2012 12th International Conference on Control, Automation and Systems
Oct. 17-21, 2012 in ICC, Jeju Island, Korea

# Integrated Vehicle Mass Estimation Using Longitudinal and Roll Dynamics

Daeil Kim[1], Seibum B. Choi[2] and Jiwon Oh[3]

[1] School of Mechanical Aerospace & Systems Engineering, Division of Mechanical Engineering, KAIST, Daejeon, Korea
(Tel : +82-42-350-4160; E-mail: kemadd@kaist.ac.kr)

[2] School of Mechanical Aerospace & Systems Engineering, Division of Mechanical Engineering, KAIST, Daejeon, Korea
(Tel : +82-42-350-4160; E-mail: sbchoi@kaist.ac.kr)

[3] School of Mechanical Aerospace & Systems Engineering, Division of Mechanical Engineering, KAIST, Daejeon, Korea
(Tel : +82-42-350-4160; E-mail: jwo@kaist.ac.kr)

**Abstract**: Information of vehicle mass is crucial for vehicle safety control. Vehicle mass value is treated as a constant value for ESP(Electronic Stability Program) and other vehicle safety controls, but it varies with the number of passengers or the weight of load vehicle carries. This paper suggests an integrated mass estimation algorithm based on a recursive least square method and parameter adaptation. First, using the longitudinal dynamics and recursive least square method, the mass estimation algorithm is designed. Second, two sorts of mass estimation algorithms are designed using the roll dynamics. The first algorithm is designed using adaptation law from roll angle observer and Lyapunov stability analysis and the second algorithm is designed using recursive least square method. Then, mass estimation algorithm is finally integrated with two algorithms to estimate vehicle mass for all sorts of situations. The simulation results through CarSim and Matlab/Simulink show the performance of the proposed vehicle mass estimation algorithm.

**Keywords:** Mass estimation, recursive least square, Lyapunov stability analysis, adaptation law.

## 1. INTRODUCTION

Nowadays, vehicle stability problems have come to a very important issue. For this reason, ESP(Electronic Stability Program) has been studied and developed by many researchers and it helps the number of the vehicle accidents to be decreased. And also, a rollover prevention control method is being studied because it is very serious and recognized as the most threatening accidents.

According to the National Highway Traffic safety Administration's(NHTSA) analysis [1] from 1998 to 2009, the number of total accidents is decreased with assistance of ESP, but the number of rollover accidents has not. The percentage of rollover occurrence of total accidents is increased from 30.6% to 35.4 % from 1998 to 2009. Even though the percentage of rollover occurrence of total accidents is not very big, it has a badly large contribution to severe fatal injuries.

One important problem of vehicle dynamics controllers is that they are sensitive to vehicle parameters like vehicle mass and vehicle height of center of gravity, and these parameters are variable factors affected by the number of passengers or the weight of load vehicle carries. However, these values are treated as constants for a vehicle safety control.

For those reasons, several studies have been executed to estimate vehicle mass [2]-[6]. For example, Vahidi et al. [2] utilized the recursive least square method with multiple forgetting factors in order to estimate a vehicle mass and time-varying road grade simultaneously. And, Huh et al. [3] suggested integrated mass estimation algorithm based on the longitudinal dynamics, lateral dynamics and vertical suspension dynamics. However, these mass estimation schemes have some limitations. For example, the algorithm [2] considers only longitudinal dynamics, so only with no steering condition, vehicle mass is updated. And for the algorithm [3], there are some difficulties to estimate vehicle mass because of unknown parameter like cornering stiffness which is varied significantly when vehicle goes to nonlinear region.

In order to overcome those limitations, this paper suggests an integrated vehicle mass estimation algorithm for variable driving situations. The integrated algorithm includes two sub-estimations based on the longitudinal and roll dynamics [7]. The first algorithm is designed using the longitudinal dynamics and the recursive least square scheme [8] and the second algorithm is designed using the roll dynamics with the adaptation scheme and the recursive least square method. Finally, through multiple observer synthesis [9], two algorithms are integrated to estimate vehicle mass for all sorts of situations.

## 2. RECURSIVE LEAST SQUARE(RLS) METHOD WITH FORGETTING FACTOR

In least square problem, the unknown parameters of a mathematical model should be chosen in such a way that the sum of the squares of the differences, between the observed values and the computed values, is a minimum. The unknown parameter should be chosen such that the least squares loss function $V(\hat{\theta},t)$ is minimized,

$$V(\hat{\theta},t) = \frac{1}{2}\sum_{i=1}^{t}\left(y(i) - \phi^T(i)\hat{\theta}\right)^2 \tag{1}$$

Solving for the parameter to minimize the above loss function, we get the solution as follows:

$$\hat{\theta} = \left(\sum_{i=1}^{t}\phi(i)\phi^T(i)\right)^{-1}\left(\sum_{i=1}^{t}\phi(i)y(i)\right) \tag{2}$$

However, because of the chance of the variability of the unknown parameter, it is more suitable for using a forgetting factor $\lambda$ for loss function as follows:

$$V(\hat{\theta},t) = \frac{1}{2}\sum_{i=1}^{t}\lambda^{t-i}\left(y(i) - \phi^T(i)\hat{\theta}(t)\right)^2 \tag{3}$$

It is desirable to make the computations recursively to alleviate computation of burden, since it is more efficient to have the observations obtained sequentially in real time. Therefore, RLS algorithm will be used in this paper to update the estimation of the unknown parameter $\theta(t)$ at time $t$, using the results obtained at time $t-1$ and regression vector $\phi(t)$. The design procedure of the RLS algorithm at each step $t$ is given as follows.

Step 1 : Measure the output $y(t)$ and calculate the regression vector $\phi(t)$.

Step 2 : Calculate the update gain $K(t)$ which is called weighting factors that tell how the correction and previous estimate should be combined:

$$\begin{aligned} K(t) &= P(t)\phi(t) \\ &= P(t-1)\phi(t)\left(\lambda I + \phi^T(t)P(t-1)\phi(t)\right)^{-1} \end{aligned} \tag{4}$$

and calculate the covariance matrix $P(t)$ defined as

$$P(t) = \left(I - K(t)\phi^T(t)\right)P(t-1)\phi(t)\frac{1}{\lambda}\,. \tag{5}$$

Step 3 : Update the unknown parameter vector $\hat{\theta}(t)$ as

$$\hat{\theta}(t) = \hat{\theta}(t-1) + K(t)\left(y(t) - \phi^T(t)\hat{\theta}(t-1)\right)\,. \tag{6}$$

The correction term $\left(y(t) - \phi^T(t)\hat{\theta}(t-1)\right)$ is proportional to the difference between the measurement $y(t)$ and prediction of the previous estimate.

The parameter $\lambda$, called the forgetting factor, reflects the variance of the time-varying parameters. This concept is based on the fact that the old data is gradually discarded, and it gives more weight on recent information.

## 3. MASS ESTIMATION USING LONGITUDINAL DYNAMICS

The mass estimation using longitudinal dynamics is a model based approach. This is available only when a vehicle accelerates without turning. Therefore we need a longitudinal model as follows:

$$m\dot{v} = F_x - F_b - F_{aero} - F_{grade} \tag{7}$$

where $m$ is the total mass of the vehicle, $v$ the vehicle velocity and $F_x$ the total longitudinal tire force which is transmitted from engine torque at the flywheel to tire. It is given by

$$F_x = \frac{T_e - J_e\dot{\omega}}{r_g} \tag{8}$$

$T_e$ is the engine torque which must be scaled down because of the possible torque losses. $J_e$ is the powertrain inertia and $r_g$ the wheel radius divided by the gear ratio and the final drive ratio:

$$r_g = \frac{r_w}{g_d g_f} \tag{9}$$

where $r_w$ is the wheel radius, $g_d$ the gear ratio and $g_f$ the final drive ratio. $F_b$ is the brake force generated by brake at the wheels, and $F_{aero}$ is the aerodynamic drag force defined as :

$$F_{aero} = \frac{1}{2}C_d\rho A\upsilon^2 \tag{10}$$

$C_d$ is the aerodynamic drag coefficient, $\rho$ the air density and $A$ the frontal area of the vehicle. $F_{grade}$ is the integrated force due to the rolling resistance of the road and road grade which is defined as:

$$F_{grade} = mg(\mu\cos\theta + \sin\theta) \tag{11}$$

where $g$ is the gravity acceleration and $\mu$ the rolling resistance coefficient.

Equation (7) can be rearranged in a regression form :

$$y(t) = \phi^T(t)\theta(t) \tag{12}$$

with

$$\begin{aligned} y(t) &= a_x + g\mu\cos\theta \\ \phi^T(t) &= \frac{T_e - J_e\dot{\omega}}{r_g} - F_b - F_{aero} \\ \theta(t) &= \frac{1}{m} \end{aligned} \tag{13}$$

where $y(t)$ is the measured output, $\phi(t)$ the known variables, and $\theta(t)$ the unknown parameter which needs to be estimated. Here, $g\mu\cos\theta$ term is assumed to be a constant since this is a very small value compared with other terms, and sensor measurement $a_x$ includes $g\sin\theta$ term.

Here, a forgetting factor is used since vehicle mass can change with the number of passengers or the weight of load vehicle carries. However, the forgetting factor is set very close to 1 since vehicle mass doesn't change fast.

## 4. MASS ESTIMATION USING ROLL DYNAMICS

This section deals with the mass estimation using roll dynamics. The first scheme is about adaptation law and the second scheme is about recursive least square. These schemes are based on the roll dynamics

### 4.1 Mass estimation using roll dynamics by adaptation law

This section deals with the mass estimation using roll dynamics from the measured roll rate and the vehicle dynamic model.

The simplified linear second order roll dynamics of a vehicle is described as follows:

$$I_x\ddot{\theta} = ma_{ym}h - k_t\theta - c_t\dot{\theta} \tag{14}$$

where the bouncing motion of the sprung mass is neglected. Here, $m$ is the sprung mass and $h$ the distance between vehicle roll center and c.g.. Also, it is assumed that c.g. height is known.

Using the roll dynamics equation (14), a roll dynamics observer is defined using the measured roll rate $\dot{\theta}$ and the estimated sprung mass $\hat{m}$ as follows:

$$I_x\ddot{\hat{\theta}} = \hat{m}a_{ym}h - k_t\hat{\theta} - c_t\dot{\hat{\theta}} + k_o(\dot{\theta} - \dot{\hat{\theta}}) \tag{15}$$

Defining the estimation errors of the roll angle and the sprung mass as follows:

$$\tilde{\theta} = \theta - \hat{\theta} \tag{16}$$

$$\tilde{m} = m - \hat{m} \tag{17}$$

Then, subtracting (15) from (14), the error dynamics of the observer is described as follows:

$$I_x\ddot{\tilde{\theta}} = \tilde{m}a_{ym}h - k_t\tilde{\theta} - c_t\dot{\tilde{\theta}} - k_0\dot{\tilde{\theta}} \tag{18}$$

Here, we can neglect $I_x\ddot{\tilde{\theta}}$ term. Because $\ddot{\tilde{\theta}}$ is a very small value compared with the other terms. So we can rewrite equation (18) as follows:

$$\tilde{m}a_{ym}h - k_t\tilde{\theta} - c_t\dot{\tilde{\theta}} - k_0\dot{\tilde{\theta}} = 0 \tag{19}$$

Or, equivalently as:

$$(c_t + k_0)\dot{\tilde{\theta}} + k_t\tilde{\theta} = \tilde{m}a_{ym}h \tag{20}$$

The stability of the roll dynamics observer is proved through Lyapunov stability analysis. Also a vehicle sprung mass adaptation algorithm is derived through the same analysis. Let a positive definite scalar function V be given by

$$V = \frac{1}{2}\tilde{\theta}^2 + \frac{1}{2}\frac{1}{k_a}\tilde{m}^2 > 0 \tag{21}$$

Taking the derivatives of the Lyapunov function (21) and combining it with equation (20):

$$\begin{aligned}\dot{V} &= \tilde{\theta}\dot{\tilde{\theta}} - \frac{1}{k_a}\tilde{m}\dot{\hat{m}} = \tilde{\theta}\left(\frac{\tilde{m}a_{ym}h - k_t\tilde{\theta}}{c_t + k_0}\right) - \frac{1}{k_a}\tilde{m}\dot{\hat{m}} \\ &= -\frac{k_t}{c_t + k_0}\tilde{\theta}^2 + \left(\frac{\tilde{\theta}a_{ym}h}{c_t + k_0} - \frac{\dot{\hat{m}}}{k_a}\right)\tilde{m}\end{aligned} \tag{22}$$

Then, update $\hat{m}$ such that:

$$\dot{\hat{m}} = \frac{k_a a_{ym} h}{c_t + k_0}\tilde{\theta} \tag{23}$$

Now, equation (22) can be written as follows:

$$\dot{V} = -\frac{k_t}{c_t + k_0}\tilde{\theta}^2 \tag{24}$$

Equation (24) is negative semi-definite for the constant observer gain $k_0$ greater than $-c_t$. Now, it can be proved that the error of the roll angle estimation will converge to zero under the condition of the persistence of excitation. In addition, the error dynamic equation (20) shows that the sprung mass estimation error converges to zero , as well.

### 4.2 Mass estimation using roll dynamics by recursive least square

This section deals with the mass estimation using the recursive least square method from roll dynamics by the measured roll rate and the vehicle dynamic model.

The recursive least square method is more suitable for online parameter estimate.

Start with the simplified the roll dynamic equation derived before:

$$I_x\ddot{\theta} = ma_{ym}h - k_t\theta - c_t\dot{\theta} \tag{25}$$

Equation (25) can be rearranged in a regression form:

$$\begin{aligned} y(t) &= a_{ym}h \\ \phi^T(t) &= I_x\ddot{\theta} + c_t\dot{\theta} + k_t\theta \\ \theta(t) &= \frac{1}{m}\end{aligned} \tag{26}$$

where $y(t)$ is the measured output, $\phi(t)$ the known variables, and $\theta(t)$ the unknown parameter which needs to be estimated.

While applying use the RLS algorithm derived before, the forgetting factor is used again since the vehicle mass can change with the number of passengers or the weight of load vehicle carries. However, the forgetting factor is set very close to 1 since the mass doesn't change fast.

## 5. INTEGRATED MASS ESTIMATION USING MULTIPLE OBSERVER SYNTHESIS

Mass estimation using just longitudinal dynamics is valid only when the steering angle is nearly zero, and mass estimation using roll dynamics when the vehicle turns.

The estimation schemes are integrated using a proper weighting factor. This multiple observer synthesis is defined as follows:

*Multiple observer synthesis*

$$= MAX\left(\begin{array}{l} sat\left(\frac{1}{2\varepsilon_{ra}}(|roll\ angle| - \Gamma_{ra} + \varepsilon_{ra})\right), \\ sat\left(\frac{1}{2\varepsilon_{rr}}(|roll\ rate| - \Gamma_{rr} + \varepsilon_{rr})\right)\end{array}\right) \tag{27}$$

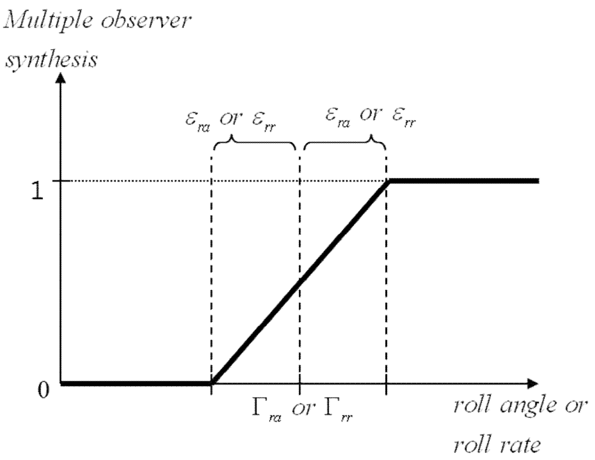


Fig. 1 Multiple observer synthesis

Figure 1 illustrates how the multiple observer synthesis is formulated as a function of the roll angle values. When the roll angle is small, mass estimation using longitudinal dynamics is more accurate. Therefore, the weighting is mostly on longitudinal based mass estimation. However, as roll angle gets bigger, mass estimation using longitudinal dynamics starts to become inaccurate. Hence, in this case, it is more efficient to rather use the mass estimation using roll dynamics. Such shift thus normally causes the estimated value to be less sensitive for all sorts of driving scenarios.

Incorporating the multiple observer synthesis in the mass estimation schemes as defined, final mass estimation value is written as follows:

$$\hat{m}_{final} = MOS \cdot \hat{m}_{roll} + (1 - MOS) \cdot \hat{m}_{longitudinal} \tag{28}$$

where MOS means the weighting factor derived from the multiple observer synthesis.

## 6. SIMULATION RESULTS

In this section, simulations are conducted to demonstrate the proposed mass estimation algorithm using commercial vehicle simulation softwares, Carsim and Simulink.

For the simulations, a vehicle model is selected, whose total mass is 1530kg and sprung mass is 1370kg.

In the first scenario, the vehicle goes straight with sine wave velocity profile from 65km/h to 75km/h to demonstrate the longitudinal mass estimation.

Figure 2 shows that the estimated value converges to a real value as time goes by.

In the second scenario, after sinusoidal steering by ± 60deg, steering angle decreases gradually. Figure 3 shows the steering profile. Adaptation law and recursive least square method are used to estimate the vehicle mass. Vehicle velocity is maintained at 80km/h.

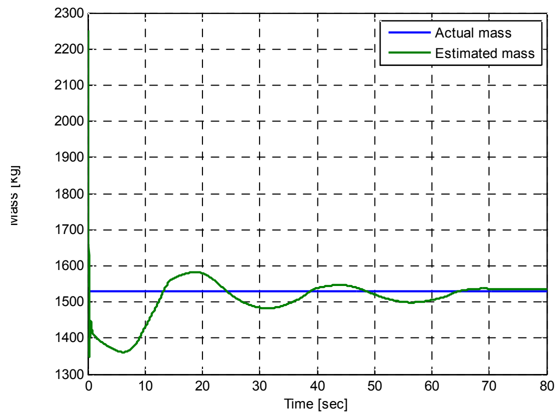


Fig. 2 Mass estimation using the recursive least square method by longitudinal dynamics

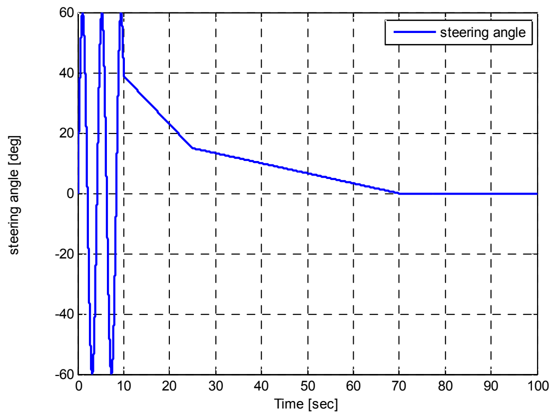


Fig. 3 Steering profile

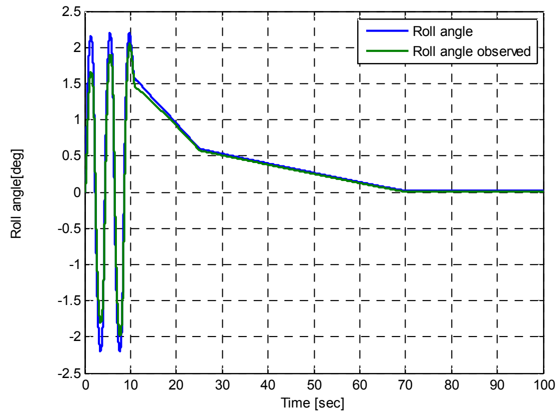


Fig. 4 Roll dynamics observer

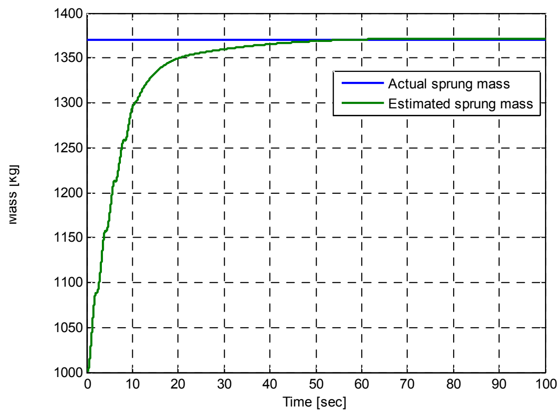


Fig. 5 Vehicle mass estimation using adaptation law by roll dynamics

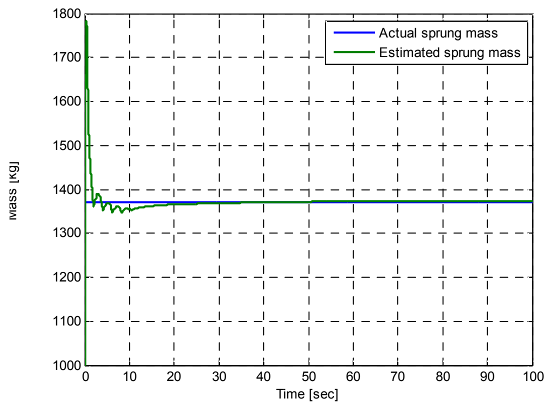


Fig. 6 Vehicle mass estimation using recursive least square by roll dynamics

Figure 4 shows that the simulation result of the roll dynamic observer and the roll angle observer is tracking the true roll angle. Initially, the observer shows some tracking error due to the difference of the initial mass value with the true mass value. However, as time goes by, the mass value converges to true value, and the roll angle observer tracks the true roll angle well.

Figure 5 and Figure 6 show the estimated mass value for the adaptation law and the recursive least square method in the second scenario. The simulation results show that each proposed scheme using roll dynamics estimates the vehicle mass effectively. For figure 6, initial mass estimation value is quite excessive due to the problem of initial covariance matrix setting. However, as time goes by, the estimated mass value tracks the true mass value.

In the final driving scenario, the vehicle goes straight with sinusoidal wave velocity from 65km/h to 75km/h for 50 seconds, then 60deg sinusoidal steering angle is added on the vehicle. This scenario demonstrates how this integrated mass estimation algorithm is beneficial.

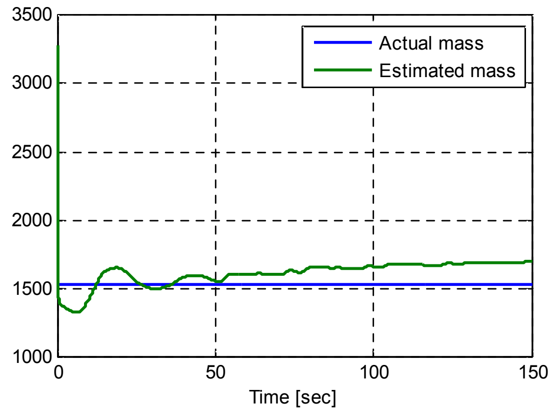


Fig. 7 Vehicle mass estimation using only longitudinal dynamics

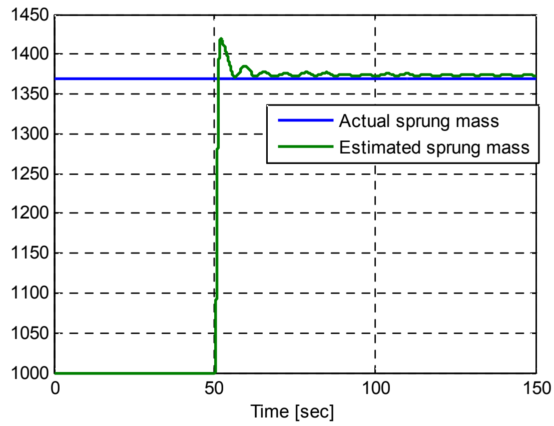


Fig. 8 Vehicle mass estimation using only roll dynamics

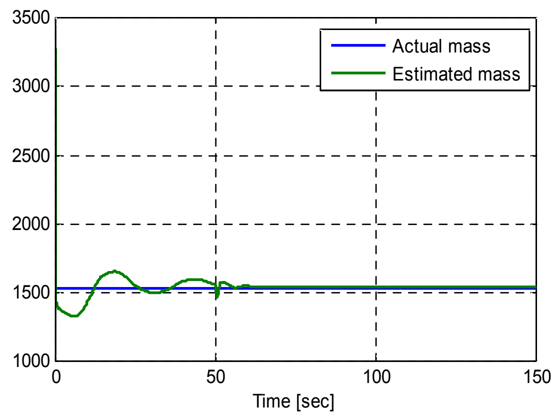


Fig. 9 Vehicle mass estimation using integrated algorithm

Figure 7 shows that the estimated mass value considering only longitudinal dynamics has some drifting issue after steering input is added. Figure 8 shows that it is impossible to estimate mass without any excitation of roll dynamics. However, figure 9 shows that the mass estimation by the proposed integrated algorithm using a recursive least square method is very efficient. This integrated method can estimate the vehicle mass very quickly without any drifting problem.

## 7. CONCLUSION

The vehicle mass is an important parameter and plays a crucial role for the vehicle safety control. This paper has focused on the development of an integrated algorithm to estimate vehicle mass for all sorts of driving situations. In order to estimate a vehicle mass, a recursive least square method is used to both longitudinal and roll dynamics, and an adaptive observer has been designed to observe the roll angle and update the vehicle mass for roll dynamics.

The performance of the developed algorithm has been investigated by simulations using Carsim and Matlab/Simulink. The simulation results have confirmed that the development algorithm performs well for all sorts of driving situations.

## ACKNOWLEDGMENT

This work was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government(MEST) (No.2012-0000991). The authors would also like to thank S.H. You, J.Y. Seo, S.K. Min, S.G. Kang, and J.K. Kim of Hyundai Motor co. for their tireless efforts in developing the contents.

## REFERENCES

[1] U.S. National highway traffic safety administration, fatality analysis reporting system encyclopedia, online at : http:// www-fars.nhtsa.dot.gov/Main/index.aspx

[2] Vahidi, A., Stefanopoulou, A. and Peng, H., “Recursive Least Squares with Forgetting for Online Estimation of Vehicle Mass and Road Grade: Theory and Experiments, *Vehicle System Dynamics* , 43(1), pp. 31-55, 2005

[3] Huh, K.S., Lim, S., Jung, J., Hong, D., Han, S. and Han, K., “Vehicle Mass Estimator for Adaptive Roll Stability Control”, *SAE Technical Paper ,* 2007-01-0820, 2007

[4] Fathy, H.K., Kang, D.S. and Stein, J.L., “Online Vehicle Mass Estimation Using Recursive Least Squares and Supervisory Data Extraction”, American Control Conference, Seattle, Washington, pp.1842-1848, 2008

[5] Zhu, G.G., Taylor, D.O. and Vailey, T.L., “Recursive Vehicle Mass Estimation System”, U.S. Patent no.6,438,510, 2002

[6] Winstead, V. and Kolmanovsky, I.V., “Estimation of Road Grade and Vehicle Mass via Model Predictive Control”, IEEE Conference on Control Applications, Toronto, pp.1588-1593, 2005

[7] Rajamani, R., “Vehicle Dynamics and Control”, Newyork: Springer Verlag, 2005

[8] Astrom, K.J. and Wittenmark, B., “Adaptive Control”, Addison-Wesley, 1995

[9] Oh, J.W., and Choi, S.B., “Design Of A New Composite Observer for Vehicle Velocity and Attitude Estimation”, IASTED International Conference Control and Application, Vancouver, pp.102-109, 2011