C_f}\right)_n + z_f \tag{6a}$$

and

$$\frac{1}{C_r} = \left(\frac{1}{C_r}\right)_n + z_r \tag{6b}$$

where $(1/C_f)_n$ and $(1/C_r)_n$ are the nominal values, and z_f and z_r are the unknown parts.¹³

By substituting equation (6) into equation (5), we obtain

$$F_{yf}\left[\left(\frac{1}{C_f}\right)_n + z_f\right] - F_{yr}\left[\left(\frac{1}{C_r}\right)_n + z_r\right] = \delta_f - \frac{l_f + l_r}{v_x}r$$
(7)

Now define a variable z as

$$z \equiv \delta_f - \frac{l_f + l_r}{v_x} r - F_{yf} \left(\frac{1}{C_f}\right)_n + F_{yr} \left(\frac{1}{C_r}\right)_n$$
$$= F_{yf} z_f - F_{yr} z_r \tag{8}$$

To make certain that the system is causal, a low-pass filter is applied to equation (8) according to

$$\dot{z} = -\gamma (z - F_{yf} z_f + F_{yr} z_r)$$

where γ is the filter gain.

In a similar manner, the estimated z can be expressed as

$$\dot{\hat{z}} = -\gamma \left(\hat{z} - F_{yf} \hat{z}_f + F_{yr} \hat{z}_r \right)$$
(10)

Now an update law for \hat{z}_f and \hat{z}_r is designed to estimate the unknown parts of the cornering stiffness according to

$$\dot{\hat{z}}_f = \gamma \gamma_f F_{yf} \varepsilon \tag{11a}$$

$$\dot{\hat{z}}_r = -\gamma \gamma_r F_{yr} \varepsilon \tag{11b}$$

where γ_f and γ_r are the adaptation gains and $\varepsilon \stackrel{\Delta}{=} z - \hat{z}$.

Stability of the system can be proved easily using an analysis of the Lyapunov function. Define $\tilde{z}_f = z_f - \hat{z}_f$ and $\tilde{z}_r = z_r - \hat{z}_r$, with the radially unbounded, decrescent and positive definite Lyapunov candidate function chosen as

$$V = \frac{1}{2} \left(\varepsilon^2 + \frac{\tilde{z}_f^2}{\gamma_f} + \frac{\tilde{z}_r^2}{\gamma_r} \right)$$
(12)

with

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \varepsilon \dot{\varepsilon} + \frac{\tilde{z}_{f}\dot{\tilde{z}}_{f}}{\gamma_{f}} + \frac{\tilde{z}_{r}\dot{\tilde{z}}_{r}}{\gamma_{r}} \\
= \varepsilon \dot{\varepsilon} - \frac{\gamma \tilde{z}_{f}\dot{\tilde{z}}_{f}}{\gamma_{f}} - \frac{\gamma \tilde{z}_{r}\dot{\tilde{z}}_{r}}{\gamma_{r}} \\
= \gamma \varepsilon \left(-\varepsilon + F_{yf}\tilde{z}_{f} - F_{yr}\tilde{z}_{r}\right) - \gamma F_{yf}\tilde{z}_{f}\varepsilon + \gamma F_{yr}\tilde{z}_{r}\varepsilon \\
= -\gamma \varepsilon^{2} \\
\leqslant 0$$
(13)

Applying the Barbalat lemma, the system actually turns out to be asymptotically stable, given that the persistent excitation condition²⁷ applies to F_{yf} and F_{yr} . Although the actual lateral tyre force characteristics are time varying, it is assumed that the estimation targets \dot{z}_f and \dot{z}_r are slowly varying and thus close to zero. In many cases, systems are modelled as time-invariant systems whose parameters are to be estimated through adaptation. This is possible by considering the effects of the parameter variations as unmodelled perturbations, so that the robust adaptive control techniques used for time-invariant plants in the presence of bounded disturbances and unmodelled dynamics work effectively for the actual plants when their parameters are smooth and vary slowly with time.²⁸

Here, the threshold between the parameters that vary discontinuously and greatly with high frequency and those that are smooth and vary slowly with time may be obscure. Thus, the suggested adaptive scheme is shown to work via careful selection of adaptive gains and applications on various samples of empirical data.

The adaptive scheme hitherto dealt with raises an issue that the cornering stiffness changes as a function of the vertical load, and the amount of its fluctuation is quite substantial. Increasing the adaptation gain is not an option, since doing so results in a noise issue. Thus, the former adaptive scheme is altered to update the cornering stiffness normalized by the vertical load. For this purpose, by neglecting the aerodynamic drag and using the estimated states, the front vertical load is estimated as

$$F_{zf} = \frac{mgl_r\cos\hat{\theta} - mgh\sin\hat{\theta} - mh\hat{v}_x + ml_r\hat{v}_z - I_y\dot{q}_m}{L}$$
(14a)

and the rear vertical load as

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(9)

$$F_{zr} = \frac{mgl_f\cos\hat{\theta} + mgh\sin\hat{\theta} + mh\dot{\hat{v}}_x + ml_f\dot{\hat{v}}_z + I_y\dot{q}_m}{L}$$
(14b)

where \hat{v}_x , \hat{v}_z and $\hat{\theta}$ are the estimated longitudinal velocity, the vertical velocity²⁹ and the pitch angle respectively.

Analogous to the previously shown linear tyre model, the model used is

$$F_{yf} = -\bar{C}_f F_{zf} \alpha_f \tag{15a}$$

$$F_{yr} = -\bar{C}_r F_{zr} \alpha_r \tag{15b}$$

where \bar{C}_f and \bar{C}_r are the normalized cornering stiffnesses, i.e.

$$\bar{C}_f = \frac{C_f}{F_{zf}} \tag{16a}$$

$$\bar{C}_r = \frac{C_r}{F_{zr}} \tag{16b}$$

After similar processes as in the non-normalized case, we obtain

$$\beta_f = -\frac{F_{yf}}{F_{zf}\bar{C}_f} + \delta_f - \frac{l_f}{v_x}r$$
(17a)

$$\beta_r = -\frac{F_{yr}}{F_{zr}\bar{C}_f} + \frac{l_r}{v_x}r$$
(17b)

which are equated to become

$$\frac{F_{yf}}{F_{zf}\bar{C}_f} - \frac{F_{yr}}{F_{zr}\bar{C}_r} = \delta_f - \frac{l_f + l_r}{v_x}r$$
(18)

This time, modelling \overline{C}_f and \overline{C}_r instead of C_f and C_r as the nominal and unknown parts, we find that

$$\frac{1}{\bar{C}_f} = \left(\frac{1}{\bar{C}_f}\right)_n + \zeta_f \tag{19a}$$

and

$$\frac{1}{\bar{C}_r} = \left(\frac{1}{\bar{C}_r}\right)_n + \zeta_r \tag{19b}$$

where

$$\left(\frac{1}{\overline{C}_f}\right)_n = F_{zf} \left(\frac{1}{C_f}\right)_n$$

and

$$\left(\frac{1}{\bar{C}_r}\right)_n = F_{zr} \left(\frac{1}{C_r}\right)_n$$

are the nominal values and where ζ_f and ζ_r are the unknown parts.

Substituting the above gives

$$\frac{F_{yf}}{F_{zf}} \left[\left(\frac{1}{\bar{C}_f} \right)_n + \zeta_f \right] - \frac{F_{yr}}{F_{zr}} \left[\left(\frac{1}{\bar{C}_r} \right)_n + \zeta_r \right] = \delta_f - \frac{l_f + l_r}{v_x} r$$
(20)

Now the variable ζ can be defined as

$$\zeta \equiv \frac{F_{yf}}{F_{zf}} \zeta_f - \frac{F_{yr}}{F_{zr}} \zeta_r$$

= $\delta_f - \frac{l_f + l_r}{v_x} r - \frac{F_{yf}}{F_{zf}} \left(\frac{1}{\bar{C}_f}\right)_n + \frac{F_{yr}}{F_{zr}} \left(\frac{1}{\bar{C}_r}\right)_n$ (21)

so that it can be expressed in terms of the known states.

Then $\hat{\zeta}$ is defined likewise and a low-pass filter is applied to it according to

$$\dot{\hat{\zeta}} = -\eta \left(\hat{\zeta} - \frac{F_{yf}}{F_{zf}} \hat{\zeta}_f + \frac{F_{yr}}{F_{zr}} \hat{\zeta}_r \right)$$
(22)

where η is the filter gain.

Then the resulting update laws for ζ_f and ζ_r are obtained as

$$\dot{\hat{\zeta}}_f = \eta_f \frac{F_{yf}}{F_{zf}} \varepsilon_n \tag{23a}$$

$$\dot{\hat{\zeta}}_r = -\eta_r \frac{F_{yr}}{F_{zr}} \varepsilon_n \tag{23b}$$

where η_f and η_r are the adaptation gains and $\varepsilon_n = \zeta - \hat{\zeta}$.

A system stability check of the altered corneringstiffness adaptation scheme is omitted, since it is nearly identical with that of the non-normalized case.

Bicycle-model-based observer

The bicycle model representation of the vehicle dynamics is expressed in a state-space format as

$$\dot{x} = Ax + Bu \tag{24}$$

where

$$x = \begin{bmatrix} \beta \\ r \end{bmatrix}, \quad u = \delta_f$$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{2(C_f + C_r)}{mv_x} & \frac{-2(C_f l_f - C_r l_r)}{mv_x^2} - 1 \\ \frac{-2(C_f l_f - C_r l_r)}{I_z} & -\frac{2(C_f l_f^2 + C_r l_r^2)}{I_z v_x} \end{bmatrix}$$

and

$$B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} \frac{2C_f}{mv_x} \\ \frac{2C_f d_f}{I_z} \end{bmatrix}$$

For clarity, the cornering stiffnesses C_f and C_r that appear in the bicycle model (24) henceforth refer to the values obtained by cornering-stiffness adaptation. Also, the influence due to the pure vertical heave dynamics of the vehicle is considered negligible in this section of the paper.

By using the lateral acceleration sensor measurement, an observer based on the bicycle model is designed. Introducing the lateral acceleration, the relationship

$$u_y = \dot{v}_y + rv_x \tag{25}$$

holds.

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Now replacing the right-hand side of equation (25) with the terms that appear in the expression for the bicycle model,³⁰ the lateral acceleration is newly defined in terms of the state variables of the bicycle model as

$$a_{y} = a_{11}v_{x}\beta + (a_{12} + 1)v_{x}r + b_{1}v_{x}\delta_{f}$$
(26)

Before proceeding, it must be clarified that the lateral acceleration sensor measurement may not always refer to the lateral acceleration in equations (25) and (26). These two are equivalent to each other only in the absence of road angles and suspension angles. The suspension angle creates a false contribution to the vehicle lateral dynamics, since the effect of gravity changes the measurement taken by the sensor. On the other hand, the road terrain other than a nearly horizontal surface, especially in the case of the bank angle, actually causes the vehicle lateral dynamics to change. Hence, while the effect of the gravity reading due to the suspension angle must be eliminated, the effect of the static road angle must be maintained in the lateral acceleration sensor reading to reflect truly what is meant by equation (26).

To meet such requirements, the suspension angle and the road angle information must be available separately. For this purpose, the suspension angle is either measured from the suspension travel or estimated through the open-loop spring-damper system modelling of the vehicle. This information is then subtracted from the estimated roll and pitch angles to obtain the road angles. Once this process is carried out, the lateral acceleration necessary for the bicycle-model-based observer is computed through the relationship

$$a_{y} = a_{y,sensor} + (-g\sin\phi\cos\theta + g\sin\phi'\cos\theta') \quad (27)$$

where

$$egin{aligned} \phi &= \phi' + \phi_{sus} \ heta &= heta' + heta_{sus} \end{aligned}$$

Here, ϕ , ϕ' , ϕ_{sus} , θ , θ' and θ_{sus} indicate the total roll, the static road bank, the pure suspension roll, the total pitch, the static road inclination and the pure suspension pitch angle respectively.

Now, using the above-mentioned expression for the lateral acceleration, the output equation can be set up as

$$\hat{y} = C\hat{x} + Du \tag{28}$$

where

$$\hat{y} = \begin{bmatrix} \hat{r} \\ \hat{a}_y \end{bmatrix}, \quad x = \begin{bmatrix} \hat{\beta} \\ \hat{r} \end{bmatrix}$$
$$C = \begin{bmatrix} 0 & 1 \\ a_{11}v_x & (a_{12} + 1)v_x \end{bmatrix}$$
$$D = \begin{bmatrix} 0 \\ b_1v_x \end{bmatrix}$$

This allows the observer design

$$\dot{\hat{x}} = A\hat{x} + Bu + K(y - \hat{y}) \tag{29}$$

with

$$K = \begin{bmatrix} K_1 & K_2 \\ K_3 & K_4 \end{bmatrix}$$

Expanding the above leads to the state-space equation

$$\begin{bmatrix} \dot{\hat{\beta}} \\ \dot{\hat{r}} \end{bmatrix} = \begin{bmatrix} a_{11} - K_2 a_{11} v_{car} & a_{12} - K_1 - K_2 (a_{12} + 1) v_{car} \\ a_{21} - K_4 a_{11} v_{car} & a_{22} - K_3 - K_4 (a_{12} + 1) v_{car} \end{bmatrix} \begin{bmatrix} \hat{\beta} \\ \hat{r} \end{bmatrix} + \begin{bmatrix} b_1 - K_2 b_1 v_{car} \\ b_2 - K_4 b_1 v_{car} \end{bmatrix} \delta_f + \begin{bmatrix} K_1 & K_2 \\ K_3 & K_4 \end{bmatrix} \begin{bmatrix} r \\ a_y \end{bmatrix}$$
(30)

where $\hat{\beta}$, \hat{r} , r and a_y are the side-slip angle estimation, the yaw rate estimation, the measured sensor yaw rate

and the compensated lateral acceleration measurement respectively. Also

$$K = \begin{bmatrix} K_{1} & K_{2} \\ K_{3} & K_{4} \end{bmatrix}$$
$$= \begin{bmatrix} \frac{I_{z}(C_{f}l_{f}-C_{r}l_{r})}{2C_{f}C_{r}(l_{f}+l_{r})^{2}}p_{o}^{2}-1 & \frac{1}{v_{car}} \\ -2p_{o} & \frac{m(C_{f}l_{f}^{2}+C_{r}l_{r}^{2})}{I_{z}(C_{f}l_{f}-C_{r}l_{r})} \end{bmatrix}$$
(31)

Restricting p_o to be a negative tuning constant, the system stability is confirmed, since the observer state matrix in equation (30) turns out to be strictly Hurwitz, as long as K_2 and K_4 are prevented from becoming ill conditioned. This is achieved by switching their values to zero whenever their denominators are near zero. Finally, the bicycle-model-based observer estimation of the lateral velocity is obtained next using

$$\hat{v}_{y,bic} = v_{car} \tan \beta \tag{32}$$

Euler angle observer

Reference angle obtained by velocity kinematics. To estimate the roll and pitch angles, logic that combines two types of reference is used: one calculated from the velocity kinematics equations and the other from the pseudointegral kinematic estimations.

Here, the velocity kinematics are given as

$$\begin{bmatrix} \dot{v}_{x} \\ \dot{v}_{y} \\ \dot{v}_{z} \end{bmatrix} = \begin{bmatrix} 0 & r & -q \\ -r & 0 & p \\ q & -p & 0 \end{bmatrix} \begin{bmatrix} v_{x} \\ v_{y} \\ v_{z} \end{bmatrix} + \begin{bmatrix} a_{x} \\ a_{y} \\ a_{z} \end{bmatrix} + g \begin{bmatrix} \sin \theta \\ -\sin \phi \cos \theta \\ -\cos \phi \cos \theta \end{bmatrix}$$
(33)

Thus, exploiting the longitudinal and lateral kinematics leads to the expressions

$$\phi'_{ref} = f_{\phi'_{ref}} \left(\hat{v}_x, \hat{v}_y, \hat{v}_z, \theta'_{ref} \right)$$

= $\sin^{-1} \left(\frac{-\hat{v}_y + a_y - r\hat{v}_x + p\hat{v}_z}{g\cos\theta'_{ref}} \right)$ (34)
 $\theta'_{ref} = f_{\theta'_{ref}} \left(\hat{v}_x, \hat{v}_y, \hat{v}_z \right)$

$$=\sin^{-1}\left(\frac{\dot{\hat{v}}_x - a_x - r\hat{v}_y + q\hat{v}_z}{g}\right)$$
(35)

Here, the velocities are estimated using the kinematic filter with multiple lateral velocity references from the cornering-stiffness adaptation in equation (17), the model-based observer in equation (32) and the kinematic observer in equation (33). For further details, see the paper by Oh and Choi.²⁹

Reference angle obtained by pseudointegration. The kinematic equations that express the time derivatives of the Euler angles are given by

$$\dot{\phi} = p + (q\sin\phi + r\cos\phi)\tan\theta$$
 (36a)

$$\dot{\theta} = q\cos\phi - r\sin\phi \tag{36b}$$

These are the foundation on which the vehicle roll and pitch angle pseudointegration is performed. Here, the yaw angle differential equation is excluded, since estimation of the vehicle yaw angle with respect to the global coordinate system is not within the scope of this research.

Before proceeding, the transient flag index TF must be defined. This variable is a normalized value between 0 and 1 obtained by summing the scaled variances of the 6D IMU signals. A high TF indicates that the vehicle is going through a transient state in its motion, whereas a low TF corresponds to the steady state.

Using the above, the pseudointegral angle estimation can be described in terms of the system

$$\hat{\phi}_{int}^* = p + \left(q\sin\hat{\phi}_{int} + r\cos\hat{\phi}_{int}\right)\tan\hat{\theta}_{int}$$
(37)

$$\hat{\theta}_{int}^{*} = q \cos \hat{\phi}_{int} - r \sin \hat{\phi}_{int}$$

$$\hat{\phi}_{int} = \text{TF} \ \hat{\phi}_{int}^{*}$$
(38)

+
$$(1 - \text{TF})f_{\phi'_{ref}}(v_{car}, \hat{v}_{y, bic}, 0, f_{\theta'_{ref}}(v_{car}, \hat{v}_{y, bic}, 0))$$

(39)

$$\hat{\theta}_{\text{int}} = \text{TF} \ \hat{\theta}_{\text{int}}^* + (1 - \text{TF}) f_{\theta'_{ref}} (v_{car}, \hat{v}_{y, bic}, 0)$$
(40)

Through obtaining the pseudointegral estimation of the angles according to the above-mentioned system, it calculates $\hat{\phi}_{int}$ and $\hat{\theta}_{int}$ via pure integration only when TF is close to 1. The integrated angles tend to return immediately back to the stable reference values $f_{\phi'_{ref}}(v_{car}, \hat{v}_{y,bic}, 0)$ and $f_{\theta'_{ref}}(v_{car}, \hat{v}_{y,bic}, 0)$ to avoid the drift issue whenever TF ≈ 0 and integration is not necessary. It must be noted that $f_{\phi'_{ref}}$ and $f_{\theta'_{ref}}$ mentioned here in the pseudointegral estimation part are not exactly identical with those defined in equations (34) and (35). While the reference angles in equations, (34) and (35) are functions of the final velocity estimations, those in equations (39) and (40) are selected to be functions of v_{car} and $\hat{v}_{y,bic}$. This selection is sensible, since a low TF guarantees that v_{car} and $\hat{v}_{y,bic}$ are reliable data.

Reference angle formation. The conventional attitude observer designed by Tseng et al.²³ chooses to use equations (34) and (35) alone as the reference feedback for the attitude observers, neglecting the lateral and vertical velocity components. This causes estimation inaccuracy under severe manoeuvring conditions. Also, it requires that the vehicle yaw rate satisfies the non-zero condition, in order to utilize the reference angle information ϕ'_{ref} and θ'_{ref} effectively, since the observer discards the feedback term when the yaw rate is nearly zero. However, it is difficult to meet this condition, because the vehicle does not always go through transient states. The second part of the paper by Tseng et al.²³ deals with the "attitude observer without steady



Figure 3. RS acquisition. RS: reference selector.

state error" which does not rely on the reference angle to estimate the vehicle attitudes during the steady state. This, however, is possible only with angular rate sensors of ideal accuracy.

This predicament is resolved in the novel attitude observer by using information on the newly designed reference angle during both the transient state and the steady state. The reference angles of the proposed work consist of both components from the pseudointegral estimation of the roll and pitch angles and the reference values obtained by velocity kinematics in equations (34) and (35) by adjusting the reference selector (RS) between them. RS is defined as

$$\mathbf{RS} = \operatorname{sat}\left\{ \max\left[\frac{1}{2\varepsilon_{t}} \left(C_{f} - \Gamma_{f}F_{zf} + \varepsilon_{t}\right), \frac{1}{2\varepsilon_{t}}\right] \right\}$$

$$(C_{r} - \Gamma_{r}F_{zr} + \varepsilon_{t}) \right\}$$
(41)

Figure 3 illustrates how the RS is formulated as a function of the cornering stiffnesses. When either C_f or C_r is in the linear region, indirect use of the reference lateral velocity $\hat{v}_{v,ref}$ by taking the velocity estimations as the reference is reliable. However, when the tyres show non-linear characteristics, maintaining such indirect use of the reference lateral velocity for the angle estimation causes an unnecessary involvement of the pseudointegral velocity estimation with all six sensor measurements and their possible errors, because velocity estimation by integration fully requires the 6D IMU. Hence, in this case, it is wiser rather to use the pseudointegral estimation of the angles only, which takes only three sensor measurements of the gyroscope. This shift thus naturally causes the observer to be less sensitive to the sensor errors. This is enabled by decreasing the RS which increases the portion of $\hat{\phi}_{int}$ and θ_{int} in the reference angles.

Incorporating the RS into the observer design as explained, the two references designed in equations (34), (35), (39) and (40) are merged according to

the velocity estimation errors and the use of all six IMU sensors, and so it is favourable to discard its result when the vehicle exhibits highly non-linear tyre dynamics.

Figures 5 and 6 show the cornering-stiffness adaptation results and calculation of the RS respectively, and they indeed show that the reference selection process is designed to use the reference selectively with higher reliability. While the vehicle experiences a severe double-lane-change manoeuvre, the cornering-stiffness estimations fall, which then cause the RS to drop in accordance. As intended, this happens only during the time period when the tyres experience non-linear characteristics so that, for the generation of the observer reference feedback, more emphasis can be laid on the pseudointegration than the use of velocity kinematics which requires an increased burden on the corneringstiffness adaptation for accurate velocity tracking.

Considering these, the tactic of using the RS as a function of the estimated cornering stiffnesses to obtain the reference angles is indeed effective.

Principal kinematic angle observer. The principal kinematic angle observer forms the last part of the integrated vehicle angle estimation process. This scheme improves the estimation performance of the high-frequency components of the estimation targets especially in their transient states. Also, it eliminates the noise in the reference angle formed in the previous section without causing a phase lag in the signal.

With the modified reference angles designed in equations (42) and (43), the attitude observer is designed as

$$\dot{\hat{\phi}} = p + \left(q\sin\hat{\phi} + r\cos\hat{\phi}\right)\tan\hat{\theta} + \alpha_r \left(\phi_{ref} - \hat{\phi}\right)$$
(44)

$$\hat{\theta} = q\cos\hat{\phi} - r\sin\hat{\phi} + \alpha_p \left(\theta_{ref} - \hat{\theta}\right)$$
(45)

where α_r and α_p are positive tuning constants.



Figure 6. Calculation of the RS (low pass filtered).



Figure 4. Roll and pitch reference angles (low pass filtered).

$$\phi_{ref} = \operatorname{RS} f_{\phi'_{ref}}(\hat{v}_x, \hat{v}_y, \hat{v}_z, \theta'_{ref}) + (1 - \operatorname{RS})\hat{\phi}_{int} \qquad (42)$$

$$\theta_{ref} = \mathbf{RS} f_{\theta'_{ref}} (\hat{v}_x, \hat{v}_y, \hat{v}_z) + (1 - \mathbf{RS}) \hat{\theta}_{int}$$
(43)

Two reference angles obtained from the velocity kinematics and pseudointegration are compared in Figure 4. As expected, the estimation obtained by pseudointegration involves an instantaneous drift issue during the transient state and a discrete change in the signal as the TF changes. However, its accuracy and ability to track the rapidly changing estimation targets are useful. On the other hand, although the estimation acquired using the velocity kinematics compromises high bandwidth accuracy, it constantly tracks the actual estimation target without a drift issue when the cornering stiffnesses do not deviate from their nominal values (i.e. when the linear bicycle model suffices to describe the vehicle dynamics). As mentioned earlier, inaccuracy during a severe vehicle manoeuvre may be caused by



Figure 5. Cornering-stiffness adaptation result (low pass filtered).



Figure 7. Roll and pitch reference angles and estimation results (low pass filtered).

The estimation results for a double-lane-change case are shown in Figure 7. As expected, the principal kinematic angle observer gives the final estimation through effectively filtering the inaccuracy and noise mixed in the reference angles without causing any phase lag issue. At the same time, the reference angles contribute to repressing the signal drift involved in the kinematics. Further evaluation under various cases is dealt with in the following experiment section.

Experiment results

Test environments

Using a real production sport utility vehicle, Hyundai Tucsan ix, experiments are conducted to show the estimation performance of the suggested vehicle attitude observer. The specification data of the car used for experiment are given in Table 1.

The gyroscope and accelerometer used for this angle observer algorithm are ADW22307 and ADXL103

respectively from Analog Devices, Inc. Also, for test verification purposes, a high-accuracy GPS-inertial navigation system (INS) RT3100 from Oxford Technical Solutions Ltd is used. Figure 8 shows this instrument mounted on the test vehicle.

The 6D IMU is mounted at the CG of the vehicle, and the RT3100 sensor according to Table 2. Here, the RT3100 sensor is internally adjusted so that it gives the sensor measurements at the CG.

In Table 3, the list of conditions in which the vehicle attitude observer is tested is given. Each case involves a rigorous change in the vehicle dynamics, and/or fairly high value(s) of the roll and/or pitch angles. By conducting the experiment in various types of situation, the robustness of the proposed attitude observer is verified.

Test results and analysis

In Figure 9, the roll and pitch angle estimation results for the severe double-lane-change manoeuvres are displayed. While trying to maintain a vehicle velocity of 70 km/h, three sets of manoeuvres are made to show the estimation robustness in both the transient state and the steady state during and between each manoeuvre.

Observation of Figure 9 reveals that the proposed attitude observer effectively estimates the roll and pitch angles, even in the condition when the vehicle is going through a continued series of severe lateral accelerations through double lane changes.

The circle turn manoeuvre induces a constant roll angle of the vehicle with lateral weight shifting. The observer effectively handles the situation well, and both the roll angle and the pitch angle are estimated with fairly high accuracy. The test results are displayed in Figure 10.

In the bumpy road experiment, the ability of the observer to estimate the rapidly changing roll and pitch angles is tested. While the road condition exerts a sinusoidal excitation on the tyres, the roll and pitch angles

Table 1. Specifications for the Tucsan ix 2WD gasoline Theta II 2.0 test vehicle.

| Feature (units) | | Front | | Rear | |
|-------------------|-----------------------------|-------|-------|------|-------|
| | | Left | Right | Left | Right |
| Dimension (mm) | Wheelbase | 2640 | | | |
| | Overhang | 800 | | 890 | |
| | Track | 1585 | | 1586 | |
| | Overall Length | 4410 | | | |
| | Overall Width | 1820 | | | |
| | Height (unloaded) | 1655 | | | |
| Weight (kgf) | Kerb weight | 450 | 417 | 326 | 330 |
| 8 (8) | 3 | 867 | | 656 | |
| | | 1523 | | | |
| | Gross vehicle weight (2 up) | 487 | 458 | 360 | 368 |
| | 5 (1) | 945 | | 728 | |
| | | 1673 | | | |
| Wheel radius (mm) | | 336 | 338 | 340 | 340 |
| | | 337 | | 340 | |

Figure 8. Oxford Technical Solutions Ltd RT3100 GPS-INS mounted on the test vehicle. GPS: Gobal Positioning System; INS: inertial navigation system.

Table 2. Mounting positions of the instruments.

| Distance between instruments | | Distance (mm) | |
|------------------------------|-------------------------|---------------|--|
| Length (x axis) | RT3100–rear axle center | 1000 | |
| U () | RT3100-6D IMU | 930 | |
| Height (z axis) | RT3100–antenna | 600 | |
| , | RT3100-6D IMU | 430 | |

6D: six-dimensional; IMU: inertial measurement unit.

vigorously change. As Figure 11 shows, the observer algorithm effectively estimates the vehicle attitudes even during this extended period of the transient state.

The next test scenario, the results of which are illustrated in Figure 12, focuses on a sudden steering input. A significant amount of side-slip is induced by intentionally causing the vehicle to spin out, or to be oversteered, on the surface with a low friction coefficient. Up to a certain level of severity in the lateral dynamics, the reference angle obtained from velocity kinematics effectively estimates the angles.

However, in the presence of extremely high side-slip such as in case IV, the velocities, too, are obtained by integration, which give rise to reference angle inaccuracy, worsened by sensor errors. In this case of high side-slip and highly non-linear tyre characteristics, the attitude observer effectively discards the information obtained from the velocity estimations and rather exploits pseudointegration for accurate attitude estimation; its effectiveness can be seen in the result.

In this special case of high slip, the absence of such tactics to avoid error and ignoring the lateral dynamics, or even merely using the bicycle-model-based lateral dynamics information obtained with fixed cornering stiffnesses, may cause a significant amount of error in the estimation performance of the previous study,²³ and even divergence from the measured estimation target, as shown in Figure 12.

The fifth scenario (case V), the results of which are shown in Figure 13, deals with the condition in which all the factors that may cause the attitude observation accuracy to deteriorate are combined. Here, the vehicle experiences a constant turn at the same time as a severe static bank angle and a severe sine steering input. Even in this harsh condition, the proposed observer accurately estimates the angles, whose magnitudes increase to over 20°.

The comparison between the roll and pitch angle estimation performances of the conventional observer designed and described in the paper by Tseng et al.²³ and that of the newly designed attitude observer is given in Figure 14. This test for comparison is conducted on wet asphalt, on which the vehicle is driven downhill which involves a bank angle and speed bumps, in order to see the effect of changing vehicle orientation on the estimation performance over an extended period of time.

If the conventional observer is made to rely more on the reference signals, it certainly drifts less but loses its estimation accuracy during the transient state. Relatively free from this trade-off, the new angle observer manages the drift issue better without losing the transient state estimation accuracy, as shown in the plot. This is accounted for by incorporation of the velocity estimation into the reference angle generation. Furthermore, as mentioned above, the non-zero yaw rate requirement utilized by Tseng et al.²³ causes a drift issue, and thus a steady state error, when the yaw rate is near zero.

| Table 3. | Test | scenarios. |
|----------|------|------------|
|----------|------|------------|

| Case | Driver control | Bank | Incline | Road condition |
|----------|----------------------|---|---|-----------------|
| Case I | DLC | None | None | Dry asphalt |
| Case II | Circle turn | None | None | Wet asphalt |
| Case III | Bumpy road | Sinusoidal | Sinusoidal | Dry asphalt |
| Case IV | Spin out | None | None | Snow and/or ice |
| Case V | Bank turn sine steer | $0^{\circ} \rightarrow 20^{\circ} \rightarrow 0^{\circ} \rightarrow 20^{\circ} \rightarrow 0^{\circ}$ | Irregular (depends on the vehicle position) | Dry asphalt |
| Case VI | Lane keeping | Random | Random | Wet asphalt |





Figure 9. Test results for the DLC test on dry asphalt (case I): (a) vehicle states; (b) vehicle roll angle estimation; (c) vehicle pitch angle estimation.

SWA: steering-wheel angle.



Figure 10. Test results for a circle turn test on wet asphalt (case II): (a) vehicle states; (b) vehicle roll angle estimation; (c) vehicle pitch angle estimation. SWA: steering-wheel angle.

Conclusion

Without having to use the GPS or a high-cost INS, the suggested attitude observer has proved its worth through effectively estimating the vehicle roll and pitch angles dynamically in both the transient state and the steady state by using the low-cost 6D IMU and the reference angle obtained from the stable velocity observer. In doing so, a novel method to generate the reference angle, to combine their results effectively as a function of cornering-stiffness estimation and to use it in the kinematic observer is suggested.



Figure 11. Test results for a bumpy road test on dry asphalt (case III): (a) vehicle states; (b) vehicle roll angle estimation; (c) vehicle pitch angle estimation.

SWA: steering-wheel angle.



Figure 12. Test results for a spin-out test on snow (case IV): (a) vehicle states; (b) vehicle roll angle estimation; (c) vehicle pitch angle estimation.

SWA: steering-wheel angle.

Summarizing the paper, the original contributions distinguished from the previously reported papers are the following: elimination of the need for GPS, elimination of the signal drift issue that exists over an extended period and resolution of the inaccuracy issue during severe vehicle manoeuvres and high slips. With numerous real-car-based experiments, the proposed attitude estimation performance is tested and is verified to be robust. The experiments are conducted under various conditions involving rigorous manoeuvres, so that the suggested algorithm is ready for actual production car application.



Figure 13. Test results for a bank turn sine steer test on dry asphalt (case V): (a) vehicle states; (b) vehicle roll angle estimation; (c) vehicle pitch angle estimation.

SWA: steering-wheel angle.



Figure 14. Test results for lane keeping on irregular terrain (case VI): (a) vehicle states; (b) vehicle roll angle estimation; (c) vehicle pitch angle estimation.

SWA: steering-wheel angle.

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Appendix

Notation

| a_x | longitudinal acceleration measured at the |
|----------------|---|
| a | lateral acceleration measured at the centre |
| u_y | of gravity |
| a | vertical acceleration measured at the |
| u_Z | centre of gravity |
| C | front-type cornering stiffness |
| C_f | rear-tyre cornering stiffness |
| E_r | front-tyre lateral force |
| F | rear-tyre lateral force |
| F c | front-type lateral force |
| F | rear-tyre vertical force |
| g | acceleration due to gravity |
| 8 Iz | moment of inertia about the z axis |
| le | distance between the centre of gravity and |
| 5 | the front axle |
| l_r | distance between the centre of gravity and |
| | the rear axle |
| L | distance between the front axle and the |
| | rear axle |
| т | mass of the vehicle |
| р | roll rate measured at the centre of gravity |
| q | pitch rate measured at the centre of |
| | gravity |
| r | yaw rate measured at the centre of gravity |
| V _X | longitudinal velocity at the centre of |
| | gravity |
| v_y | lateral velocity at the centre of gravity |
| V_Z | vertical velocity at the centre of gravity |
| α_f | front-tyre slip angle |
| α_r | rear-tyre slip angle |
| β | side-slip angle at the centre of gravity |
| δ_f | front-tyre steering angle |
| θ | pitch angle |
| ϕ | roll angle |