# Precision Motion Control Based on a Periodic Adaptive Disturbance Observer

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*Abstract*—This paper proposes a periodic adaptive disturbance observer (PADOB) for a precision position control of a PMLSM (Permanent Magnet Linear Synchronous Motor), which is based on a periodic adaptive learning control(PALC). It updates the output of a classical linear DOB to compensate modeling errors between a nominal plant and actual plant and external disturbance forces such as the friction and detent force. Therefore, it can improve problems occured by inaccurate parameters of the nominal plant and the instability problem occurred by updating parameters of nominal model in the DOB directly. Also, the complicated procedures to design the initial conditions in PALC are not needed. Through simulation test and experiments of the PMLSM, the validity of PADOB is illustrated.

# I. INTRODUCTION

Recently, the precision control has been widely required for indutrial products such as semiconductor manufacturing equipment, LCD pannel transportation equipment, and X-Y gantry devices. A Permanent Magnet Linear Synchronous Motor (PMLSM) has been used for such works required accurate position or speed control. Unlike screw-type rotary motors, indirect mechanical transmissions such as chains, gear boxes and screw coupling are eliminated, and the effects of contact-types of nonlinearities and disturbances such as backlash and coupling frictional forces are reduced. These advantages make possible to achieve high-speed/high-accuracy positioning control. However, other nonlinear effects such as the detent and friction force must be compensated to establish high precision motion control. The detent force is occurred by the interaction between the permanent magnets and the steel teeth of the primary section. It is a position-dependent periodic disturbance and the pitch of permanent magnets becomes a period of the detent force.[1] The friction force is a velocitydependent nonlinear disturbance.

Much efforts have been devoted to compensate these disturbances.[2] The numerical models of detent and friction force have been used to eliminate the effect of these disturbances. In [3], a simple sinusoidal model and high-order harmonic models were used to describe the detent force. The amplitude and phase of the model were updated by a certain adaptation scheme. In [4] and [5], the static and dynamic friction models such as Coulomb and Lugre friction model were used to compensate the actual friction force. Here, the friction coefficients are estimated to obtain the actual friction

model. However, these models are not enough to describe the actual detent and friction force. Also, it is difficult to guarantee asymptotically convergence of estimated parameters because there exist several parameters to be estimated and all operationg conditions are always not satisfied with the persistance of excitation conditions.[6] The disturbance observer (DOB) is another compensation scheme that the numerical models of these disturbance are not required. It uses only the nominal model of the used actuator.[7] The difference between the output of nominal plant and measured output is considered as lumped disturbance. However, it is very dependent on parameters of the nominal model because there exist parametric errors between the used nominal model and actual plant. Therefore, above mentioned schemes are extremely constrictive to the used model. To improve these problems, the periodic adaptive learning control (PALC) has been proposed as the learning control strategies without the model.[8][9][10] Adaptive learning compensators of these disturbances in PMLM were designed in [8]. The key idea is to use the state-periodic characteristics of these disturbances, which are a position-dependent detent force and a velocitydependent friction force. The past information of more than one period along the state axis is used to update the current adaptation learning law. In order words, the initial conditions of disturbances for one period must be designed. Therefore, it needs the complicated design procedure of initial condition using certain adaptation schemes or nonlinear functions.

In this paper, a periodic adaptive disturbance observer (PADOB) is proposed. The output of the classical linear DOB is used as initial conditions in adaptation laws. It eliminates complicated procedure to design initial conditions in PALC. Also, it compensates not only the parametric error of used nominal plant but also external disturbances such as detent and friction force. And it improves the instability problem occurred by updating parameters of nominal model directly in the DOB.

This paper is organized as follows: In Section II, the used PMLSM model and control problem are introduced. In Section III, the proposed PADOB are presented. In Section IV, the performance of PADOB is verified by the simulation and experiment illustration. Concluding remarks are given in Section V.



Fig. 1. Permanent Magnet Linear Synchronous Motor

## **II. PROBLEM FORMULATION**

In this section, the PMLSM model and control problem are formulated. Also, the properties of used PALC are presented.

# A. PMLSM Model

The used PMLSM is depicted in Fig. 1 and it is represented by the dynamic equation as follows, [8]

$$m\ddot{x}(t) = -\frac{k_f k_e}{R} \dot{x}(t) + \frac{k_f}{R} u_v(t) - F_{ext}$$
(1)

where, x is the position of the primary section, m the mass of the primary section,  $k_f$  the force constant,  $k_e$  the backemf constant, R the resistance,  $u_v(t)$  the input voltage,  $F_{ext}$  the detent and friction force. For simplicity, the load and small external disturbance are ignored.

The control objective is to track the given desired position,  $x_d$  and the corresponding desired velocity,  $\dot{x}_d$  with tracking error as small as possible. The performed task is assumed as follows:

Assumption 1: The task is performed on the periodic trajectory under same conditions repetitively.

Assumption 2: The disturbance effects for each period are nearly identical because of good tracking performance.

Assumption 3: The measurement noise and high frequency disturbance are attenuated by low pass filters.

Suppose that the external disturbance force such as the detent and friction force are described as follows,

$$F_{det}(x) = a(x), \quad F_{fric}(\dot{x}) = b(t)sgn(\dot{x})$$

The equation of the PMLSM, (1) is rewritten as follows:

$$M\ddot{x}(t) = -B\dot{x}(t) + u(t) - \tau(t)$$
<sup>(2)</sup>

where *M* is the mass of the primary section (M := m), u(t) the control input  $(u(t) := k_f/R \cdot v(t))$ , and B defines as the viscous friction coefficient  $(B := k_f k_e/R)$ , and  $\tau(t)$  the lumped disturbance  $(\tau = a(x) - b(t)sgn(\dot{x}))$ .

# B. Properties of PALC

In this section, the defined properties of PALC are introduced, which are applied to the proposed PADOB.

Property 1: (Total pass trajectory)

The total passed trajectory is given as follows,

$$s(t) = \int_0^t \frac{|dx|}{dr} dr = \int_0^t |v(r)| dr$$

where x is the position, and v the velocity. Since s is the summation of absolute position increase along the time axis,



Fig. 2. Classical linear DOB structure

*s* is a monotonously growing signal. Physically it is the total passed trajectory; hence, it has the following property:

$$s(t_1) \ge s(t_2), \quad iff \quad t_1 \ge t_2$$

Property 2: (Trajectory periodicity)

From Assumption 1, the passed position has a periodicity on the total passed trajectory:

$$x(t) = s(t) - m's_p$$

where x is the position,  $s_p$  the moving distance for a period and m' the integer part of  $s(t)/s_p$ . When it assumes that the tracking performance is good, the position and velocity can be presented in time-domain as follows,

$$x(s) = x(s - s_p) \quad \rightarrow \quad x(t) = x(t - P_t), \quad \dot{x}(t) = \dot{x}(t - P_t)$$

where,  $P_t$  is a time period of performed task.

Property 3: (Disturbance periodicity)

From Assumption 2 and Property 2, the lumped disturbance has the following properties:

$$\tau(t) = \tau(t - P_t)$$

## III. PERIODIC ADAPTIVE DISTURBANCE OBSERVER

#### A. Disturbance Observer

The DOB has been proposed to eliminate the disturbance which is difference between the actual system and nominal model. The nominal model represents the desired model based on the desired control specifications. It makes the actual system become a given nominal model. Fig. 2 depicts the classical linear DOB structure. The output of DOB,  $\hat{\tau}(t)$ is an estimated disturbance which consists of the parametric errors between the actual plant and the nominal model of DOB and disturbance forces such as friction and detent force. The symbols in Fig. 2 are defined as follows,

x(t) and  $\dot{x}(t)$ : the position and velocity of mover

P(s): the transfer function of actual plant

 $P_n(s)$ : the transfer function of nominal model

u(t): the control input

d(t): the disturbance including detent and friction force Q(s): a low-pass filter

- $\tau(t)$ : the estimated disturbance without filtering
- $\hat{\tau}(t)$ : the estimated disturbance with a low-pass filter
- $\zeta(t)$ : the sensor noise

The transfer functions and disturbance are presented as follows,

$$P(s) = 1/(Ms + B) \tag{3}$$

$$P_n(s) = 1/(M_n s + B_n) \tag{4}$$

$$d(t) = a(x) + b(t)sgn(\dot{x})$$
(5)

where  $M_n$  and  $B_n$  are nominal values of the nominal model. The output of the plant, x(t) is represented by

$$x(t) = \frac{PP_n}{D}u(t) + \frac{PP_n(1-Q)}{D}d(t) - \frac{PQ}{D}\zeta(t)$$
(6)

where  $D = P_n + (P - P_n)Q$ 

Assume that the transfer functions in (6) are stable. In the low frequency range(i.e.,  $Q(j\omega) \approx 1$ ), the output  $x(j\omega)$ becomes similar to  $P_n(j\omega)u(j\omega) - \zeta(j\omega)$ , but from Assumption  $3(\zeta \approx 0)$ , we have the nominal input-output relation, i.e.,  $x(j\omega) \approx P_n(j\omega)u(j\omega)$ , which is desired. More detailed DOBs are presented in numerous literatures.[11][12]

### B. PADOB

The DOB has been known as a powerful scheme as referred to numerous literatures. However, in case that parameters of nominal model are inaccurate, the performance can be worse. Therefore, the information of nominal model must be updated. To update parameters of the nominal model directly can occur instability problems because poles and zeros of the transfer function of nominal model are changed. This paper has focused on updating not parameters of nominal model, *M* and *B* but output of DOB,  $\tau$ (t).

1) Lumped Disturbance: The lumped disturbance is calculated by output of DOB, which is used for initial conditions of the periodic adaptation mechanism. In Fig. 2, the estimated disturbance without filtering can be presented as follows,

$$\tau(t) = u(t) - \frac{1}{P_n(s)}(x(t) + \zeta(t))$$
(7)

For simplicity, variables in time-domain and s-domains are used simultaneously. Substituting (4) into (7),

$$\tau(t) = u(t) - M_n \ddot{x}(t) - B_n \dot{x}(t)$$
(8)

where  $\zeta = 0$  by Assumption 3.

And, the output of the transfer function of the actual plant is presented as follows,

$$x(t) = \frac{1}{Ms^2 + Bs} \left( u(t) - d(t) \right)$$
(9)

Rearranging (9), the control input, u(t) is rewritten as follows,

$$u(t) = d(t) + M\ddot{x}(t) + B\dot{x}(t)$$
(10)



Fig. 3. Periodic Adaptive DOB structure

Substituting (5) and (10) into (8), the estimated disturbance can be presented as follows,

$$\tau(t) = \Delta M\ddot{x}(t) + \Delta B\dot{x}(t) + a(x) + b(t)sgn(\dot{x})$$
(11)

where, the inertia error is  $\Delta M = M - M_n$  and the viscous friction coefficient error is  $\Delta B = B - B_n$ . The modeled disturbance is satisfied with property 2 and 3 in Section II-B.

2) Periodic Adaptation Mechanism and Controller: The structure of PADOB is described in Fig. 3. The system is controlled by following two step:

- When *s* < 2*s*<sub>*p*</sub>, the system is controlled by using the classical linear DOB to be bounded input bounded output(BIBO).
- When  $s \ge 2s_p$ , the system is controlled by the proposed PADOB. By the periodic adaptation mechanism, the unknown disturbances are estimated.

First, consider the case when  $s < 2s_p$ . The control law is designed as follows:

$$u_{1}(t) = u_{fb_{1}}(t) + u_{ff_{1}}(t)$$

$$u_{fb_{1}}(t) = -K_{s_{1}}S(t) - M_{n}\lambda_{1}\dot{e}_{x}(t) - B_{n}\lambda_{2}e_{x} + \tau_{0}(t)$$

$$u_{ff_{1}}(t) = M_{n}\ddot{x}_{d}(t) + B_{n}\dot{x}_{d}(t)$$
(12)

where

$$S(t) = \dot{e}_x(t) + \lambda_1 e_x(t) + \lambda_2 \int e_x(t)dt, \quad e_x(t) = x(t) - x_d(t)$$

 $x_d$  is the desired position,  $K_{s_1}$ ,  $\lambda_1$  and  $\lambda_2$  are tuning parameter ( $K_{s_1} > 0$ ,  $\lambda_1 > 0$  and  $\lambda_2 > 0$ ). And  $\tau_0$  is the output of DOB. Substituting (12) into (2) and differentiating it, the closed-

loop error dynamics are obtained as follows,

$$M\ddot{e}_{x}(t) + B\dot{e}_{x}(t) + M_{n}\lambda_{1}\dot{e}_{x}(t) + B_{n}\lambda_{2}e_{x}(t) + K_{s_{1}}\dot{S}(t)$$
  
= -[\Delta M\vec{x}\_{d}(t) + \Delta B\vec{x}\_{d}(t) + \Delta \tau(t)] (13)

In (13),  $\Delta \tau(t) = \tau_0(t) - a(x) - b(t)sgn(\dot{x})$ . And the desired trajectories,  $x_d(t)$ ,  $\dot{x}_d(t)$ , and  $\ddot{x}_d(t)$  are bounded in practice. Also, assuming that  $|\Delta M|$ ,  $|\Delta B|$ , and  $|\Delta \tau|$  are all bounded, the system satisfies the BIBO stable.

Next, consider the case when  $s \ge 2s_p$ . The designed control v law is as follows,

$$u_{2}(t) = u_{fb_{2}}(t) + u_{ff_{2}}(t) + u_{PADOB}(t)$$
(14)  

$$u_{fb_{2}}(t) = -K_{s_{2}}S(t) - M_{n}(\lambda_{1}\dot{e}_{x}(t) + \lambda_{2}e_{x}(t)) + B_{n}\dot{x}(t)$$
  

$$u_{ff_{2}}(t) = M_{n}\ddot{x}_{d}(t)$$
  

$$u_{PADOB}(t) = \hat{\tau}(t)$$

where  $K_{s_2}$  is a tuning parameter ( $K_{s_2} > 0$ ).

And, the adaptation laws of  $u_{PADOB}(t)$  are designed as follows,

$$\hat{\tau}(t) = \begin{cases} \hat{\tau}(t-P_t) - K_a S(t) & if \quad s \ge 2 \cdot s_p \\ \tau_0(t) & if \quad 0 < s < 2 \cdot s_p \end{cases}$$
(15)

Here,  $K_a$  is adaptation gains ( $K_a > 0$ ).

The initial conditions of the disturbances to be estimated,  $\tau_0(t)$  is obtained by the output of DOB, The control input (14) and adaptation laws (15) guarantee that the system is asymptotically stable. The Lyapunov stability analysis is performed to prove it.

Consider the following positive Lyapunov candidate function at s(t), whose corresponding time is t:

$$V(s(t)) = V(t) = \frac{1}{2}S(t)^2 + \frac{1}{2K_a M_n} \int_{t-P_t}^t e_{\tau}^2(r) dr \qquad (16)$$

where

$$e_{\tau}(t) = \tau(t) - \hat{\tau}(t)$$

Then, the difference of the positive Lyapunov candidate functions at two discrete time point (t and  $t - P_t$ ) can be calculated as:

$$\Delta V = V(t) - V(t - P_t) = \frac{1}{2}S^2(t) - \frac{1}{2}S^2(t - P_t) + \frac{1}{2K_a M_n} \int_{t-P_t}^t \left[e_\tau^2(r) - e_\tau^2(r - P_t)\right] dr$$
(17)

For simplicity, let the first term on the right hand side be denoted by A and the second integral term by B. Then, A is calculated as follows,

$$A = \frac{1}{2}S^{2}(t) - \frac{1}{2}S^{2}(t - P_{t}) = \int_{P_{t}}^{t} S(r)\dot{S}(r)dr \qquad (18)$$
$$= \int_{t - P_{t}}^{t} \frac{1}{M_{n}} \left[ -e_{\tau}(r)S(r) - K_{s_{2}}S^{2}(r) \right]dr$$

For a proof, see the appendix.

Using Property 1 and 3, B can be calculated as follows,

$$B = \frac{1}{2K_{a}M_{n}} \int_{t-P_{l}}^{t} \left[ e_{\tau}^{2}(r) - e_{\tau}^{2}(r-P_{l}) \right] dr$$
  

$$= \frac{1}{2K_{a}M_{n}} \int_{t-P_{l}}^{t} \left[ \tau(r) - \hat{\tau}(r) \right]^{2} - \left[ \tau(r-P_{l}) - \hat{\tau}(r-P_{l}) \right]^{2} dr$$
  

$$= \frac{1}{2K_{a}M_{n}} \int_{t-P_{l}}^{t} \left[ \hat{\tau}(r-P_{r} - \hat{\tau}(r)) \right] \cdot \left\{ 2 \left[ \tau(r) - \hat{\tau}(r) \right] + \left[ \hat{\tau}(r) - \hat{\tau}(r-P_{\tau}) \right] \right\} dr$$
  

$$= \frac{1}{2K_{a}M_{n}} \int_{t-P_{l}}^{t} \alpha(r) \cdot \left\{ 2e_{\tau}(r) - \alpha(r) \right\} dr$$
(19)

where,

$$\alpha(r) = \hat{\tau}(r - P_r) - \hat{\tau}(r)$$

Substituting (14) and (15) into (18) and (19), (17) is rewritten as follows,

$$\Delta V = A + B$$
  
=  $\int_{t-P_t}^{t} -\frac{1}{M_n} K_{s_2} S^2(r) - \frac{K_a}{2M_n} S^2(r) dr$   
=  $\int_{t-P_t}^{t} -\frac{1}{M_n} \left( K_{s_2} + \frac{K_a}{2} \right) S^2(r) dr$  (20)

The difference of the Lyapunov candidate function becomes  $\Delta V(t) \leq 0$ . From LaSalle's invariant set theorem, the asymptotical stability is proved. From (20), only S = 0 makes  $\Delta V = 0$ . Using the  $S(t) = \dot{e}_x + \lambda e_x$ , if  $e_x(0) = 0$ , only  $e_x = 0$  makes S = 0. Also, since  $e_x = 0$ , we have  $\dot{e}_x = 0$ . Therefore,  $e_x$  and  $\dot{e}_x$  are asymptotically stable at equilibrium points.

## C. Gain Design

The gains of the proposed PADOB controller are obtained by follow relation,

$$\dot{S}(t) = -K_{s_1}S(t) \tag{21}$$

Therefore, the error dynamics is as follows,

$$\ddot{e}_x + (\lambda_1 + K_{s_1})\ddot{e}_x + (\lambda_2 + K_{s_1}\lambda_1)\dot{e}_x + K_{s_1}\lambda_2e_x = 0$$
 (22)

For simplicity of gain tuning process, all poles are designed to be same value. When the desired pole sets p, the gains are as follows,

$$K_{s_1} = p, \qquad \lambda_1 = 2p, \qquad \lambda_2 = p^2 \tag{23}$$

The gain values must be selected by the bandwidth of used PMLSM.

# IV. SIMULATION AND EXPERIMENT ILLUSTRATIONS

# A. Simulation

The simulation test has been performed to verify the performance of PADOB. The result is compared with one of PI controller with a classical linear DOB.

For the simulation test, the following reference trajectory is used:

$$x_d(t) = 0.1 - 0.1 \cdot \cos(\pi t)$$

The friction force is modelled as follows:

$$F_{fric}(\dot{x}) = \left\{ 10 + 10e^{(-\dot{x}/0.1)^2)} + |\dot{x}| \right\} sgn(\dot{x})$$

And, the detent force is modelled as follows:

$$F_{det}(x) = 4\sin(\frac{2\pi x}{T_c}) + 2\sin(\frac{4\pi x}{T_c}) + \sin(\frac{6\pi x}{T_c}) + 0.5\sin(\frac{8\pi x}{T_c}) + 0.25\sin(\frac{10\pi x}{T_c}) + 0.125\sin(\frac{12\pi x}{T_c})$$

where  $T_c$  is the pitch of the permanent magnet. The parameters of actual plant are M = 12kg, and B = 72.38N/m/s. The



Fig. 4. Case1:  $M_n = 10$  and  $B_n = 60$  (simulation)



Fig. 5. Frequency Response

simulation has been performed when there exist parametric errors of used nominal model as follows:

• 
$$M_n = 10$$
 and  $B_n = 60.00$  (Fig. 4)

In Fig 4, the tracking performance of PADOB is superior than one of PI controller with DOB. The tracking error has become smaller and smaller as time goes on because disturbances have been updated. It shows that the PADOB has a better performance than the PI controller with DOB although parametric errors exist in the nominal model. The performance of PI controller with DOB becomes worse if the control gain make be higher to reduce the tracking error because it occurs serious chattering of the control input. Therefore, simulation results show that the performance of PADOB is superior.



Fig. 6. Experiment 1

### B. Experiment

To determine an nominal model that should be used for a linear DOB, an input signal which consists of sinusoidal signals from 0.1Hz to 100Hz is applied to the PMLSM. The identified frequency response is described in Fig 5. The nominal model(red line in Fig.5) is obtained as follows,

$$P_n(s) = \frac{40}{4s^2 + 0.001s} \tag{24}$$

The first experiment is performed on the same desired trajectory as one in simulation. The position and current control execution time are 0.1*ms* and 80*us*. The resolution of used linear encoder is 1*um*. The results are shown in Fig. 6. The switching of control input by PADOB is executed at 12*sec* to show the learning effect of the proposed PADOB. After 12*sec*, the traking error becomes smaller and smaller as updating the external disturbance.

The second experiment is performed on short desired trajectory as follows,

$$x_d(t) = 0.05 - 0.05 \cdot \cos(\pi t)$$

The control input is switched at 12*sec*. The result shows that the tracking performance is higher that one of the controller using only a linear DOB.

#### V. CONCLUSION

In this paper, a periodical adaptive disturbance observer (PADOB) was developed, which uses the periodic adaptive learning control (PALC) based on the output of classical linear DOB as initial conditions of adaptation mechanicsm. The PADOB compensates not only the modeling error but

also detent and friction force through updating the output of DOB. Therefore, the complicated design procedure of initial condition of PALC were improved. From simulation and experiment results, it is shown that the PADOB provides a superior tracking performance. However, the performance of PADOB can be restricted by bandwidth of Q-filter. As future works, the effect of Q-filter (LPF) must be studied.

## APPENDIX

In this appendix, we prove (18),

$$A = \frac{1}{2}S^{2}(t) - \frac{1}{2}S^{2}(t - P_{t}) = \int_{P_{t}}^{t} S(r)\dot{S}(r)dr$$
$$= \int_{t - P_{t}}^{t} \frac{1}{M_{n}} \left[ -e_{\tau}(r)S(r) - K_{s_{1}}S^{2}(r) \right] dr$$

Here,  $\dot{S}$  is calculated as follows, (for simplicity, (t) is omitted)

$$\begin{split} \dot{S} &= \ddot{e}_x + \lambda_1 \dot{e}_x + \lambda_2 \int e_x dt = (\ddot{x} - \ddot{x}_d) + \lambda_1 \dot{e}_x + \lambda_2 e_x \\ &= \frac{1}{M} \left\{ -B\dot{x} + u - a(x) - bsgn(\dot{x}) \right\} - \ddot{x}_d + \lambda_1 \dot{e}_x \\ &= \frac{1}{M} \left\{ -B\dot{x} - K_{s_2}S - M_n\lambda_1 \dot{e}_x - M_n\lambda_2 e_x + B_n\dot{x} + M_n\ddot{x}_d \\ &+ \hat{a}(x) + (\Delta M\ddot{x} - \Delta M\ddot{x}) + \Delta M\ddot{x} + \hat{b}sgn(\dot{x}) + \Delta B\dot{x} \\ &- a(x) - bsgn(\dot{x}) \right\} - \ddot{x}_d + \lambda_1 \dot{e}_x + \lambda_2 e_x \\ &= \frac{1}{M} \left\{ -\Delta B\dot{x} - \Delta M\ddot{x} - a(x) - bsgn(\dot{x}) + \Delta M\ddot{x} - K_{s_2}S(t) \\ &+ \Delta B\dot{x} + \Delta M\ddot{x} + \hat{a}(x) + \hat{b}sgn(\dot{x}) - \Delta M\ddot{x}_d + \Delta M(\lambda_1 \dot{e}_x + \lambda_2 e_x) \right\} \\ &= \frac{1}{M} \left\{ -e_{\Delta B}\dot{x} - e_{\Delta M}\ddot{x} - e_a - e_b sgn(\dot{x}) - K_{s_2}S(t) + \Delta M\dot{S} \right\} \\ &= \frac{1}{M} \left\{ -e_{\tau} - K_{s_2}S + \Delta M\dot{S} \right\} \end{split}$$

$$(25)$$

where

$$\hat{\tau} = \hat{a}(x) + \Delta \hat{M} \ddot{x} + \hat{b} sgn(\dot{x}) + \Delta \hat{B} \dot{x}$$

$$e_{\tau} = e_{AB} \dot{x} + e_{AM} \ddot{x} + e_{a} + e_{b} sgn(\dot{x})$$

Rearranging (25),

$$(1 - \Delta M/M)\dot{S} = (1/M)\{-e_{\tau} - K_{s_2}S(t)\}$$
(26)

From (26),  $\dot{S}(t)$  can be presented as follows,

$$\dot{S} = (1/M_n) \{ e_{\tau} - K_{s_2} S(t) \}$$
(27)

Therefore, A is obtained by substituting (27) into (18).

#### ACKNOWLEDGMENT

This work was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government(MEST). (No.2012-0000991)



Fig. 7. Experiment 2

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