# Sliding Sector Mode Control with Linear Parameter Varying Diesel Engine Model

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#### ABSTRACT

Automotive industry has been developing technologies to reduce exhaust emission. Moreover, developing an environmentally friendly diesel engine will become the most important issue in the future engine technology. A modern well-tuned diesel engine no longer smokes and the noise has been attenuated to more acceptable levels. In spite of these efforts, emission of particulates and nitrogen oxides (NOx) have not been equally successful. Therefore, the accurate control method of diesel engines is coming up in solving the problems related with NOx and PM. The diesel engine is a highly nonlinear system and the modeling parameters are changed significantly for different operating points. Therefore, it is very difficult to control the diesel engine system accurately. Every engine system model is actually composed of a variety of maps. And then, actuators like dc motors are controlled for transferring the appropriate flow rate to the manifolds. This method must be simple and robust for real world operating conditions. However, simple controllers don't work well in every working condition. Moreover, engine systems like the common rail system have some manufacturing error. Those problems make the engine model. The time varying diesel engine model can reflect every engine operating point and schedule the controller for abnormal environments. In this study, a sliding sector mode control method is suggested for reducing the well known chattering problem of sliding mode control. Sliding sector mode is defined for several sliding hyperplanes which are verified by Liapunov inequality. A general sliding mode controller is designed for several but fixed engine operating points.

#### INTRODUCTION

Diesel engine is introduced in 1922 by Mercedes-Benz having the poor acceleration response, smoke, and vibrations. And then, the introduction of turbo charging system in1970s helped extend diesel engine usage amongst the common user. In 1990s, diesel engine was mounted in modern cars especially SUV and sedan. Current generation diesel engines have the high performance in fueling and the low Co2 levels. However, the vibration and noise from the autoignition had remained as a reason to avoid the diesel automobiles. In the 21<sup>st</sup> century, thanks to the high technology in diesel fueling systems, diesel engines have been developed extravagantly. Moreover, the electric control systems have been supported by the development of the computer process. A modern well-tuned diesel engine no longer smokes and the noise has been attenuated to more acceptable levels. In spite of these efforts, emission of particulates and the oxides of Nitrogen (NOx) have been not proved successfully. From the research in reducing NOx and PM, the diesel engine system uses the Exhaust Gas Recirculation (EGR). This system makes the intake manifold fractionalizing. The another system is VGT(Variable Geometry Recirculation) which controls the fresh air flow rate from the compressor by changing the vane. Most diesel engine systems are composed of the two or three controls like EGR valve and VGT valve and fueling valve which are interacted between subsystems.

In diesel engine modeling techniques, there are many constraints to seek controllable and simple model which improves the various engine processes, such as air flow, thermal management, charge mixing. Generally, diesel engine modeling may not exhaustive and very accurate. In real conditions, the recessive parameters or dynamics will be disappeared or transformed into the other things. Due to these reasons, mean-value models are been used by several researchers [1-5] in diesel engine modeling.

Moreover, diesel engine is divided into nonlinear and linear model. Linear models rely on empirical data from the engine of interest. Flower et al.[6,7] used an identification scheme called P.R.B.S(Pseudo Random Binary Sequence) to obtain a discrete engine transfer function. Their purpose is to research the reasonably linear discrete engine model. Linearized models have the advantage of simplicity and short run times. However, the intermittent nature of diesel engines like transient operating may make the whole system unstable. So, linearization techniques require extensive empirical input in order to cover the entire operating range.

Nonlinear engine models can be separated into the following groups, the Quasi-Steady method, Filling and Empting method and characteristics method. Filling and empting method's feature is that the inlet and exhaust manifolds are considered as separated thermodynamic control volumes linked by gas flow through valves. And then, quasi-steady method is used when equations for the conservation of mass and energy are solved. Characteristics method is a mathematical technique for solving hyperbolic partial Page 1 of 17

differential equations such as those obtained for unsteady, compressible fluid flow [8,9,10].

In control theory, on account of the durability and reliability, PID control method is used in real situation. However, it is difficult to realize the high performance and to cover whole condition of engine in PID control. Therefore, a variety of control methods are developed as robust control or nonlinear control. The robust control has the benefit in diesel engine control. Engine model is highly coupled system which has the approximation errors in linearization. The robust controller can deal with this error using an appropriate uncertainty description. Moreover, robust controllers are actually guaranteed to yield the performance achieved on the uncertain model set. Using LMIs, the robust control can handle parameter-varying plants [11].

Another controller is sliding control which has the robust properties in disturbance and signal noise. These benefits are remarkable in diesel engines. Therefore, many studies have been come into action [12]. The problem of sliding mode control is chattering in the sliding surface and disturbance matching in actual situation. Sliding sector method is introduced by Furuta [13] to reduce the chattering problem.

This paper introduces new sliding sector mode control with LPV model which is already introduced by Peter [14] by using DC-servo motor control. This system matrix changed from the viscous, Coulomb and Stribeck frictions is a very simple case and there is no optimized solution in solving the coefficients of the sliding surfaces. This paper is comprised of three chapters. First chapter introduces diesel engine modeling from the air flow point of view. And, the modeling is transformed to the regular form with LPV form. Second chapter introduces how to make the sliding surfaces with Minimum and Maximum intake and exhaust manifolds pressures which are needed in designing the sliding sector mode. Final chapter shows the sliding sector mode control in any operating points of diesel engine like a random condition of 1270rpm and 100% road.

#### **DIESEL ENGINE MODELING**

The diesel engine mechanical parts and air flow directions are described in fig.1. According to the directions of air mass flow, mechanical parts are composed of intake manifold, engine, exhaust manifold, turbo system and EGR valve.



Figure1- diesel engine model

The mean value model of a typical diesel engine plant with EGR and VGT was followed in equations (1). This model comprises two open thermodynamic volumes, representing the intake and exhaust manifolds, two variable area orifices model, the flows through the EGR valve and the variable geometry turbine. A first order lag models the power transfer between the turbine and the compressor.

$$\begin{aligned} \frac{dm_{1}}{dt} &= W_{c1} + W_{21} - W_{1e} - W_{12} \\ \frac{dm_{2}}{dt} &= W_{12} + W_{e2} - W_{21} - W_{2t} \\ \dot{F}_{1} &= \frac{W_{21}(F_{2} - F_{1}) - F_{1}W_{c1}}{m_{1}} \\ \dot{F}_{2} &= \frac{W_{e2}(F_{e2} - F_{2}) - W_{12}(F_{1} - F_{2})}{m_{2}} \end{aligned}$$
(1)  
$$\begin{aligned} \frac{dT_{1}}{dt} &= \frac{W_{21}(h_{21} - u_{1}) - W_{c1}(h_{c1} - u_{1}) - (W_{1e} - W_{12})R_{1}T_{1} - m_{1}\chi_{F1}\dot{F}_{1}}{c_{v1}m_{1}} - \frac{\dot{Q}_{1}}{c_{v1}m_{1}} \\ \frac{dT_{2}}{dt} &= \frac{W_{e2}(h_{e2} - u_{2}) - W_{12}(h_{12} - u_{2}) - (W_{21} - W_{2t})R_{2}T_{2} - m_{2}\chi_{F2}\dot{F}_{2}}{c_{v2}m_{2}} - \frac{\dot{Q}_{2}}{c_{v2}m_{2}} \\ \frac{dP_{c}}{dt} &= \frac{1}{\tau_{tc}}(-P_{c} + \eta_{tm}P_{t}) \end{aligned}$$

The subscripts of the equations (1) are expressed as follows

1 : intake manifold 2 : exhaust manifold

c : compressor e : engine

ij : flow from volume i to volume j

F : Fraction of air mass to the total mass of air EGR mixture

However, this model is so high order system that is difficult to control the system. Moreover, the fractions of air in the manifold and exhaust manifold are not measured easily. These parameters must be estimated or removed by using the assumption of target values which satisfies the regulations of NOx and PM. The temperatures of the intake and exhaust manifold are dominated by pressures of each mechanical parts, which can be omitted. Therefore, equations of this model are reduced into the 3<sup>rd</sup> order dominant system as follows.

$$\dot{P}_{1} = k_{1} \left( \frac{\eta_{c}}{C_{p} T_{a}} \frac{P_{c}}{\left( \left\{ \frac{P_{1}}{P_{a}} \right\}^{\mu} - 1 \right)} + W_{egr} - k_{e} P_{1} \right),$$
where  $k_{1} = \frac{R_{a} T_{1}}{V_{1}}, \quad k_{e} = \frac{\eta_{v} N V_{d}}{R_{a} T_{1}}$ 

$$\dot{P}_{2} = k_{2} (k_{e} P_{1} + W_{f} - W_{egr} - W_{2t}), \qquad k_{2} = \frac{R_{a} T_{2}}{V_{2}}$$

$$\dot{P}_{c} = \frac{1}{\tau} \left( -P_{c} + \eta_{im} \eta_{iis} C_{pa} T_{2} \left( 1 - \left( \frac{P_{a}}{P_{2}} \right)^{\mu} \right) W_{2t} \right), \qquad (2)$$

The 3<sup>rd</sup> order diesel engine dynamics are composed of intake manifold pressure, exhaust manifold pressure and power of compressor term. The pressure terms are made from the remained gas flow rate from inlet to outlet. The considering factors in this study is the time varying parameters like pressure and temperature which are changed from engine load and rpm. Before linearizing the system, this 3<sup>rd</sup> order equation must be reflected by time varying characteristic. Actually, LPV form is applied to make linearized system equations and gain scheduling method. LPV systems are linear systems whose describing matrices depend on an exogenous time-varying parameter vector. The exogenous parameter is real parameter and is measured or estimated when the systems are only operating. This parameter is determined from the significance of the whole systems. In other words, the dominant parameter in system operating region have to be selected. In this study, the exogenous parameters are pressures in intake and exhaust manifold whose values determine the system's stability and performance and the other parameters. Temperature parameters are less sensitive in whole systems and changed from the pressure parameters. LPV form is made as follows.

$$\dot{P}_{i} = \frac{RT_{i}}{V_{i}} \frac{\eta_{c}}{C_{p}T_{a}} \frac{1}{\left(\left\{\frac{P_{1}}{P_{a}}\right\}^{\mu} - 1\right)} P_{c} + \frac{RT_{i}}{V_{i}} W_{xi} - \frac{N}{V_{i}} \frac{\eta_{v}V_{d}}{2 \cdot 60} P_{i}$$
$$\dot{P}_{x} = \frac{RT_{x}}{V_{x}} \frac{N}{T_{i}} \frac{\eta_{v}V_{d}}{2 \cdot 60} P_{i} - \frac{RT_{x}}{V_{x}} W_{xi} - \frac{RT_{x}}{V_{x}} W_{xt} + \frac{RT_{x}}{V_{x}} W_{f}$$
$$\dot{P}_{c} = -\frac{1}{\tau} P_{c} + \frac{\eta_{m}}{\tau} \eta_{i} c_{p} T_{x} \left(1 - \left(\frac{P_{a}}{P_{x}}\right)^{\mu}\right) W_{xi}$$
(3)

The  $3^{rd}$  order diesel engine system in LPV form has the three states and two inputs. The rest pressure parameters, except for the states and inputs, are regarded as the time varying parameters. In order to make the simplest system, pressure parameters are changed into the ratio between the atmosphere pressure value and manifold pressure values. Finally, the time varying parameters and input states are as follows.

$$\rho_{1} = \left( \left\{ \frac{P_{1}}{P_{a}} \right\}^{\mu} - 1 \right), \quad \rho_{2} = \left( 1 - \left( \frac{P_{a}}{P_{x}} \right)^{\mu} \right)$$
$$X_{1} = P_{1}, \quad X_{2} = P_{2}, \quad X_{3} = P_{3}$$
$$U_{1} = W_{xt}, \quad U_{2} = W_{xt}$$
$$\dot{X} = A(\rho)X + B(\rho)U \tag{4}$$

The systems are changed into the regulation problems, the error state dynamic equations are derived from the desired operation points. The desired states can be evaluated by solving the state equations at steady state and fundamental values in diesel engine properties.

$$EGR_{ref} = \frac{W_{EGR}^{d}}{W_{C}^{d} + W_{EGR}^{d}} = \phi_{g}$$

$$AFR_{d} = \frac{W_{a}^{d}}{W_{f}^{d}},$$

$$F_{1} = \phi_{g}F_{2}$$

$$F_{2} = F_{e}$$
(5)

According to the driver's demand, the RPM and engine road is determined from data mapping and the engine ECU (Electric Control Unit) calculates the moderate fuel rate as follows to the equation (5). And then, the fraction which value is satisfied in NOx and PM regulations is suggested from the experiment data. From now on, the desired states are calculated by using previous values. These equations are suggested from Devesh Upadhyay [15].

Steady states value for desired intake manifold pressure

$$\begin{split} P_{2} &= K_{2}(K_{e}P_{1} + W_{f} - W_{egr} - W_{2t}) = 0 \\ K_{e}P_{1} &= W_{EGR}^{d} + W_{2t}^{d} + W_{f}^{d} = 0 \\ also \ at \ s.s \\ W_{e2} &= W_{1e} + W_{f} \\ W_{1e} &= W_{c} + W_{egr} \\ W_{2t}^{d} &= W_{c}^{d} + W_{f}^{d} \\ hence \\ P_{1} &= \frac{W_{EGR}^{d} + W_{c}^{d}}{K_{e}} \end{split}$$

Steady states value for desired exhaust manifold pressure

$$\dot{P}_{c} = \frac{1}{\tau} \left(-P_{c} + \eta_{m} \eta_{iis} C_{pa} T_{2} \left(1 - \left(\frac{P_{a}}{P_{2}}\right)^{\mu}\right) W_{2t}\right) = 0$$

$$P_{c}^{d} = \eta_{m} \eta_{iis} C_{pa} T_{2} \left(1 - \left(\frac{P_{a}}{P_{2}}\right)^{\mu}\right) W_{2t}$$

$$\left(\frac{P_{a}}{P_{2}}\right)^{\mu} = 1 - \frac{P_{c}^{d}}{\eta_{m} \eta_{iis} C_{pa} T_{2}}$$
(7)

Steady states value for desired power of compressor

$$\dot{P}_{i} = K_{1} \left( \frac{\eta_{c}}{C_{p}T_{a}} \frac{P_{C}}{\left( \left\{ \frac{P_{1}}{P_{a}} \right\}^{\mu} - 1 \right)} + W_{egr} - K_{e}P_{1} \right)$$
we get
$$P_{C}^{d} = \frac{C_{p}T_{a}}{\eta_{c}} \left( \left\{ \frac{P_{1}}{P_{a}} \right\}^{\mu} - 1 \right) (K_{e}P_{1} - W_{egr})$$

$$P_{C}^{d} = \frac{C_{p}T_{a}}{\eta_{c}} \left( \left\{ \frac{P_{1}}{P_{a}} \right\}^{\mu} - 1 \right) (W_{C}^{d})$$
(8)

From the desired values, the error state dynamics systems are derived as follows.

$$\begin{split} \dot{e}_{1} &= \frac{RT_{i}}{V_{i}} \frac{\eta_{c}}{C_{p}T_{a}} \frac{1}{\left(\left\{\frac{P_{1}}{P_{a}}\right\}^{\mu} - 1\right)} (e_{3} + P_{c}^{d}) + \frac{RT_{i}}{V_{i}} (U_{2} + U_{2}^{d}) \\ &- \frac{N}{V_{i}} \frac{\eta_{v}V_{d}}{2 \cdot 60} (e_{1} + P_{1}^{d}) \\ \dot{e}_{2} &= \frac{RT_{x}}{V_{x}} \frac{N}{T_{i}} \frac{\eta_{v}V_{d}}{2 \cdot 60} (e_{1} + P_{1}^{d}) - \frac{RT_{x}}{V_{x}} (U_{1} + U_{1}^{d}) - \frac{RT_{x}}{V_{x}} (U_{2} + U_{2}^{d}) \\ &+ \frac{RT_{x}}{V_{x}} W_{f} \\ \dot{e}_{3} &= -\frac{1}{\tau} (e_{3} + P_{c}^{d}) + \frac{\eta_{m}}{\tau} \eta_{i} c_{p} T_{x} \left( 1 - \left(\frac{P_{a}}{P_{x}}\right)^{\mu} \right) (U_{1} + U_{1}^{d}) \\ from \quad e_{1} &= P_{1} - P_{1}^{d}, \quad e_{2} &= P_{2} - P_{2}^{d}, \quad e_{3} &= P_{c} - P_{c}^{d} \\ &U_{1} &= U_{1t} - U_{1}^{d}, \quad U_{2} &= U_{2t} - U_{2}^{d} \end{split}$$

Now, the desired values of each state are known from the previous study, the remained states are composed of regulation problems.

$$\begin{bmatrix} \dot{e}_{1} \\ \dot{e}_{2} \\ \dot{e}_{3} \end{bmatrix} = \begin{bmatrix} -A_{11}(\rho_{1}) & 0 & A_{13} \\ A_{21} & 0 & 0 \\ 0 & 0 & -A_{33} \end{bmatrix} \begin{bmatrix} e_{1} \\ e_{2} \\ e_{c} \end{bmatrix} + \begin{bmatrix} 0 & B_{12} \\ -B_{21} & -B_{21} \\ B_{31}(\rho_{2}) & 0 \end{bmatrix} \begin{bmatrix} U_{1} \\ U_{2} \end{bmatrix}$$
(10)

(9)

Finally, this LPV regulated form has to be transformed to the regular form. The regular form is derived from the B matrix SVD methods in fixed operating point. However, values of this system matrix are changed into the road and RPM, the regular form is transformed using diffeomorphic state space transformation. Distribution delta value has to be satisfied as follows. g matrix is composed of the B matrix span values.

$$\Delta = span\{g_1, g_2\}, g_1 = (0 - B_{21} B_{31})^T, g_2 = (B_{12} - B_{21} 0)^T$$

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$$\Delta : d\lambda g_1 = 0 = \frac{\partial \lambda}{\partial P_2} + \frac{\partial \lambda}{\partial P_c}$$
$$d\lambda g_2 = 0 = \frac{\partial \lambda}{\partial P_1} + \frac{\partial \lambda}{\partial P_2}$$
(11)

The suggested lambda values are as follows

$$\lambda(P) = \frac{1}{B_{12}}P_1 + \frac{1}{B_{21}}P_2 + \frac{1}{B_{31}}P_c = z_3$$
(12)

The reasons in selecting the new state variable  $z_2$ =lambda value is that the second error state, pressure of exhaust manifold, can be expressed by removing the two input states which is coupled in mathematically. Therefore, system in regular form is simpler than any other form. The regular form in new state variables is as follows.

$$\begin{bmatrix} \dot{z}_{1} \\ \dot{z}_{2} \\ \dot{z}_{3} \end{bmatrix} = \begin{bmatrix} \left( -A_{11} - \frac{B_{31}}{B_{12}} \right) & -\frac{B_{31}}{B_{21}} & A_{13}B_{31} \\ A_{21} & 0 & 0 \\ \left( \frac{A_{21}}{B_{21}} - \frac{A_{11}}{B_{12}} \right) & 0 & \left( \frac{A_{13}}{B_{12}} - \frac{A_{33}}{B_{31}} \right) \end{bmatrix} \begin{bmatrix} z_{1} \\ z_{2} \\ z_{c} \end{bmatrix} \\ + \begin{bmatrix} 0 & B_{12} \\ -B_{21} & -B_{21} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} U_{1t} \\ U_{2t} \end{bmatrix} + \begin{bmatrix} 0 \\ B_{21} \\ 0 \end{bmatrix} W_{f} \\ \begin{bmatrix} \dot{z}_{1} \\ \dot{z}_{2} \\ \dot{z}_{3} \end{bmatrix} = \begin{bmatrix} \overline{A}_{11} & \overline{A}_{12} & \overline{A}_{13} \\ \overline{A}_{21} & 0 & 0 \\ \overline{A}_{31} & 0 & \overline{A}_{33} \end{bmatrix} \begin{bmatrix} z_{1} \\ z_{2} \\ z_{3} \end{bmatrix} + \begin{bmatrix} 0 & \overline{B}_{12} \\ -\overline{B}_{21} & -\overline{B}_{21} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} U_{1t} \\ U_{2t} \end{bmatrix} + \begin{bmatrix} 0 \\ \overline{B}_{21} \\ 0 \end{bmatrix} W_{f}$$
(13)

#### SLIDING MODE CONTROL WITH LPV

Sliding mode control has the advantage in dealing with the disturbance and uncertainty parameters. This advantage may be acted on controlling the actual diesel engine environment which has the transition of temperature, pressure, humidity, sensor noise, vibration and impacts. A variety of factors may influence engine model parameters. However, the focus of this study is completion of the sliding sector mode with time varying parameters that is the first qualification in controlling diesel engine. Most of all, to design the sliding sector mode with LPV, the operating range and exogenous parameter's range must be known. The reason is that the system stability in local and global is guaranteed in this sufficient condition.

$$\rho_i(t) \in \left[ \begin{array}{c} \rho_i, \overline{\rho}_i \\ - \end{array} \right] \tag{14}$$

In linear sliding surface, the pole placement method in obtaining the sliding surface coefficient is generally used. However, to obtain the optimized coefficient of sliding surface, LMI method in Lyapunov inequality equation in full state nominal plant is used. In fact, the guaranteed cost sliding mode design becomes a quadratic cost minimization problem. Given a matrix O>0, let the set be defined as follows.

$$\Omega(Q) = \{ P | (A - BK)^T P + P(A - BK) + Q = 0, P > 0, K \in \Re^{m \times n} \}$$
(15)

P is positive matrix and all the stabilizing sliding function coefficients are given by

$$c_1 = P_{22}^{-1} P_{12}^{T} \qquad (16)$$

For some  $P \in \Omega(Q)$ , where  $P_{ij}$ 's are defined as

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix} \in \begin{bmatrix} \mathfrak{R}^{(n-m)\times(n-m)} & \mathfrak{R}^{(n-m)\times m} \\ \mathfrak{R}^{m\times(n-m)} & \mathfrak{R}^{m\times m} \end{bmatrix}$$
(17)

Also for any  $P \in \Omega(Q)$ , the matrix  $c_1 = P_{22}^{-1} P_{12}^{T}$  is the stabilizing sliding function coefficient.

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The proof of the sliding mode surface design for linear multivariable systems is introduced by Kim [16].

The sliding surface coefficients are two values because of two input variables. Moreover, to design the sliding sector mode, two sliding surface controllers which are Minimum performance sliding mode and Maximum performance sliding surface respectively are needed. The system specification is as follows.

Engine size	6000cc (L6) diesel
Intake manifold	0.00192mm^3
Exhaust manifold	0.000635mm^3
Operating range	1000~2200RPM

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Table L	engine	specification
I doit I	Chighne	specification

From the specifications and desired state values, the transition range of the Minimum and Maximum exogenous parameters and sliding surface coefficients are calculated by Matlab. The values are as follows.

$$\rho_{1} = 0.1013 \sim 0.3091$$

$$\rho_{2} = 0.1224 \sim 0.2864$$

$$c_{1\_\min} = -0.0208, c_{2\_\min} = 1.0126$$

$$c_{1\_\max} = -0.903, c_{2\_\max} = 0.0227$$
(18)

The Minimum performance linear sliding surface design is as follows. The two sliding surfaces are existed first and second z state equations are with different input parameters.

$$V = 0.5S^{T}S$$

$$\dot{V} < S^{T}\dot{S}$$

$$S_{1} = c_{1}z_{3} + z_{1}$$

$$\dot{S}_{1} = c_{1}\dot{z}_{3} + \dot{z}_{1} = c_{1}\overline{A}_{31}z_{1} + c_{1}\overline{A}_{33}z_{3} + \overline{A}_{11}z_{1}$$

$$+ \overline{A}_{12}z_{2} + \overline{A}_{13}z_{3} + \overline{B}_{12}U_{2t} \qquad (19)$$

$$U_{2t} = \frac{1}{-\overline{B}_{12}} (c_{1}\overline{A}_{31}z_{1} + c_{1}\overline{A}_{33}z_{3} + \overline{A}_{11}z_{1} + \overline{A}_{12}z_{2}$$

$$+ \overline{A}_{13}z_{3} + Q_{1}sign(S_{1}))$$

$$S_{2} = c_{2}z_{3} + z_{2}$$

$$U_{1t} = \frac{1}{B_{21}} (c_{2}\overline{A}_{31}z_{1} + c_{2}\overline{A}_{33}z_{3} + \overline{A}_{21}z_{1} - \overline{B}_{12}U_{2t}$$

$$+ \overline{B}_{12}W_{f} + Q_{2}sign(S_{2}))$$

In sliding mode s(t)=0,

$$S_{1} = c_{1}z_{3} + z_{1}, S_{1} = 0$$

$$S_{2} = c_{2}z_{3} + z_{2}, S_{2} = 0$$

$$z_{1} = -c_{1}z_{3}, z_{2} = -c_{2}z_{3}$$

$$\therefore \dot{z}_{3} = -\overline{A}_{31}c_{1}z_{3} - \overline{A}_{33}c_{2}z_{3}$$
(20)

The desired state values and parameters related with the operating range are as follows. Fuel rate is offered by the experiment data and turbo system efficiency is map data files from manufacturer.

rpm	Fuel rate	P_i_d	P_x_d	P_c_d	Compressor	Turbine
					Efficiency	Efficiency
1000	0.0054	118	125	1.55	60%	58%

Table 2- 1000rpm desired state values

The assumption is that every state can be measured and disturbance is not considered. The results in Minimum performance sliding mode are as follows. The performance is determined by checking the regulating speed and error rate to the zero value.







Figure 6. Min. input W\_xi

The results is shown that the intake, exhaust pressure and compressor power states are regulated to the zero value exactly. And exhaust pressure's regulation speed is very fast comparing to the other states which have delay term generated from the mechanical system characteristic of turbine system. However, the problem is that the peak point occurred in the initial area. This phenomenon may be occurred from the initial values of states and parameters. Input values are shown that the gas flow rate from exhaust manifold to the turbine is more than gas flow rate from exhaust manifold to the EGR valve, which proves that the fraction of gas is low and a lot of gas in exhaust manifold are gone to the turbine system in low load and rpm.

In second, the Maximum performance sliding mode control is designed. Most procedures are similar to the previous design except for the operating range.

rpm	Fuel rate	P_i_d	P_x_d	P_c_d	Compressor	Turbine
					Efficiency	Efficiency
1400	0.01	303	352	16.2	71.5%	51%

Table 3- 1400rpm desired state values

The results in Maximum performance sliding mode control are as follows.



Figure 8. Max. exhaust manifold pressure



Figure 9. Max. compressor power



Figure 11. Max. input W\_xi

The results in maximum performance sliding mode control are that the every state is regulated to the zero. However, the little gap between desired values and state values is occurred. Because the desired state values are not exactly calculated from the steady state condition. Nonlinear characteristics are existed in initial nonlinear dynamics that make the error states. Moreover, due to the influence of the input value, the exhaust manifold pressure doesn't have the fast response. Initial values of inputs may need to be calibrated.

## SLIDING SECTOR MODE DESIGN

Using the sliding mode design in previous chapter, the sliding sector mode is designed. Most of all, the weighting function related with the exogenous parameter's transitions is important. The weighting function influences the whole system like gain scheduling. The nominal system dynamics with LPV form is as follows.

$$\dot{x}(t) = A(\rho(x))x(t) + B(\rho(x))u(t)$$
$$y(t) = Cx(t) + Du(t)$$
(21)

And then, there exists the hypercube composed of the time varying parameter vectors.

Where 
$$\rho(\mathbf{X}) \in \Omega$$
  $\Omega = [a_1, b_1] \times [a_2, b_2] \times ... \times [a_N, b_N] \in \mathfrak{R}^N$  hypercube (22)

Finally, the model is represented by using weighting coefficients.

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$$\dot{x}(t) = \sum_{r=1}^{R} w^{r}(\rho(x)) A^{r} x(t) + \sum_{r=1}^{R} w^{r}(\rho(x)) B^{r} u(t)$$
(23)

Where  $w^r(\rho\rho(x) \in [0,1])$  are weighting coefficients. Control signal is the weighed sum of the control signal of the component systems.

$$u = \sum_{r=1}^{R} w^{r} (\rho(x)) u^{r}$$
(24)

In general, four sliding surface design are needed for sliding sector. Because two exogenous parameters have the Minimum and Maximum value respectively. However, exogenous parameters tend to have the similar characteristic in diesel engine operating region. Another speaking, theses parameters are coupled in dynamics of diesel engines. So, weighting values are composed of two-degree look up table as follows.



Figure 12. Weighting coefficients from two exogenous parameters

To prove the sliding sector mode, two operating points are suggested. The first operating point is as follows.

rpm	Fuel rate	P_i_d	P_x_d	P_c_d
1270	0.0068	155	195	3.57
Table 4- 1270rpm desired state values				



Figure 13. Sliding sector\_1 intake manifold pressure



Figure 14.Sliding sector\_1 exhaust manifold pressure







Figure 16. Sliding sector\_1 input W\_xt



Figure 17. Sliding sector\_1 input W\_xi

The results of sliding sector mode with low rpm and high load conditions are that the most values are regulated to the desired values. However, the error is existed in steady states. The reason is that the accuracy of weighting coefficients is not perfect. In other words, the engine characteristic that the Maximum torque operating points and Maximum power operating points are not matching linearly is not feed into weighting coefficient. However, the last weighting coefficients values are 0.886, 0.114 respectively which shows that the sliding sector mode is similar to the Minimum performance sliding mode control method.

Second condition is the maximum power operating points in target engine.

rpm	Fuel rate	P_i_d	P_x_d	P_c_d
2200	0.0093	257	318	15.2

Table 5- 2200rpm desired state values



Figure 18. Sliding sector\_2 intake manifold pressure



Figure 19. Sliding sector\_2 exhaust manifold pressure







Figure 21. Sliding sector\_2 input W\_xt



Figure 22. Sliding sector\_2 input W\_xi

The results of sliding sector mode with high rpm and high road condition are that the error of states in steady state condition is increased. As previously stated, the error is occurred from the weighting coefficient values which are not linear in exogenous parameters. Especially, the error of high rpm and load is bigger than that of row rpm and high load. The reason is that the engine characteristics are shown in high rpm and load and second weighting coefficient is more weighting at the high rpm and load.

# CONCLUSIONS

In this paper, to apply the actual dynamics of diesel engine, the sliding sector mode is realized by using LPV characteristics and two sliding mode control. Transforming the LPV form, the engine model is simple and easily controlled. And the optimal linear sliding hyperplanes are designed by using the quadratic cost minimization problem. Moreover, two sliding mode controls can provide the basis of sliding sector. Finally, the sliding sector with the weighting coefficients which reflect the diesel engine properties is designed. Most of all, through the efforts of the simplest weighting coefficients, the sliding sector mode can be simple and easily controlled. In future works, the disturbances have to be adapted. In fact, the advantage of the sliding mode is the robustness in disturbances. And then, the discontinuous factor term is added in controllers. As previously stated, actual diesel engine model is very complicated and system order is very high. So, the high order system plant has to be adapted. Last work is that the weighting coefficient has to be revised. In high load and rpm range, the engine characteristic is not reflected to the weighting. Revising the weighting values, the actual diesel engine model control will be performed in the future.

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