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Lateral Acceleration Compensation of a Vehicle Based on Roll Angle Estimation

Kwanghyun Cho, Hyunwoo Son, Seibum B. Choi and Sukchang Kang

Abstract— **This paper demonstrates a method for compensating the gravity component of the lateral acceleration through the estimation of the roll angle. The lateral acceleration has a direct influence on the road disturbance and suspension motion by driver's steering input. It is difficult to differenciate the bias(gravity component) induced by the road bank disturbance and suspension motion from the actual lateral acceleration measured by a sensor. Although the roll rate sensor can estimate the roll angle via the integration, it has several limitations concerning its use. Because there is no roll rate sensor or roll angle sensor in usual vehicles and the integration may cause the sensor drift problem. In this paper, the state index that implicates whether the state of the vehicle is in the transient region or steady state region, is defined. The roll angle is estimated by integrating the suspension angle rate and kinematic roll angle through switching of the state index. The gravity component induced by roll angle is substracted from the measured lateral acceleration components. And the side-slip angle is estimated to verify the influence of the compensated lateral acceleration. These works are verified using CarSim program with variety of road environments and steering inputs.**

I. INTRODUCTION

The vehicle stability control systems have been developed for the safety of a vehicle. The stability control systems have to observe the vehicle states such as side-slip angle, vehicle velocity and roll angle to keep the vehicle in the stable region. However, usual vehicles do not use any sensors to measure the lateral velocity or roll angle directly because of the cost issue. Therefore the robust vehicle state estimation has to be implemented. The performance of the vehicle state estimator or observer depends on the vehicle model or sensors. These factors must be filtered or compensated because they are subject to be affected by road environment or model parameters.[1][2] Especially, the acceleration has a direct influence on the road disturbance and a driver's steering input. Although the pure acceleration component must be used to estimate the state of vehicles, the one measured by the longitudinal/lateral accelerometer has both the pure lateral dynamics components and the gravity components. Acceleration under the influence of gravity components makes the estimation of the vehicle state like side-slip angle more difficult because the biased component from gravity makes the error larger in using integration.

This paper focuses on the compensation of the lateral accelerometer. The lateral accelerometer is widely used to estimate the vehicle state such as the side-slip angle, roll angle, bank angle and etc.[3] A method for compensating the gravity components of the lateral acceleration is proposed using the estimation of the roll angle. The lateral acceleration has a direct influence on the road disturbance and suspension motion and it is represented by the roll angle. Therefore the gravity component by road bank disturbance or steering input can be eliminated if the roll angle is precisely estimated. [4][5][6] Although the roll rate sensor can estimate the roll angle through integration, it has several limitations to use. This is accounted by the absence of roll rate and roll angle sensors in mass production vehicles and the integration method may cause the sensor signal drift problem. The roll angle consists of the combination of road bank angle and suspension angle. In case of the vehicle in a steady state region, the roll angle is represented by the kinematic roll angle. In case of the vehicle in transient region, it is represented by the suspension angle. In this paper, the state index that implicates whether the state of vehicle is in transient region or steady state region, is defined. The roll angle is estimated by integrating the kinematic roll angle and the suspension roll rate through switching of the state index. The gravity component induced by the roll angle is subtracted from the measured lateral acceleration components. And the side-slip angle is estimated using a simple integration method to verify the influence of the compensated lateral acceleration.[8]

In the following sections, the used models and method for the compensation of the lateral acceleration are introduced. In section II, the estimation of the roll angle is discussed. In section III, the compensation method of the lateral acceleration is discussed. In section IV, the developed estimator is verified in simulation to verify the appropriateness using CarSim program with the variety of road environments and steering inputs.

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II. ROLL ANGLE ESTIMATION

A. Estimation of Suspension Roll Angle

The roll angle of the vehicle is represented by the inertial force for the lateral dynamics and road bank angle as shown in Fig 1. The suspension roll model is expressed equivalently using a damper and a spring. Assume that

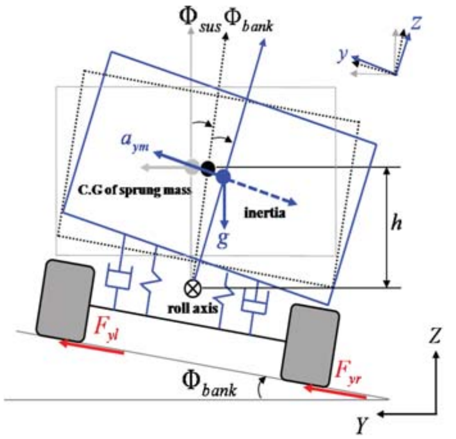


Fig. 1: Vehicle Roll Model(ISO 8855 coordinate)

- The roll axis is fixed
- The roll angle does not change depending on payload
- The roll stiffness and damping coefficient are constants
- There is no vertical motion of the vehicle

Using these assumptions and moment balance, the suspension dynamic model can be written as,

$$I_r\ddot{\Phi}_{sus} = m a_{ym} h - d_{sus}\dot{\Phi}_{sus} - k_{sus}\Phi_{sus} \tag{1}$$

$$I_r = I_{C.G} + mh^2 \tag{2}$$

where I_r is moment inertia of the roll axis, $I_{C.G}$ roll moment in C.G, Φ_{sus} suspension angle, m mass of vehicle, h height of roll axis, d_{sus} roll damping coefficient, a_{ym} measured lateral acceleration and k_{sus} roll stiffness coefficient.

In this paper, the suspension angle is estimated by an open-loop method under the assumptions that a roll rate sensor cannot be used and the vehicle motion is in a steady state region. From 1, the steady state suspension angle is described as follows,

$$\Phi_{sus} = \frac{hm}{k_{sus}} a_{ym} \tag{3}$$

B. Estimation of Static Bank Angle

The measurements taken by the yaw rate sensor and lateral accelerometer can be written as,

$$a_y + g\sin(\Phi_{roll}) = \dot{v}_y + r v_x \tag{4}$$

$$\Phi_{roll} = \sin^{-1}\left(\frac{\dot{v}_y + r v_x - a_y}{g}\right) \tag{5}$$

where g is gravity, v_y lateral velocity, r yaw rate, and v_x longitudinal velocity.

In (5), $\dot{v}_y$ is not used directly because it cannot be measured. In case the lateral motion of the vehicle hardly changes, Equation (5) becomes as follow,

$$\Phi_{ss} = \sin^{-1}\left(\frac{r v_x - a_y}{g}\right) \tag{6}$$

where Φ_{ss} is defined as static bank angle.

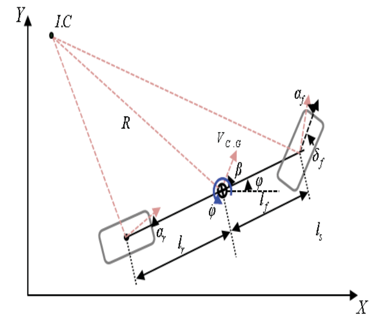


Fig. 2: Bicycle Model

C. Estimation of Dynamic Bank Angle using DFC

In case of the static bank angle, the estimation performance of the roll angle is very poor in a transient region where the vehicle stability controller is operated. Tseng proposed a method that offers de-coupling information between the lateral dynamics and the road bank disturbances.[5] It is called 'DFC(Dynamic Factor)' and is based on the static bank angle. Tseng uses the bicycle model as shown in Fig. 2. The state space representation of the bicycle model can be described as follows,

$$\dot{x} = Ax + Bu + Ed \tag{7}$$

$$x = \begin{bmatrix} \beta \\ r \end{bmatrix}, \qquad u = \delta_f, \qquad d = \frac{g}{v_x}\sin(\Phi_{roll}),$$

$$A = \begin{bmatrix} -\frac{2(C_f + C_r)}{m v_x} & \frac{2(C_r l_r - C_f l_f)}{m v_x^2} - 1 \\ \frac{2(C_r l_r - C_f l_f)}{I_z} & -\frac{2(C_f l_f^2 + C_r l_r^2)}{I_z v_x} \end{bmatrix},$$

$$B = \begin{bmatrix} \frac{2C_f}{m v_x} \\ \frac{2C_f l_f}{I_z} \end{bmatrix}, \qquad E = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

where β is side-slip angle, δ_f steering angle, C_f/C_r front/rear tire cornering stiffness, l_f/l_r length from C.G. to front/rear wheel, and I_z moment of inertia.

In (7), the transfer functions from Φ_{roll} and δ_f to a_y and r can be derived in a steady state region.

$$H_{\Phi_{roll} \mapsto a_y} = \frac{v_x^2 k_u g}{L + k_u {v_x}^2} - g \tag{8}$$

$$H_{\Phi_{roll} \mapsto r} = \frac{v_x k_u g}{L + k_u {v_x}^2} \tag{9}$$

$$H_{\delta_f \mapsto a_y} = \frac{v_x^2}{L + k_u {v_x}^2} \tag{10}$$

$$H_{\delta_f \mapsto r} = \frac{v_x}{L + k_u {v_x}^2} \tag{11}$$

where k_u is understeer coefficient, $k_u = (l_r m/L)/C_f - (l_f m/L)/C_r$, L the length of wheel base.

Tseng showed that the influence of steering and road bank to yaw rate and lateral acceleration can be super-positioned as follows,

$$a_y = H_{\delta_f \mapsto a_y} \cdot \delta_f + H_{\Phi_{roll} \mapsto a_y} \cdot \Phi_{roll} \tag{12}$$

$$r = H_{\delta_f \mapsto r} \cdot \delta_f + H_{\Phi_{roll} \mapsto r} \cdot \Phi_{roll} \tag{13}$$

The roll angles depending on a_y, r are derived from (12) and (13).

$$sin(\hat{\Phi}_{a_y}) = H_{\Phi_{roll} \mapsto a_y}{}^{-1}(a_{ym} - H_{\delta_f \mapsto a_y} \cdot \delta_f) \tag{14}$$

$$sin(\hat{\Phi}_r) = H_{\Phi_{roll} \mapsto r}{}^{-1}(r - H_{\delta_f \mapsto r} \cdot \delta_f) \tag{15}$$

Tseng proposed DFC(dynamic factor) as

$$\begin{aligned} DFC = H_{\Phi_{roll} \mapsto a_y} \cdot (sin\hat{\Phi}_{a_y} - sin\hat{\Phi}_{ss}) \\ + v_x \cdot H_{\Phi_r oll \mapsto r} \cdot (sin\hat{\Phi}_r - sin\hat{\Phi}_{ss}) \end{aligned} \tag{16}$$

It implies that the magnitude of $\dot{v}_y$ is related to the lateral dynamics component directly. Using (16), dynamic road bank angle can be derived as follows,

$$sin(\hat{\Phi}_{dyn}) = sin(\hat{\Phi}_{ss}) \cdot max[0, 1 - |DFC|] \tag{17}$$

D. Estimation of Roll Angle Using a State Index

In case of DFC method, the suspension angle is eliminated when DFC is larger than 1. It is considered only when the vehicle is in a steady state region. The performance of DFC method is poor when the steering input changes significantly, which means the vehicle is in the transient region. Tseng proposed the summing of the suspension angle and the road bank angle multiflied by a weighting factor which depends on the frequency to solve this problem.[7]

$$\Phi_{roll} = \Phi_{dyn\_freq} + \Phi_{ss\_freq} \tag{18}$$

However, this method involves a limitation for the real-time implementation due to the complexity of computation.

In this paper, a state index is defined which represents whether the state of vehicle is in a steady state or not. A state index, λ, is derived from the relation among the estimated roll angles such as Φ_{ss}, Φ_{a_y} and Φ_r and vehicle motion factors such as δ_f and r.

$$\lambda = f\big(\dot{\hat{\Phi}}_{ss}, \dot{\hat{\Phi}}_{a_y}, \dot{\hat{\Phi}}_r\big) \cdot g\big(\hat{\Phi}_{a_y}, \hat{\Phi}_r\big) \cdot h\big(\delta_f, \dot{\delta}_f, r\big) \cdot k\big(\delta_f, r\big) \tag{19}$$

where,

$$f = \begin{cases} 1 & if \quad \left| \frac{d}{dt}\big(|\dot{\hat{\Phi}}_{a_y} - \dot{\hat{\Phi}}_{ss}| - |\dot{\hat{\Phi}}_r - \dot{\hat{\Phi}}_{ss}|\big) \right| < F_{th} \\ & during \quad t_{TH1} \\ 0 & otherwise \end{cases}$$

$$g = \begin{cases} 1 & if \quad \hat{\Phi}_{a_y} \cdot \hat{\Phi}_r < 0 \quad during \quad t_{TH2} \\ 0 & otherwise \end{cases}$$

$$h = \begin{cases} 1 & if \quad \left|\delta_f\right| < G_{TH1} \quad and \quad \left|\dot{\delta}_f\right| < G_{TH2} \\ & and \quad |r| < G_{TH3} \quad during \quad t_{TH3} \\ 0 & otherwise \end{cases}$$

$$k = \begin{cases} 1 & if \quad \delta_f \cdot r > K_{TH} > 0 \quad during \quad t_{TH4} \\ 0 & otherwise \end{cases}$$

In (19), the index f is a flag that indicates whether the vehicle is in a steady state or not using the pattern difference among Φ_{ss}, Φ_{a_y} and Φ_r. The index g is a flag that indicates the existence of road bank angle in a steady state. The index h denotes whether the vehicle is on a straight and flat surface or not. The index k denotes whether the vehicle on the banked road is in a steady state or not. In conclusion, the state index is 1 if the vehicle is in steady state and 0 otherwise. The steady state means that the lateral motion of a vehicle is almost nonexistent because the steering input is nearly constant or the variation of it is very small enough to neglect. And F_{th}, $G_{TH1,TH2,TH3}$ and K_{TH} are some thresholds(constant), and $t_{TH1,TH2,TH3}$ are time thresholds.

The total roll angle is estimated using the static bank angle and suspension angle rate based on the state index switching.

$$\hat{\Phi}_{roll} = (1 - \lambda)\Big(\int_{t_1}^{t_2} \dot{\hat{\Phi}}_{sus} dt + \hat{\Phi}^*_{ss}\Big) + \lambda \cdot \hat{\Phi}_{ss} \tag{20}$$

where, $\hat{\Phi}^*_{ss}$ is the static bank angle when the state index switches 1 to 0. It is used as an initial condition to integrate the suspension roll rate during the transient state when the index is 0.

III. LATERAL ACCELERATION COMPENSATION

The measured lateral acceleration has both lateral dynamics components and gravity components due to road bank angle and suspension angle as described in Fig. 1. In (4), the gravity components should be eliminated from the measured lateral acceleration. In this paper, the lateral acceleration is compensated using (21).

$$a_y = a_{ym} - g \cdot sin(\hat{\Phi}_{roll}) \tag{21}$$

Vehicle side-slip angle is estimated using a simple integration method to verify whether the compensation of the lateral acceleration is useful to estimate the side-slip angle. The integration method to estimate the side-slip angle is as follows,

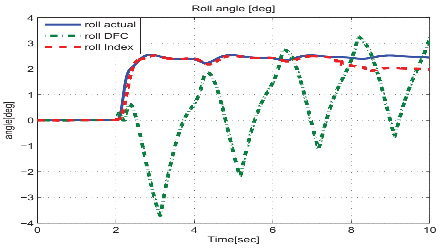
$$\dot{\beta} = \frac{a_{ycomp} - r \cdot vx}{v_x} = \frac{a_{ym} - g \cdot sin(\hat{\Phi}_{roll})}{v_x} - r \tag{22}$$

$$\beta = \int \dot{\beta} dt \tag{23}$$

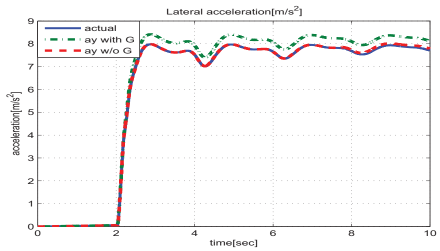
where, a_{ycomp} is the compensated lateral acceleration.

IV. SIMULATION

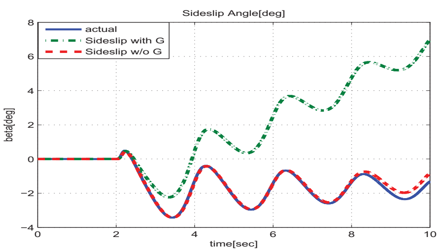
The performance of the designed roll angle estimator is evaluated under several driving conditional parameters like surface type, steering input and bank angle using Carsim, a commercial vehicle dynamics simulation program. The parameters of the used vehicle model and the cases of the



(a) Roll Angle

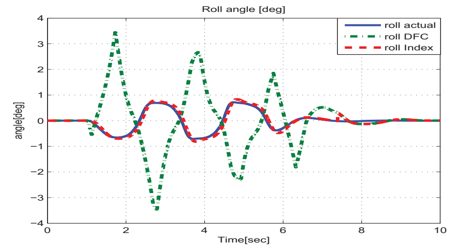


(b) Lateral Acceleration

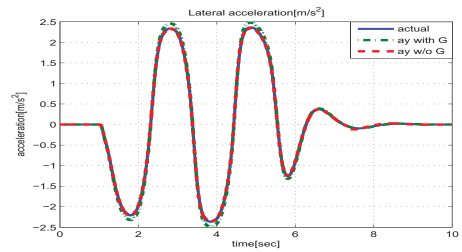


(c) Side-slip Angle

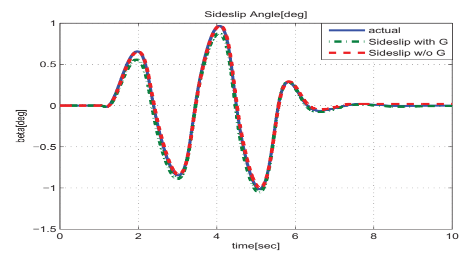
Fig. 3: Case.I



(a) Roll Angle



(b) Lateral Acceleration



(c) Side-slip Angle

Fig. 4: Case.II

simulations are shown in TABLE I and TABLE II. In these results, the actual roll angle(actual value by CarSim), the roll angle using DFC, and the roll angle using state index are compared to verify the performance of the roll angle estimation. The pure actual lateral acceleration(actual value by CarSim), the measured lateral acceleration and compensated lateral acceleration are compared to show the influence of the gravity components caused by the road bank angle and suspension angle on the measured lateral acceleration. Also, the side-slip angle estimated by a simple integration method is shown to verify the influence of the compensated lateral acceleration on the vehicle state estimation. In here, actual means the actual value by CarSim.

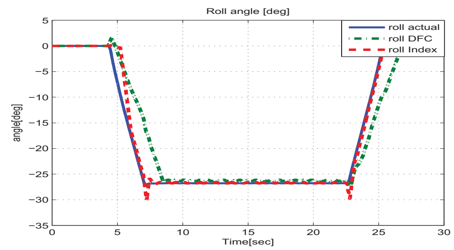
The simulated results for the dynamically applied steering input on a flat surface (road without bank angle) are shown in Fig. 3 and Fig. 4. These cases show the influence of the

TABLE I: Parameters of the vehicle model

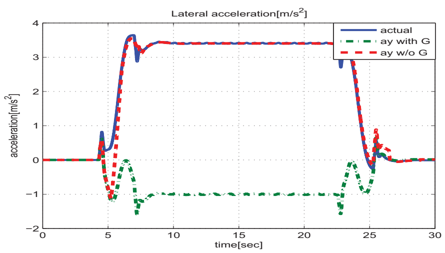
Parameter	Symbol	Value	Unit
Vehicle mass	m	1370	kg
Moment of inertia	I_z	4190	$kg \cdot m^2$
Front Cornering stiffness	C_f	2000	N/deg
Rear Cornering stiffness	C_r	1600	N/deg
Distance from CG to front	l_f	1.110	m
Distance from CG to rear	l_r	1.666	m

TABLE II: Simulation Cases

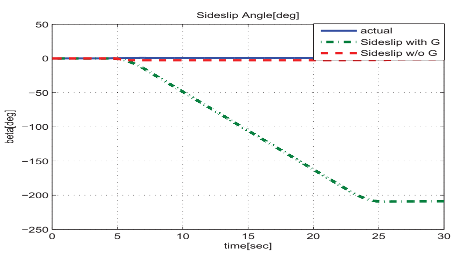
Case.I	Step steering / high mu(0.85) / 80kph / flat surface
Case.II	Slalom steering(0.5Hz) / low mu(0.25) / 140kph / flat surface
Case.III	Straight and Bank Turn / high mu(0.85) / 80kph / bank(0 $\sim$ 25deg)
Case.II	Double lane change steering/high mu(0.85 / 80kph / bank(0 $\sim$ 16deg)



(a) Roll Angle

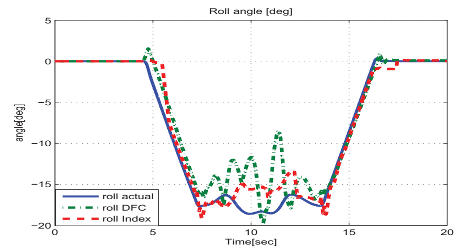


(b) Lateral Acceleration

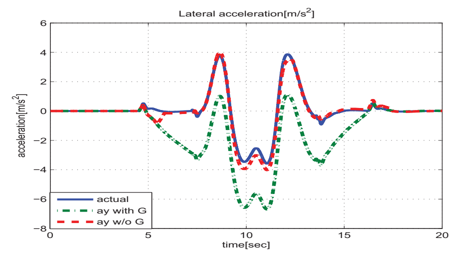


(c) Side-slip Angle

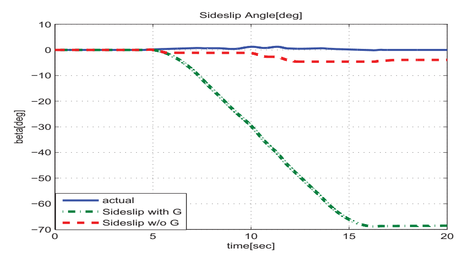
Fig. 5: Case.III



(a) Roll Angle



(b) Lateral Acceleration



(c) Side-slip Angle

Fig. 6: Case.IV

suspension angle accounted by steering input-excluding the effect of the road bank angle- on the roll angle estimation. In Case I, the vehicle is on a high μ surface with step steering input. In Case II, the vehicle is on a low μ surface with a slalom steering input. For the roll angle estimation described in Fig. 3(a) and Fig. 4(a), the performance of the state index method is superior. Since the state index method considers the effect of the suspension angle occurred due to steering input while the DFC method considers only bank angle. The differences between the measured and compensated lateral accelerations have turned out to be approximately 0.5 m/s^2 and 0.1 m/s^2 as shown in Fig. 3(b) and Fig. 4(b), which are the biased gravity components caused by the suspension angle. These biased components cause the integration to estimate the side-slip angle to diverge as shown in Fig. 3(c).

Case.III shown in Fig. 5 and Case.IV shown in Fig. 6 take the road with bank angle into account. These cases show the influence of both the suspension angle and the road bank angle on the roll angle estimation. In Case III, the vehicle goes straight on the flat surface and turns left on the banked curve road. In Case IV, the vehicle is going on the flat-banked-flat road with straight-double lane change-straight steering input. In the estimation of the roll angle, the performance of the state index method is superior than that of the DFC method as described in Fig. 5(a) and Fig. 6(a) for the same reson previously mentioned. The differences between the measured and compensated lateral accelerations have turned out to be approximately 4.5 m/s^2 and 2 m/s^2 as shown in Fig. 5(b) and Fig. 6(b), which are the biased gravity components caused by both the bank angle and the suspension angle. The differences attained in Case.III and Case.IV are larger than those attained in Case.I and Case.II because of road bank angle disturbance. The results of the side-slip angle estimation show that the larger difference between the measured and the compensated lateral acceleration makes the performance of the side-slip angle estimation worse as shown in Fig. 5(c) and Fig. 6(c).

In conclusion, the simulation results show that the lateral acceleration must be compensated precisely to be used directly to estimate the vehicle states like side-slip angle. Also it shows that the compensation performance of the lateral acceleration depends on the roll angle estimation accuracy.

V. CONCLUSION

The performance of the vehicle stability control system can be improved by precise observation of vehicle states such as side-slip angle, vehicle velocity, pitch and roll angle. High level sensors are able to measure the vehicle states more precisely, but it involves extra costs. Therefore, the robust vehicle state estimation algorithm has to be implemented. This work requires more precise vehicle model and filtered or compensated sensor signals. Of many sensors equipped on a vehicle, accelerometers are used widely to estimate vehicle states. This paper focuses on the lateral accelerometer equipped on most vehicles. The lateral accelerometer is used to estimate the lateral motion of a vehicle such as side-slip angle or roll angle. However, the signal from this sensor must be compensated to be used because the lateral acceleration is subject to be affected by gravity due to road bank angle and suspension angle.

In this paper, a roll angle estimation algorithm using the state index has been proposed. The estimated roll angle has been used to subtract the gravity components from the measured lateral acceleration. The estimated side-slip angle has been compared to verify the influence of the compensated lateral acceleration on the vehicle state estimation. The simulation results show that the performance of the proposed roll angle estimation algorithm is superior than that of the DFC method when dynamic steering input is applied. Also, it has been verified that the compensated lateral acceleration can be used directly to estimate the vehicle side-slip angle.

VI. ACKNOWLEDGMENTS

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REFERENCES

[1] H.E. Tseng, et al., "The Development of Vehicle Stability Control at Ford", *IEEE/ASME Transactions on Mechatronics*, Vol.4, No.3, pp.223-234, 1999

[2] A. Van Zanten, R. Erhardt, and G. Pfaff, "VDC-the vehicle dynamics control system of Bosch", *ATZ Automobiltechnische Zeitschrift*, Vol.96, No.11, pp.8, 1994

[3] A. Hac and M.D. Simpson, "Estimation of Vehicle Side Slip Angle and Yaw Rate", *SAE International*, 2000

[4] Y. Fukada, "Slip-angle estimation for vehicle stability control", *Vehicle System Dynamics*, Vol.32, No.4, pp.375-388, 1999

[5] H.E. Tseng, "Dynamic Estimation of Road Bank Angle", *Vehicle System Dynamics*, Vol.36, No.4-5, pp.307-328, 2001

[6] H.E. Tseng, Li Xu and Davor Hrovat, "Estimation of Land Vehicle Roll and Pitch Angles", *Vehicle System Dynamics*, Vol.45, No.5, pp.433-443, May 2007

[7] H.E. Tseng, Li Xu, "Robust Model-Based Fault Detection for Roll Rate Sensor", *Proceedings of th 42nd IEEE Conference on Decision and Control*, Vol.2, pp.1968-1973, December 2003

[8] Y. Fukada, "Estimation of Vehicle Slip-Angle with Combination Method of Model Observer and Direct Integration", *Proceedings of the 4th International Symposium on Advanced Vehicle Control(AVEC)*, pp.201-206, 1998