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Vehicle Side-Slip Angle Estimation of Ground Vehicles Based on a Lateral Acceleration Compensation

KWANGHYUN CHO¹, (Member, IEEE), HYUNWOO SON², YAFEI WANG¹⁰³, (Member, IEEE), KANGHYUN NAM¹⁰⁴, (Member, IEEE), AND SEIBUM CHOI¹⁰⁵, (Member, IEEE)

¹Manufacturing Technology Center, Samsung Electronics Company Ltd., Suwon 443-742, South Korea

²Plant Intelligent Team, Samsung SDS Company Ltd., Seoul 05510, South Korea

³School of Mechanical Engineering, Shanghai Jiao Tong University, Shanghai 200240, China

⁴School of Mechanical Engineering, Yeungnam University, Gyeongsan 712-749, South Korea ⁵Korea Advanced Institute of Science and Technology, Daejeon 34141, South Korea

Corresponding authors: Kanghyun Nam (khnam@yu.ac.kr) and Seibum Choi (sbchoi@kaist.ac.kr)

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ABSTRACT For vehicle stability control system to function properly under a variety of changing road conditions and driver's inputs, precise estimations of vehicle states are necessarily required. In particular, information on the side-slip angle is critical to vehicle handling and safety control. Since commercial sensors measuring the side-slip angle are not cost effective, estimation methods that use available sensor measurements and vehicle dynamics models are necessarily required. This paper proposes a novel methodology to estimate the side-slip angle which is used as an index of vehicle stability. A side-slip angle observer is designed using the bicycle model and the kinematic model. Here, in order to directly use the lateral accelerometer signal, the lateral acceleration compensation method through the roll angle estimation is proposed. The estimation performance is verified under various road conditions and different driver's inputs using HIL test equipment including commercial vehicle dynamics simulation software CarSim.

INDEX TERMS Lateral acceleration compensation, ground vehicle, observers, roll angle, side-slip angle.

I. INTRODUCTION

As increasing parts of the automobiles rapidly become digitalized, numerous high-performance vehicle stability controllers are being developed [1]. Generally, the vehicle stability control system is a device which secures stability through observing the vehicle states and driver's inputs on the real-time basis. Although a vehicle requires numerous sensors for this purpose, typical vehicles use only the minimum number of the sensors of absolute necessity due to cost issue. This has inspired the development of various observers to estimate the vehicles states of interest, and research projects to extend their performances are ongoing. However, observer design accompanies numerous limitations since it is nearly impossible to account for the strong nonlinear nature of the vehicle characteristics, non-predictability of the road conditions, and irrational intentions of the drivers. For instance, lateral vehicle dynamics control system-that is, in case of

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ESP-uses the side-slip angle as an index indicating the vehicle stability along with the yaw rate. And here, unlike the yaw rate which is measured through a sensor, the side-slip angle is estimated by an observer. Of course, the side-slip angle can be either directly or indirectly measured using a GPS or an optical sensor [2]. However, the results obtained this way are severely sensitive to the variation of the surroundings and weather. Consequently, the reliability of the measurement is low, and it surely involves additional cost. To solve this issue, numerous research projects have been heading to explore an effective way to indirectly estimate the side-slip angle [3]–[12], [19]–[24]. Due to the extremely nonlinear characteristics of the suspension, the results are bound by limitations for guaranteeing robust performance in various situations. Fukada approached through an estimation method based on the lateral force directly integrated using the acceleration information and the side-slip angle observer that is designed on the basis of the linear tire model [5]. However this approach involves a theoretical problem concerning its stability, and entails limitations regarding

the vehicle dynamics since the tire characteristic exhibits severely nonlinear region during the maneuvers like J-turn. Farrelly and Wellstead proposed a kinematics model-based linear time-variant observer. [3] This model is certified for the robust performance toward the modeling uncertainty and nonlinear nature, but it neglects the analysis of the accelerometer error and road disturbance including roll and pitch angle that critically affect the observer performance. Kaminaga and Naito proposed the adaptive observer which satisfies Lyapunov stability condition to the tire stiffness variation, and verified its effectiveness even in the strongly nonlinear region, but there existed the condition of persistent excitation to be met [9]. H. Lee proposed a lateral velocity observer based on a simple bicycle model, but it involved an uncertified assumption in the process of obtaining the observer gain for the theoretical analysis, and neglected the effect of the uncertainty related to an accelerometer [11]. H. Kim and D. Pi proposed a side-slip observer using the extended kalman filter, but it used the lateral accelerometer directly and it neglected the road disturbance such as road bank angle. Also, this method has severe computational burden because it uses the extended kalman filter [23]-[25].

In this paper, to solve the issue of side-slip angle observation, a novel concept is developed based on a modified bicycle model using lateral acceleration information. It is a type of observer that combines the advantages of two conventional models: the bicycle model that is sensitive to vehicle parameters but insensitive to sensor bias due to the absence of a lateral accelerometer, and the kinematic model that includes a lateral acceleration information but does not effective on the vehicle parameters. Using the combination of both models and compensating the lateral accelerometer from the sensor offset such as the gravity component, the robust side-slip angle observer is designed. In addition, a new roll angle estimation methodology is proposed to compensate the drifting issue of a lateral accelerometer which is directly used for the observer. By obtaining the realistic lateral acceleration component which excludes gravitational effect, the issue with the lateral accelerometer-that it is easily disturbed by various road conditions and driver's inputs-is resolved. The performance of the recommended observer is verified through simulations under various conditions using HIL test equipment.

This paper consists of the following. In section II, an analysis and issue of the conventional side-slip angle observer design are addressed, and a new observer with the use of the modified bicycle model is presented. In section III, a compensation method for the roll angle is presented. In section IV, in order to verify the performance of each designed observer, HIL tests in various conditions are carried out. In section V, a summary of the paper is presented to conclude it.

II. SIDE-SLIP ANGLE OBSERVER

A. BICYCLE MODEL

The lateral motion of a vehicle is typically explained based on the bicycle model. The bicycle model shown in Fig. 1 is a linearized model under the assumptions that the vehicle



FIGURE 1. Bicycle model for a front steering vehicle.

longitudinal velocity v_x is constant and the tire cornering stiffness, C_f and C_r are symmetric laterally. It is a simple model which requires minimum amount of calculation and relatively efficient in its performance. Although it is very susceptible to the vehicle parameter and the road conditions, it does not directly use the lateral accelerometer in estimating the side-slip angle β but rather use the yaw rate sensor rwhich is relatively reliable. Therefore the bicycle model owns an advantage for it is well-guarded from the influence of the sensor error. The dynamics model based on the bicycle model is derived from the following force balance and moment balance equations.

$$ma_y = F_{yf} + F_{yr} \tag{1}$$

$$I_z \dot{r} = F_{yf} l_f - F_{yr} l_r \tag{2}$$

where *m* is vehicle mass, I_z is inertia of moment, l_f and l_r are the distance between front and rear tires and the center of inertia. If the vehicle lateral motion is derived in terms of the lateral acceleration a_y in the inertial frame, and the lateral forces of front and rear, F_{yf} and F_{yr} , are expressed in terms of the tire cornering stiffness and the tire side-slip angles, α_f and α_r , the following equations can be derived. For the sake of simplification, nominal value of the cornering stiffness is fixed at zero tire side-slip angle.

$$a_y = \dot{v}_y + rv_x \tag{3}$$

$$\alpha_f = \beta + r \frac{l_f}{v_x} - \delta_f \tag{4}$$

$$\alpha_r = \beta - r \frac{l_r}{v_x} \tag{5}$$

$$F_{yf} = -2C_f \big|_{\alpha_f = 0} \alpha_f \tag{6}$$

$$F_{yr} = -2C_r \big|_{\alpha_r = 0} \alpha_r \tag{7}$$

where v_y is lateral velocity, and δ_f is steering angle. If the longitudinal velocity of the vehicle is constant, the derivative of the side-slip angle (8) is expressed as (9).

$$\beta \approx \frac{v_y}{v_x} \tag{8}$$

$$\dot{\beta} = \frac{\dot{v}_y}{v_x} \tag{9}$$

Using the above equations, the following state-space equations of the bicycle model are derived.

$$\dot{x} = Ax + Bu, \quad A \in \mathbb{R}^{2 \times 2}, \ B \in \mathbb{R}^{2 \times 1}$$
(10)

$$y = Cx, \quad C \in \mathbb{R}^{1 \times 2} \tag{11}$$

where,

$$A = \begin{bmatrix} -\frac{2(C_f + C_r)}{mv_x} & \frac{2(C_r l_r - C_f l_f)}{mv_x^2} - 1\\ \frac{2(C_r l_r - C_f l_f)}{I_z} & -\frac{2(C_f l_f^2 + C_r l_r^2)}{I_z v_x} \end{bmatrix},$$
$$B = \begin{bmatrix} \frac{2C_f}{mv_x}\\ \frac{2C_f l_f}{I_z} \end{bmatrix},$$
$$C = \begin{bmatrix} 0 & 1 \end{bmatrix}, \quad x = \begin{bmatrix} \beta\\ r \end{bmatrix}, \text{ and } y = r.$$

B. SIMPLE BICYCLE MODEL-BASED OBSERVER

For the estimation of the side-slip angle, a simple Luenberger observer is designed from (10) and (11). Since it is a linear time-invariant system, the error value is made to approach zero by the negative pole-placement method. This satisfies the asymptotical stability.

$$\hat{x} = A\hat{x} + Bu + K_b(y - \hat{y}), \quad K_b \in \mathbb{R}^{2 \times 1}$$
 (12)

$$\hat{y} = C\hat{x}, \quad C \in \mathbb{R}^{1 \times 2} \tag{13}$$

If the influence of the model uncertainty and sensor error is neglected, the observer gain value K_b can be obtained through the error dynamics equation. To satisfy the stability of the system, all poles must have negative values. In here, all poles are designed to have the same value of p for simplicity.

$$\dot{\tilde{x}} = (A - K_b C)\tilde{x} \tag{14}$$

$$det(sI - (A - K_bC)) = (s - p)^2, \quad p < 0$$
 (15)

The gain value to meet the desired pole location, p, is calculated as follows from (15).

$$K_{b1} = -\frac{I_z p^2}{2(C_f l_f - C_r l_r)} - \frac{2(C_f l_f - C_r l_r)}{mv_r} + 1 \quad (16)$$

$$K_{b2} = -2\frac{(C_f + C_r) + (C_f l_f^2 + C_r l_r^2) + p}{I_z m v_x}$$
(17)

C. KINEMATIC MODEL

Bicycle model-based observer has the issues of sensitivity to model parametric uncertainty and oversensitivity to road conditions. In contrast, the kinematic model shows an independent nature regarding the vehicle model parameters and road conditions. However, since it uses the longitudinal/lateral accelerometer and the estimated longitudinal velocity, the kinematic model is not robust to the sensor error, and thus requires an appropriate compensation. Fig. 2 describes the kinematic model used for the vehicle motion.

$$\dot{v}_x = r(t)v_y + a_x + w_x$$
 (18)

$$\dot{v}_y = -r(t)v_x + a_y + w_y$$
 (19)



(b) vehicle motion

FIGURE 2. Kinematic model.

$$y = v_x + s_x \tag{20}$$

where w_x and w_y mean unknown uncertainties.

The kinematic model is expressed as a linear time-variant system in terms of the yaw rate r(t). Here, the measurement, y represents a longitudinal vehicle speed which is estimated based on the undriven wheel speeds. Therefore, it includes the pure longitudinal velocity, v_x and the sensor error, s_x which is the uncertainty due to the roll, pitch and others.

D. KINEMATIC MODEL-BASED OBSERVER

For the sideslip angle estimation, a Luenberger observer is designed from (18), (19) and (20) as follows.

$$\dot{\hat{x}} = (A_k(t) - K_k(t)C_k)\hat{x} + B_kt)u + K_k(t)y, \quad K_k \in \mathbb{R}^{2 \times 1} \quad (21)$$
$$\dot{\tilde{x}} = (A_k(t) - K_k(t)C_k)\tilde{x} + w - K_k(t)s_x \quad (22)$$

where,

$$A_k(t) = \begin{bmatrix} 0 & r(t) \\ -r(t) & 0 \end{bmatrix}, \quad B_k = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$
$$C_k = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad x = \begin{bmatrix} v_x \\ v_y \end{bmatrix}, \text{ and } \quad u = \begin{bmatrix} a_x \\ a_y \end{bmatrix}.$$

Since this observer $K_k(t) = [K_{k1}(t) \quad K_{k2}]^T$ is a time-variant system, the simple negative pole placement method cannot be used. Instead, a frozen-time pole placement method like what is shown in (23) is taken to calculate the stable observer gain. [13]

$$det(sI - (A(t) - K_k(t)C(t))) = (s + \alpha |r(t)|)^2, \ \alpha > 0 \ (23)$$

The primary calculated observer gains are as follows.

$$K_{k1}(t) = 2\alpha |r(t)| \tag{24}$$

$$K_{k2}(t) = (\alpha^2 - 1)r(t)$$
 (25)

From (21) and (22), the gains are observed to vary effectively in the stable region according to the change of the yaw rate. With the observer gain described as (24) and (25), a Lyapunov function V is defined as follows to evaluate the stability of error dynamics.

$$V(\tilde{v}_{x}, \tilde{v}_{y}) = \frac{\alpha^{2} \tilde{v}_{x}^{2} + \tilde{v}_{y}^{2}}{2} > 0, \quad \forall x = [\tilde{v}_{x}, \tilde{v}_{y}] \in R^{2}$$
(26)
$$\frac{dV(\tilde{v}_{x}, \tilde{v}_{y})}{dt} = -2\alpha^{3} |r(t)| \tilde{v}_{x}^{2} + (\alpha^{2} w_{x} - 2\alpha^{3} |r(t)| s_{x}) \tilde{v}_{x}$$

$$+(w_y + (1 - \alpha^2)r(t)s_x)\tilde{v}_y$$
(27)

Assuming that the acceleration sensor input of the kinematic model is properly compensated and the longitudinal velocity estimated from the undriven wheel speeds is fairly accurate, (27) can be rewritten as follows.

$$\dot{\tilde{v}}_x = -2\alpha |r(t)|\tilde{v}_x + r(t)\tilde{v}_y$$
(28)

$$\dot{\tilde{v}}_y = -\alpha^2 r(t) \tilde{v}_x \tag{29}$$

Therefore,

$$\frac{dV(t,x)}{dt} = -2\alpha^3 |r(t)| \tilde{v}_x^2 \le 0, \quad \forall x = [\tilde{v}_x, \tilde{v}_y] \in R^2 - [0,0]$$
(30)

For (28), (29) and (30), the asymptotical stability can be verified via LaSalle's theorem. The estimated lateral velocity can also be expressed in terms of side-slip angle by (8).

E. MODIFIED BICYCLE MODEL-BASED OBSERVER

The conventional bicycle model uses only the yaw rate as a measurement. This paper proposes a type of bicycle model assuming that a vehicle has a lateral accelerometer additionally mounted. The advantage of the original bicycle model in high mu condition, where it effectively estimates the side-slip angle relatively free from the sensor error, and the advantage of the kinematic model in low mu nonlinear tire dynamics region, where it is insensitive to vehicle parameters, are both combined in this model.

Using (3) and (9) can be expressed to include the side-slip angle term and the yaw rate term as follows.

$$a_y = \dot{v}_y + rv_x \approx v_x(\dot{\beta} + r) \tag{31}$$

When the conventional bicycle model is associated with $\dot{\beta}$ in (10), output (11) and (31), then a new output equation is obtained.

$$a_{y} = -\frac{2(C_{f} + C_{r})}{m}\beta - \frac{2(C_{f}l_{f} - C_{r}l_{r})}{mv_{x}}r + \frac{2C_{f}}{m}\delta_{f} \quad (32)$$

Since the modified bicycle model is a linear time-invariant system, stability of the observer can easily be satisfied via negative pole placement method.

The equation of the designed observer is as follows.

$$\hat{x} = A\hat{x} + Bu + K(y - \hat{y}), \quad K \in \mathbb{R}^{2 \times 2}$$
 (33)

$$\hat{y} = C\hat{x} + Du, \quad C \in \mathbb{R}^{2 \times 2}$$
(34)

where,

$$\begin{split} A &= \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \\ &= \begin{bmatrix} -\frac{2(C_f + C_r)}{mv_x} & \frac{2(C_r l_r - C_f l_f)}{mv_x^2} - 1 \\ \frac{2(C_r l_r - C_f l_f)}{I_z} & -\frac{2(C_f l_f^2 + C_r l_r^2)}{I_z v_x} \end{bmatrix}, \\ B &= \begin{bmatrix} \frac{2C_f}{mv_x} \\ \frac{2C_f l_f}{I_z} \end{bmatrix}, \\ C &= \begin{bmatrix} 0 & 1 \\ -\frac{2(C_f + C_r)}{m} & -\frac{2(C_f l_f - C_r l_r)}{mv_x} \end{bmatrix}, \\ K &= \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}, \quad D = \begin{bmatrix} 0 \\ \frac{2C_f}{m} \end{bmatrix}, \hat{x} = \begin{bmatrix} \beta \\ r \end{bmatrix}, \\ u &= \delta_f, \quad y = \begin{bmatrix} r \\ a_y \end{bmatrix}, \quad \hat{y} = \begin{bmatrix} \hat{r} \\ \hat{a}_y \end{bmatrix}, \end{split}$$

where, \hat{a}_y is defined from (32) with β and r replaced by $\hat{\beta}$ and \hat{r} .

Here, the error dynamics concerning the side-slip angle is derived using (10), (33) and (34) as follows.

$$\tilde{\beta} = A_{11}(1 - K_{12}v_x)\tilde{\beta} + A_{12}(1 - K_{12}v_x)\tilde{r} -(K_{11} + K_{12}v_x)\tilde{r}$$
(35)

where, $\tilde{\beta}$ and \tilde{r} are equal to $\beta - \hat{\beta}$ and $r - \hat{r}$.

To provide an independent nature about the model uncertainty, setting the observer gain K_{12} as $\frac{1}{v_x}$ makes the equation considerably simpler, and turning (35) into the following

$$\tilde{\beta} = -(1+K_{11})\tilde{r} \tag{36}$$

Other gain values are defined as follows using negative pole placement method.

$$K = \begin{bmatrix} \frac{I_z(l_f C_f - l_r C_r)p^2}{2C_f C_r(l_f + l_r)^2} - 1 & \frac{1}{v_x} \\ -2p & \frac{m(l_f^2 C_f + l_r^2 C_r)}{I_z(l_f C_f - l_r C_r)} \end{bmatrix}$$
(37)

Final design of the observer is defined as follows.

$$\dot{\hat{\beta}} = -\frac{I_z(C_f l_f - C_r l_r)p^2}{2C_f C_r L^2} \hat{r} + \left(\frac{I_z(C_f l_f - C_r l_r)p^2}{2C_f C_r L^2} - 1\right)r + \frac{a_y}{v_x}$$
(38)
$$\dot{\hat{r}} = -\frac{2C_f C_r L^2}{I_z(C_f l_f - C_r l_r)} \hat{\beta} + 2p\hat{r} - \frac{2C_f C_r L^2}{I_z(C_f l_f - C_r l_r)} \delta_f - 2pr + \frac{m(C_f l_f^2 + C_r l_r^2)}{I_z(C_f l_f - C_r l_r)} a_y$$
(39)

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As it can be seen in (38), lateral acceleration a_y is directly used to estimate the side-slip angle. To use the lateral accelerometer signal for the estimation, the compensation of the gravity component must be done by estimating the roll angle of the vehicle.

III. ROLL ANGLE ESTIMATION FOR COMPENSATING LATERAL ACCELERATION MEASUREMENT

This section discusses the estimation of the roll angle for compensating the lateral accelerometer signal. The roll motion of the vehicle causes the lateral accelerometer to read gravity. This gravity component makes the direct use of the lateral accelerometer very problematic because it causes the sensor offset. Therefore, the exact estimation of the roll angle must be performed to compensate for the lateral accelerometer sensor bias. Many researches have been performed to estimate the roll angle of the vehicle. However, they have the issue of sensing sensitivity due to the nonlinear characteristics or coupling problem with the lateral velocity observer [5], [14]. Recently, the research using GPS has been also in progress. However, application to real vehicles is still in difficulty due to the cost issue as well as the robustness of GPS signal itself [2]. In Tseng's research, the estimation of the overall road bank angle was conducted [16]-[18]. Tseng proposed a method which decouples the effect of the road bank and the lateral dynamics in steady state conditions, but its performance is deteriorated during the transient states where the suspension angle is not necessarily zero even on leveled ground like when a sudden step, slalom, or double lane change steering input is fed. This paper proposes a new roll angle estimation method using a roll index which is defined reflexing the vehicle states such as suspension angle and static bank angle through the contemplation of the previous researches. This paper assumes that the pure sensor offset except the gravity component is compensated already using sensor zeroing process [26], [27].

A. ESTIMATION OF SUSPENSION ROLL ANGLE

The rolling of a vehicle can occur due to the centripetal force caused by the lateral force and road bank angle as described in (40) and also shown in Fig. 3.

$$\Phi_{roll} = \Phi_{sus} + \Phi_{bank} \tag{40}$$

While the vehicle is making a flat surface maneuver, rolling is caused by the suspension system in motion. Suspension model is often considered as equivalent to a spring and damper system. Here, to derive the suspension model, the following assumptions are made.

- roll axis is not varying with the vehicle roll angle.
- roll stiffness and damping coefficient are constants.
- pure heave motion of the vehicle is negligible.
- The center of gravity position and mass of the vehicle are constants.

With these assumptions, a 2nd-order dynamic model of the suspension is derived as follows.

$$I_r \ddot{\Phi}_{sus} = ma_{ym}h - d_{sus}\dot{\Phi}_{sus} - k_{sus}\Phi_{sus}$$
(41)



FIGURE 3. Vehicle roll model (ISO 8855 coordinate).

$$I_r = I_{C.G} + mh^2 \tag{42}$$

In this paper, suspension roll angle is estimated via open-loop integration method assuming that a roll rate sensor is absent due to a cost issue. Also at a steady state, (41) can be further simplified as follows.

$$\Phi_{sus} \approx \Phi_{sus-ss} = \frac{hm}{k_{sus}} a_{ym} \tag{43}$$

(43) can be used to compensate for the steady state bias of the lateral acceleration sensor signal due to gravity.

B. ESTIMATION OF STATIC BANK ANGLE

(3) can be expressed as follows using the measured lateral acceleration a_{ym} and the vehicle roll angle Φ_{roll} .

$$a_{ym} + g\sin(\Phi_{roll}) = \dot{v}_y + rv_x \tag{44}$$

Therefore,

$$\Phi_{roll} = \sin^{-1}(\frac{\dot{v}_y + rv_x - a_{ym}}{g}) \approx \frac{\dot{v}_y + rv_x - a_{ym}}{g} \quad (45)$$

In (44), \dot{v}_y is not the readily available information. When the lateral motion of a vehicle is mild enough to be neglected in presence of the road bank angle, i.e. $\dot{v}_y \approx 0$, (45) is expressed as follows.

$$\Phi_{roll} = \Phi_{ss} = \sin^{-1} \left(\frac{rv_x - a_{ym}}{g} \right) \approx \frac{rv_x - a_{ym}}{g} \quad (46)$$

Here, Φ_{ss} is defined as the static bank angle.

C. ESTIMATION OF DYNAMIC BANK ANGLE USING DFC

The estimation of static bank angle degenerates rapidly in the transient region where the vehicle dynamics controller is usually activated. For this reason, Tseng proposed a method to decouple the lateral dynamics component from the road bank angle based on the static bank angle estimated during the steady state maneuver [16]. When (43) is applied to enhance (10) to banked cases, the following relationship can be derived.

$$\dot{\hat{\beta}} = -\frac{2(C_f + C_r)}{mv_x}\beta - \frac{2(C_f l_f + C_R l_r)}{mv_x^2}r + \frac{2C_f}{mv_x}\delta_f + \frac{g}{v_x}\sin(\Phi_{roll})$$
(47)
$$\dot{\hat{r}} = -\frac{2(C_f l_f + C_R l_r)}{I_z}\beta - \frac{2(C_f l_f^2 + C_R l_r^2)}{I_z v_x}r + \frac{2C_f l_f}{I_z}\delta_f$$
(48)

If the transfer functions at a steady state are obtained from (47) and (48), they are described as follows. Here, inputs are Φ_{roll} and δ_f , while outputs are a_y and r. L is the length of the wheel base.

$$H_{\Phi_{roll}\mapsto a_y} = \frac{v_x^2 k_u g}{L + k_u v_x^2} - g \tag{49}$$

$$H_{\Phi_{roll}\mapsto r} = \frac{v_x k_u g}{L + k_u v_x^2} \tag{50}$$

$$H_{\delta_f \mapsto a_y} = \frac{v_x^2}{L + k_u v_x^2} \tag{51}$$

$$H_{\delta_f \mapsto r} = \frac{v_x}{L + k_u v_x^2} \tag{52}$$

Tseng showed the validity of superpositioning for the following equations with some experimental evidences.

$$a_{y} = H_{\delta_{f} \mapsto a_{y}} \cdot \delta_{f} + H_{\Phi_{roll} \mapsto a_{y}} \cdot \Phi_{roll}$$
(53)

$$r = H_{\delta_f \mapsto r} \cdot \delta_f + H_{\Phi_{roll} \mapsto r} \cdot \Phi_{roll}$$
(54)

Here, the roll angles due to a_y and r are defined as follows.

$$\sin(\hat{\Phi}_{a_y}) = H_{\Phi_{roll} \mapsto a_y}^{-1} (a_{ym} - H_{\delta_f \mapsto a_y} \cdot \delta_f)$$
(55)

$$\sin(\hat{\Phi}_r) = H_{\Phi_{roll} \mapsto r}^{-1} (r - H_{\delta_f \mapsto r} \cdot \delta_f)$$
(56)

Tseng defined a dynamic factor (DFC) indicating the magnitude of \dot{v}_y that is directly related to the lateral dynamics component as follows.

$$DFC = H_{\Phi_{roll} \mapsto a_y} \cdot (\sin \hat{\Phi}_{a_y} - \sin \hat{\Phi}_{ss}) + v_x \cdot H_{\Phi_roll \mapsto r} \cdot (\sin \hat{\Phi}_r - \sin \hat{\Phi}_{ss})$$
(57)

In the next equation, dynamic road bank angle is defined using DFC declared in (57).

$$\sin(\hat{\Phi}_{dyn}) = \sin(\hat{\Phi}_{ss}) \cdot \max[0, 1 - |DFC| - |d\sin\hat{\Phi}_{ss}/dt|]$$
(58)

D. ESTIMATION OF ROLL ANGLE USING ROLL INDEX

The approach proposed by Tseng and shown in (58) has a problem of canceling suspension roll angle in the dynamic region where DFC is greater than 1. For this issue, a solution is given by multiplying weighting factors according to the frequency of the suspension roll angle and the road bank angle. [18]

$$\Phi_{roll} = \Phi_{dyn_freq} + \Phi_{ss_freq} \tag{59}$$

However, this method has some problem in real-time implementation, for it requires numerous calculations. This paper defines a roll index λ to indicate vehicle states, concerning the characteristic of the roll angle that consists of the combination of the road bank disturbance and the suspension roll angle caused by steering inputs. Using the roll index which indicates the transient or the steady state region, a new algorithm to estimate the overall roll angle is proposed. The roll index is defined considering the correlation of the static angles defined in (46), (55) and (56). In addition, vehicle states are distinguished by the yaw rate induced by the driver's steering inputs and road bank angle. Finally, the roll index λ is defined as follows.

$$\lambda = f\left(\dot{\Phi}_{ss}, \dot{\Phi}_{ay}, \dot{\Phi}_{r}\right) \cdot g\left(\hat{\Phi}_{ay}, \hat{\Phi}_{r}\right) \cdot h\left(\delta_{f}, \dot{\delta}_{f}, r\right) \cdot k\left(\delta_{f}, r\right)$$
(60)

where,

$$f = \begin{cases} 1 & if \quad \left| \frac{d}{dt} (|\dot{\Phi}_{a_y} - \dot{\Phi}_{ss}| - |\dot{\Phi}_r - \dot{\Phi}_{ss}|) \right| < F_{TH} \\ & \text{during} \quad t_{TH1} \\ 0 & \text{otherwise} \end{cases}$$
$$g = \begin{cases} 1 & if \quad \hat{\Phi}_{a_y} \cdot \hat{\Phi}_r < 0 \quad \text{during} \quad t_{TH2} \\ 0 & \text{otherwise} \end{cases}$$
$$h = \begin{cases} 1 & if \quad \left| \delta_f \right| < G_{TH1} \quad \text{and} \quad \left| \dot{\delta}_f \right| < G_{TH2} \\ & \text{and} \quad |r| < G_{TH3} \quad \text{during} \quad t_{TH3} \\ 0 & \text{otherwise} \end{cases}$$
$$k = \begin{cases} 1 & if \quad \delta_f \cdot r > K_{TH} \quad \text{during} \quad t_{TH4} \\ 0 & \text{otherwise} \end{cases}$$

Here, *f* is a flag which distinguishes between the steady state and the transient region, based on the difference in the overall shapes of the roll angles calculated from (46), (55) and (56), where F_{TH} is set to 25 deg/s^2 and t_{TH1} 0.4 sec. *g* is a flag that indicates the presence of the road bank angle using $\hat{\Phi}_{ay}$ and $\hat{\Phi}_r$, where t_{TH2} is set to 0.4 sec. *h* is a flag that tells whether the vehicle is running straight on a flat surface or not, using the steering input and yaw rate, where G_{TH1} , G_{TH2} and G_{TH3} are set to 0.5 deg, 0.1 deg/s and 0.3 deg/s respectively. *k* is a flag which shows that the driver has not applied an urgent steering input in the presence of a road bank angle, where K_{TH} is set to 1500 deg^2/s and t_{TH4} 1.5 sec.

Using the roll index the roll angle is estimated as follows. The proposed estimation method considers both suspension roll rate in the transient state and static bank angle in the steady state.

$$\hat{\Phi}_{roll} = (1 - \lambda) \Big(\int_{t_1}^{t_2} \dot{\Phi}_{sus-ss} dt + \hat{\Phi}_{ss}^* \Big) + \lambda \cdot \hat{\Phi}_{ss} \quad (61)$$

Here, $\hat{\Phi}_{ss}^*$ denotes the static bank angle at the moment when the bank index is switching from 1 to 0. Namely, it is used as the initial condition to integrate the suspension roll rate in the region where the roll index is 0. Also, $\hat{\Phi}_{ss}$ and $\hat{\Phi}_{sus-ss}$ are the static bank angle and the derivative of the suspension roll angle.

E. LATERAL ACCELERATION COMPENSATION

The measured lateral acceleration reflects the addition of the practical lateral dynamics component and the gravity component by the influence of the road bank disturbance and the driver's steering input. When it comes to the estimator which uses the lateral acceleration, the performance of the estimation is decreased due to the reading of the gravitational component. In this paper, the measured lateral acceleration is compensated using the estimated roll angle derived in (61) as follows.

$$a_{v} = a_{vm} + g \cdot \sin(\hat{\Phi}_{roll}) \tag{62}$$

The compensated lateral acceleration, a_y is used to estimate the side-slip angle of the vehicle at the proposed side-slip angle observer suggested in (38).

IV. PERFORMANCE VERIFICATION

In order to verify the effectiveness of the proposed method, HIL tests were carried out and the testing platform is shown in Fig. 4. The vehicle runs in a DS1006 real-time computer and the designed roll angle estimator and the side-slip angle observer are implemented in a MicroAutobox. The communications between these two equipment are based on the controller area network (CAN). To verify the performance of the proposed method under various road conditions and different driver's inputs, the vehicle is simulated under various conditions using a production simulation software, CarSim, and the vehicle parameters and simulation scenarios are shown in TABLE 1 and TABLE 2.



FIGURE 4. Description of Hardware-in-the-loop (HIL) test environment: (a) Real view of HIL test equipment. (b) Components and those functions.

To verify the performance of roll angle estimation, the actual roll angle(roll actual), that obtained by using Tseng's DFC (roll DFC), and that from the proposed roll angle estimator using the roll index are compared. To show

TABLE 1. Parameters of the vehicle model.

Parameter	Symbol	Value	Unit
Vehicle mass	m	1370	kg
Moment of inertia	I_z	4190	$ ext{kg} \cdot ext{m}^2$
Front Cornering stiffness	C_{f}	2000	N/deg
Rear Cornering stiffness	C_r	1600	N/deg
Distance from CG to front	l_{f}	1.110	m
Distance from CG to rear	l_r	1.666	m
C.G. Height above roll axis	h	0.54	m

TABLE 2. HIL test scenario.

Case.I	· Step steering
	• high mu(0.85)
	· 80kph
	· flat surface
Case.II	• Slalom steering(0.5Hz)
	· low mu(0.25)
	• 140kph
	• flat surface
Case.III	Straight and bank turn
	• high mu(0.85)
	· 80kph
	· bank angle($0 \sim 25$ deg)
Case.IV	· Double lane change steering
	• high mu(0.85
	· 80kph
	· bank angle($0 \sim 16$ deg)

the effect of compensating the gravity component out of the measured lateral acceleration, the pure lateral acceleration free from gravity component (actual), measured lateral acceleration (ay w G), and compensated acceleration(ay w/o G) are compared. Furthermore, the performance of the suggested side-slip angle observer using the modified bicycle model(mod-bic) is compared with those of different models presented in section II such as the simple bicycle model(simp-bic) and the kinematic model(kin). The proposed observer shows the effect of the lateral acceleration compensation by comparing the cases of using compensated lateral acceleration(mod-bic) and uncompensated one(mod-bic-w/g).

Fig. 5 and Fig. 6 display the results of dynamic steering maneuvers obtained in the absence of the road bank angle. Fig. 5 shows the case in which a step steer input is fed under the high mu condition like on asphalt, and Fig. 6 considers the case in which a slalom steer input is fed under the low mu condition like on ice. It is observed that the estimation using the roll index which takes the effect of the suspension angle into consideration is more accurate than that in Tseng's work where it only considers the presence of bank angle. Also, in Fig. 5(b) and Fig. 6(b), it is observed that the lateral acceleration not compensated by the estimated roll angle is biased up to $0.5m/s^2$, unlike the cases in which the lateral acceleration is compensated. This biased component leads to the divergence of the estimated side-slip angle as shown in Fig. 5(c) and Fig. 6(c). Concerning the side-slip angle



FIGURE 5. HIL test results for Case.I: (a) Roll angle estimation, (b) Lateral acceleration compensation, and (c) Side-slip angle estimation.

estimation, it is demonstrated that the modified bicycle model gives the most reliable performance regardless of the friction coefficient of the road.

Fig. 7 and Fig. 8 show the results for the case when the road bank disturbance is present. The HIL test result of the vehicle that first runs straight on the flat ground, turns left on the curved road with the road bank angle and then again proceeds onto the straight flat road is given in Fig. 7. Here, it must be addressed that the steering angle is near zero even when the vehicle is turning left, for it turns naturally due







FIGURE 6. HIL test results for Case.II: (a) Roll angle estimation, (b) Lateral acceleration compensation, and (c) Side-slip angle estimation.

to the severe bank angle. The HIL test result of the vehicle that goes through a double lane change via steering input on the banked road is demonstrated in Fig. 8. In Fig. 7(a), the roll angle is estimated for hardly any steering angle, and the roll angle estimation performance of either method, i.e. using Tseng's DFC or using the roll index, is proven to be effective. It can be seen in Fig. 7(a) that only an insignificant amount of delay exists in the estimation result obtained by the roll index model when the bank angle is present (at t=5).



FIGURE 7. HIL test results for Case.III: (a) Roll angle estimation, (b) Lateral acceleration compensation, and (c) Side-slip angle estimation.

This delay is observed to cause some error in compensating for the lateral acceleration, whose result is shown in Fig. 7(b). However, the effectiveness of compensating the lateral acceleration through deleting the gravity component from the road bank is still clearly shown. This improvement is a significant advantage when it comes to estimation of the side-slip angle as well, as shown in Fig. 7(c). Fig. 8(a) is the case in which the suspension roll angle is present. It demonstrates the case in which the suspension roll angle is present due to the steering input on the bank, unlike that in Case III. The proposed method in this paper has some error in roll angle estimation







FIGURE 8. HIL test results for Case.IV: (a) Roll angle estimation, (b) Lateral acceleration compensation, and (c) Side-slip angle estimation.

but still shows some improvement compared to the existing methods. Improvement of side-slip estimation in Fig. 8(c) is due to the better lateral acceleration compensation shown in Fig. 8(b).

V. CONCLUSION

This paper presents a side-slip angle observation method using the lateral accelerometer signal directly. To compensate the lateral accelerometer signal bias due to gravity, a vehicle roll angle estimator is designed. The performance of this observer has been verified through HIL tests under various road conditions and steering inputs. As the results show a near-perfect compensation of the lateral acceleration bias improves the performance of the side-slip angle observer significantly. However, the proposed roll angle estimator is found to have a slight amount of error concerning the roll angle estimation due to the switching delay of the roll index. In the future, the model must be supplemented by methods to change the roll index continuously, and to improve the time delay issue due to the binary nature of the roll index. Also, the effect of wide parameter variation such as the longitudinal speed must be considered to expand the application of this study to more dynamic vehicle maneuvers.

APPENDIX

NOMENCLATURE

a_x actual longitudinal acceleration. a_y actual lateral acceleration. a_{ym} measured lateral acceleration. d_{stus} roll damping coefficient. k_{stus} roll stiffness coefficient. F_{yf} lateral force of the frontal tire. F_{yr} lateral force of the rear tire. $I_{C.G}$ roll moment of inertia from C.G I_r roll moment of inertia. h height of C.G. from roll axis. r yaw moment of inertia. h height of C.G. to frontal axle. I_f distance from C.G. to frontal axle. I_r distance from C.G. to rear axle. v_x true longitudinal velocity. v_y true lateral velocity. α_f rear tire side-slip angle. α_r rear tire cornering stiffness. C_r rear tire cornering stiffness. K_b observer gain of the bicycle model based observer. k_k observer gain of the kinematic model based observer. p designed pole value. s_x longitudinal velocity error. w_x longitudinal modeling error of the kinematic model. w_y lateral modeling error of the kinematic model. β vehicle side-slip angle.	m	sprung mass.
a_y actual lateral acceleration. a_{ym} measured lateral acceleration. d_{sus} roll damping coefficient. k_{sus} roll stiffness coefficient. F_{yf} lateral force of the frontal tire. F_{yr} lateral force of the rear tire. $I_{C.G}$ roll moment of inertia from C.G I_r roll moment of inertia. h height of C.G. from roll axis. r yaw moment of inertia. h height of C.G. to frontal axle. I_r distance from C.G. to frontal axle. I_r distance from C.G. to rear axle. v_x true longitudinal velocity. v_y true lateral velocity. v_y true lateral velocity. α_r rear tire side-slip angle. C_f frontal tire cornering stiffness. C_r rear tire cornering stiffness. K_b observer gain of the bicycle model based observer. K_k observer gain of the kinematic model based observer. p designed pole value. s_x longitudinal velocity error. w_x longitudinal modeling error of the kinematic model. β vehicle side-slip angle.	a_x	actual longitudinal acceleration.
$\begin{array}{llllllllllllllllllllllllllllllllllll$	$a_{\rm v}$	actual lateral acceleration.
d_{sues} roll damping coefficient. k_{sus} roll stiffness coefficient. F_{yf} lateral force of the frontal tire. F_{yr} lateral force of the rear tire. $I_{C.G}$ roll moment of inertia from C.G I_r roll moment of inertia. h height of C.G. from roll axis. r yaw rate. L wheel base length. l_f distance from C.G. to frontal axle. l_r distance from C.G. to rear axle. v_x true longitudinal velocity. v_y true lateral velocity. α_r rear tire side-slip angle. α_r rear tire cornering stiffness. C_r rear tire cornering stiffness. K observer gain of the bicycle model based observer. k_k observer gain of the kinematic model based observer. p designed pole value. s_x longitudinal velocity error. w_x longitudinal modeling error of the kinematic model. β vehicle side-slip angle.	a_{vm}	measured lateral acceleration.
k_{sus} roll stiffness coefficient. F_{yf} lateral force of the frontal tire. F_{yr} lateral force of the rear tire. $I_{C.G}$ roll moment of inertia from C.G I_r roll moment of inertia with respect to roll center. I_z yaw moment of inertia. h height of C.G. from roll axis. r yaw rate. L wheel base length. l_f distance from C.G. to frontal axle. l_r distance from C.G. to rear axle. v_x true longitudinal velocity. v_y true lateral velocity. α_f frontal tire side-slip angle. C_f frontal tire cornering stiffness. C_r rear tire cornering stiffness. K observer gain of the bicycle model based observer. K_k observer gain of the kinematic model based observer. p designed pole value. s_x longitudinal velocity error. w_x longitudinal modeling error of the kinematic model. β vehicle side-slip angle.	d _{sus}	roll damping coefficient.
F_{yf} lateral force of the frontal tire. F_{yr} lateral force of the rear tire. $I_{C.G}$ roll moment of inertia from C.G I_r roll moment of inertia with respect to roll center. I_z yaw moment of inertia. h height of C.G. from roll axis. r yaw rate. L wheel base length. l_f distance from C.G. to frontal axle. l_r distance from C.G. to rear axle. v_x true longitudinal velocity. v_y true lateral velocity. α_f frontal tire side-slip angle. C_f frontal tire cornering stiffness. C_r rear tire cornering stiffness. K observer gain of the bicycle model based observer. K_k observer gain of the kinematic model based observer. p designed pole value. s_x longitudinal velocity error. w_x longitudinal modeling error of the kinematic model. β vehicle side-slip angle.	k _{sus}	roll stiffness coefficient.
F_{yr} lateral force of the rear tire. $I_{C.G}$ roll moment of inertia from C.G I_r roll moment of inertia with respect to roll center. I_z yaw moment of inertia. h height of C.G. from roll axis. r yaw rate. L wheel base length. l_f distance from C.G. to frontal axle. l_r distance from C.G. to rear axle. v_x true longitudinal velocity. v_y true lateral velocity. α_f rear tire side-slip angle. α_r rear tire side-slip angle. C_f frontal tire cornering stiffness. C_r rear tire cornering stiffness. K observer gain of the modified bicycle model based observer. K_b observer gain of the bicycle model based observer. p designed pole value. s_x longitudinal velocity error. w_x longitudinal modeling error of the kinematic model. β vehicle side-slip angle.	F_{vf}	lateral force of the frontal tire.
$\begin{array}{llllllllllllllllllllllllllllllllllll$	F_{vr}	lateral force of the rear tire.
$\begin{array}{ll} I_r & \mbox{roll moment of inertia with respect to roll center.}\\ I_z & \mbox{yaw moment of inertia.}\\ h & \mbox{height of C.G. from roll axis.}\\ r & \mbox{yaw rate.}\\ L & \mbox{wheel base length.}\\ I_f & \mbox{distance from C.G. to frontal axle.}\\ I_r & \mbox{distance from C.G. to rear axle.}\\ v_x & \mbox{true longitudinal velocity.}\\ v_y & \mbox{true lateral velocity.}\\ \alpha_f & \mbox{frontal tire side-slip angle.}\\ C_f & \mbox{frontal tire cornering stiffness.}\\ C_r & \mbox{rear tire cornering stiffness.}\\ K & \mbox{observer gain of the modified bicycle model}\\ & \mbox{based observer.}\\ K_k & \mbox{observer gain of the kinematic model based}\\ & \mbox{observer.}\\ p & \mbox{designed pole value.}\\ s_x & \mbox{longitudinal velocity error.}\\ w_x & \mbox{longitudinal modeling error of the kinematic model.}\\ \beta & \mbox{vehicle side-slip angle.}\\ \Phi_{roll} & \mbox{total vehicle roll angle.}\\ \end{array}$	$I_{C.G}$	roll moment of inertia from C.G
$\begin{array}{llllllllllllllllllllllllllllllllllll$	I_r	roll moment of inertia with respect to roll center.
h height of C.G. from roll axis. r yaw rate. L wheel base length. l_f distance from C.G. to frontal axle. l_r distance from C.G. to rear axle. v_x true longitudinal velocity. v_y true lateral velocity. α_f frontal tire side-slip angle. α_r rear tire side-slip angle. C_f frontal tire cornering stiffness. C_r rear tire cornering stiffness. K observer gain of the modified bicycle model based observer. K_b observer gain of the bicycle model based observer. F designed pole value. s_x longitudinal velocity error. w_x longitudinal modeling error of the kinematic model. w_y lateral modeling error of the kinematic model. β vehicle side-slip angle. β roll angle.	I_z	yaw moment of inertia.
r yaw rate. L wheel base length. l_f distance from C.G. to frontal axle. l_r distance from C.G. to rear axle. v_x true longitudinal velocity. v_y true lateral velocity. α_f frontal tire side-slip angle. α_r rear tire side-slip angle. C_f frontal tire cornering stiffness. C_r rear tire cornering stiffness. K observer gain of the modified bicycle model based observer. K_b observer gain of the bicycle model based observer. K_k observer gain of the kinematic model based observer. p designed pole value. s_x longitudinal velocity error. w_x longitudinal modeling error of the kinematic model. w_y lateral modeling error of the kinematic model. β vehicle side-slip angle. Φ_{roll} total vehicle roll angle.	h	height of C.G. from roll axis.
$ \begin{array}{ll} L & \mbox{wheel base length.} \\ l_f & \mbox{distance from C.G. to frontal axle.} \\ l_r & \mbox{distance from C.G. to rear axle.} \\ v_x & \mbox{true longitudinal velocity.} \\ v_y & \mbox{true lateral velocity.} \\ \alpha_f & \mbox{frontal tire side-slip angle.} \\ \alpha_r & \mbox{rear tire side-slip angle.} \\ C_f & \mbox{frontal tire cornering stiffness.} \\ C_r & \mbox{rear tire cornering stiffness.} \\ K & \mbox{observer gain of the modified bicycle model based observer.} \\ K_b & \mbox{observer gain of the bicycle model based observer.} \\ K_k & \mbox{observer gain of the kinematic model based observer.} \\ p & \mbox{designed pole value.} \\ s_x & \mbox{longitudinal velocity error.} \\ w_x & \mbox{longitudinal modeling error of the kinematic model.} \\ \beta & \mbox{vehicle side-slip angle.} \\ \Phi_{roll} & \mbox{total vehicle roll angle.} \\ \end{array} $	r	yaw rate.
$\begin{array}{ll} l_f & \mbox{distance from C.G. to frontal axle.}\\ l_r & \mbox{distance from C.G. to rear axle.}\\ v_x & \mbox{true longitudinal velocity.}\\ v_y & \mbox{true lateral velocity.}\\ \alpha_f & \mbox{frontal tire side-slip angle.}\\ \alpha_r & \mbox{rear tire side-slip angle.}\\ C_f & \mbox{frontal tire cornering stiffness.}\\ K & \mbox{observer gain of the modified bicycle model}\\ & \mbox{based observer.}\\ K_b & \mbox{observer gain of the bicycle model based}\\ & \mbox{observer.}\\ K_k & \mbox{observer gain of the kinematic model based}\\ & \mbox{observer.}\\ p & \mbox{designed pole value.}\\ s_x & \mbox{longitudinal velocity error.}\\ w_x & \mbox{longitudinal modeling error of the kinematic model.}\\ \phi_{roll} & \mbox{vehicle side-slip angle.}\\ \Phi_{roll} & \mbox{total vehicle roll angle.}\\ \end{array}$	L	wheel base length.
$ \begin{array}{ll} l_r & \mbox{distance from C.G. to rear axle.} \\ v_x & \mbox{true longitudinal velocity.} \\ v_y & \mbox{true lateral velocity.} \\ \alpha_f & \mbox{frontal tire side-slip angle.} \\ \alpha_r & \mbox{rear tire side-slip angle.} \\ C_f & \mbox{frontal tire cornering stiffness.} \\ C_r & \mbox{rear tire cornering stiffness.} \\ K & \mbox{observer gain of the modified bicycle model based observer.} \\ K_b & \mbox{observer gain of the bicycle model based observer.} \\ K_k & \mbox{observer gain of the kinematic model based observer.} \\ p & \mbox{designed pole value.} \\ s_x & \mbox{longitudinal velocity error.} \\ w_x & \mbox{longitudinal modeling error of the kinematic model.} \\ w_y & \mbox{lateral modeling error of the kinematic model.} \\ \beta & \mbox{vehicle side-slip angle.} \\ \Phi_{roll} & \mbox{total vehicle roll angle.} \\ \end{array}$	l_f	distance from C.G. to frontal axle.
$\begin{array}{lll} v_x & \mbox{true longitudinal velocity.} \\ v_y & \mbox{true lateral velocity.} \\ \alpha_f & \mbox{frontal tire side-slip angle.} \\ \alpha_r & \mbox{rear tire side-slip angle.} \\ C_f & \mbox{frontal tire cornering stiffness.} \\ C_r & \mbox{rear tire cornering stiffness.} \\ K & \mbox{observer gain of the modified bicycle model} \\ & \mbox{based observer.} \\ K_b & \mbox{observer gain of the bicycle model based} \\ & \mbox{observer.} \\ K_k & \mbox{observer gain of the kinematic model based} \\ & \mbox{observer.} \\ p & \mbox{designed pole value.} \\ s_x & \mbox{longitudinal velocity error.} \\ w_x & \mbox{longitudinal modeling error of the kinematic model.} \\ w_y & \mbox{lateral modeling error of the kinematic model.} \\ \beta & \mbox{vehicle side-slip angle.} \\ \Phi_{roll} & \mbox{total vehicle roll angle.} \\ \end{array}$	ľ _r	distance from C.G. to rear axle.
$\begin{array}{lll} v_y & \mbox{true lateral velocity.} \\ \alpha_f & \mbox{frontal tire side-slip angle.} \\ \alpha_r & \mbox{rear tire side-slip angle.} \\ C_f & \mbox{frontal tire cornering stiffness.} \\ C_r & \mbox{rear tire cornering stiffness.} \\ K & \mbox{observer gain of the modified bicycle model} \\ & \mbox{based observer.} \\ K_b & \mbox{observer gain of the bicycle model based} \\ & \mbox{observer.} \\ K_k & \mbox{observer gain of the kinematic model based} \\ & \mbox{observer.} \\ p & \mbox{designed pole value.} \\ s_x & \mbox{longitudinal velocity error.} \\ w_x & \mbox{longitudinal modeling error of the kinematic model.} \\ w_y & \mbox{lateral modeling error of the kinematic model.} \\ \beta & \mbox{vehicle side-slip angle.} \\ \Phi_{roll} & \mbox{total vehicle roll angle.} \\ \end{array}$	v_x	true longitudinal velocity.
$ \begin{array}{lll} \alpha_f & \mbox{frontal tire side-slip angle.} \\ \alpha_r & \mbox{rear tire side-slip angle.} \\ C_f & \mbox{frontal tire cornering stiffness.} \\ C_r & \mbox{rear tire cornering stiffness.} \\ K & \mbox{observer gain of the modified bicycle model based observer.} \\ K_b & \mbox{observer gain of the bicycle model based observer.} \\ K_k & \mbox{observer gain of the kinematic model based observer.} \\ p & \mbox{designed pole value.} \\ s_x & \mbox{longitudinal velocity error.} \\ w_x & \mbox{longitudinal modeling error of the kinematic model.} \\ \phi_{roll} & \mbox{vehicle side-slip angle.} \\ \Phi_{roll} & \mbox{total vehicle roll angle.} \\ \end{array} $	v_y	true lateral velocity.
$ \begin{array}{lll} \alpha_r & \mbox{rear tire side-slip angle.} \\ C_f & \mbox{frontal tire cornering stiffness.} \\ C_r & \mbox{rear tire cornering stiffness.} \\ K & \mbox{observer gain of the modified bicycle model based observer.} \\ K_b & \mbox{observer gain of the bicycle model based observer.} \\ K_k & \mbox{observer gain of the kinematic model based observer.} \\ p & \mbox{designed pole value.} \\ s_x & \mbox{longitudinal velocity error.} \\ w_x & \mbox{longitudinal modeling error of the kinematic model.} \\ \phi_{roll} & \mbox{vehicle side-slip angle.} \\ \Phi_{roll} & \mbox{total vehicle roll angle.} \\ \end{array} $	α_f	frontal tire side-slip angle.
$\begin{array}{lll} C_f & \mbox{frontal tire cornering stiffness.} \\ C_r & \mbox{rear tire cornering stiffness.} \\ K & \mbox{observer gain of the modified bicycle model} \\ & \mbox{based observer.} \\ K_b & \mbox{observer gain of the bicycle model} \\ & \mbox{observer.} \\ K_k & \mbox{observer gain of the kinematic model} \\ & \mbox{baserver.} \\ p & \mbox{designed pole value.} \\ s_x & \mbox{longitudinal velocity error.} \\ w_x & \mbox{longitudinal modeling error of the kinematic model.} \\ & \mbox{wy} & \mbox{lateral modeling error of the kinematic model.} \\ & \mbox{vehicle side-slip angle.} \\ & \mbox{Φ_{roll}} & \mbox{total vehicle roll angle.} \\ \end{array}$	α_r	rear tire side-slip angle.
$ \begin{array}{lll} \vec{C_r} & \mbox{rear tire cornering stiffness.} \\ K & \mbox{observer gain of the modified bicycle model} \\ & \mbox{based observer.} \\ K_b & \mbox{observer gain of the bicycle model} \\ & \mbox{based observer.} \\ K_k & \mbox{observer gain of the kinematic model} \\ & \mbox{baserver.} \\ p & \mbox{designed pole value.} \\ s_x & \mbox{longitudinal velocity error.} \\ w_x & \mbox{longitudinal modeling error of the kinematic model.} \\ & \mbox{wy} & \mbox{lateral modeling error of the kinematic model.} \\ & \mbox{w}_y & \mbox{lateral modeling error of the kinematic model.} \\ & \mbox{vehicle side-slip angle.} \\ & \mbox{\Phi}_{roll} & \mbox{total vehicle roll angle.} \\ \end{array} $	C_f	frontal tire cornering stiffness.
$ \begin{array}{lll} K & & \text{observer gain of the modified bicycle model} \\ & & \text{based observer.} \\ K_b & & \text{observer gain of the bicycle model based} \\ & & \text{observer. gain of the kinematic model based} \\ & & \text{observer.} \\ p & & \text{designed pole value.} \\ s_x & & \text{longitudinal velocity error.} \\ w_x & & \text{longitudinal modeling error of the kinematic} \\ & & \text{model.} \\ w_y & & \text{lateral modeling error of the kinematic model.} \\ \beta & & \text{vehicle side-slip angle.} \\ \Phi_{roll} & & \text{total vehicle roll angle.} \\ \end{array} $	$\dot{C_r}$	rear tire cornering stiffness.
$\begin{array}{llllllllllllllllllllllllllllllllllll$	Κ	observer gain of the modified bicycle model
$ \begin{array}{ll} K_b & & \text{observer gain of the bicycle model based} \\ & & \text{observer.} \\ K_k & & \text{observer gain of the kinematic model based} \\ & & \text{observer.} \\ p & & \text{designed pole value.} \\ s_x & & \text{longitudinal velocity error.} \\ w_x & & \text{longitudinal modeling error of the kinematic} \\ & & \text{model.} \\ w_y & & \text{lateral modeling error of the kinematic model.} \\ \beta & & \text{vehicle side-slip angle.} \\ \Phi_{roll} & & \text{total vehicle roll angle.} \end{array} $		based observer.
$ \begin{array}{ll} & \mbox{observer.} \\ K_k & \mbox{observer gain of the kinematic model based} \\ & \mbox{observer.} \\ p & \mbox{designed pole value.} \\ s_x & \mbox{longitudinal velocity error.} \\ w_x & \mbox{longitudinal modeling error of the kinematic model.} \\ w_y & \mbox{lateral modeling error of the kinematic model.} \\ \beta & \mbox{vehicle side-slip angle.} \\ \Phi_{roll} & \mbox{total vehicle roll angle.} \end{array} $	K_b	observer gain of the bicycle model based
$ \begin{array}{ll} K_k & \mbox{observer gain of the kinematic model based} \\ & \mbox{observer.} \\ p & \mbox{designed pole value.} \\ s_x & \mbox{longitudinal velocity error.} \\ w_x & \mbox{longitudinal modeling error of the kinematic model.} \\ w_y & \mbox{lateral modeling error of the kinematic model.} \\ \beta & \mbox{vehicle side-slip angle.} \\ \Phi_{roll} & \mbox{total vehicle roll angle.} \end{array} $		observer.
$\begin{array}{llllllllllllllllllllllllllllllllllll$	K_k	observer gain of the kinematic model based
$\begin{array}{lll} p & \mbox{designed pole value.} \\ s_x & \mbox{longitudinal velocity error.} \\ w_x & \mbox{longitudinal modeling error of the kinematic model.} \\ w_y & \mbox{lateral modeling error of the kinematic model.} \\ \beta & \mbox{vehicle side-slip angle.} \\ \Phi_{roll} & \mbox{total vehicle roll angle.} \end{array}$		observer.
$ \begin{array}{ll} s_x & \mbox{longitudinal velocity error.} \\ w_x & \mbox{longitudinal modeling error of the kinematic model.} \\ w_y & \mbox{lateral modeling error of the kinematic model.} \\ \beta & \mbox{vehicle side-slip angle.} \\ \Phi_{roll} & \mbox{total vehicle roll angle.} \end{array} $	р	designed pole value.
w_x longitudinal modeling error of the kinematic model. w_y lateral modeling error of the kinematic model. β vehicle side-slip angle. Φ_{roll} total vehicle roll angle.	S_X	longitudinal velocity error.
model. w_y lateral modeling error of the kinematic model. β vehicle side-slip angle. Φ_{roll} total vehicle roll angle.	W_X	longitudinal modeling error of the kinematic
w_y lateral modeling error of the kinematic model. β vehicle side-slip angle. Φ_{roll} total vehicle roll angle.		model.
βvehicle side-slip angle. $Φ_{roll}$ total vehicle roll angle.	w_y	lateral modeling error of the kinematic model.
Φ_{roll} total vehicle roll angle.	β	vehicle side-slip angle.
	Φ_{roll}	total vehicle roll angle.

 Φ_{ss} static bank angle.

 Φ_{sus-ss} suspension angle at steady state.

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KWANGHYUN CHO (Member, IEEE) received the B.S. degree in electrical engineering and computer science from Kyungpook National University, Daegu, South Korea, in 2008, and the M.S. and Ph.D. degrees in mechanical engineering from the Korea Advanced Institute of Science and Technology (KAIST), Daejeon, South Korea, in 2010 and 2014, respectively. He is currently a Senior Engineer with the Mechatronics Research and Development Center, Samsung Electronics

Company Ltd., Hwaseong, Gyeonggi, South Korea. His research interest includes precision motion control based on linear motors.



HYUNWOO SON received the B.S. degree in electrical engineering and computer science from Kyungpook National University, Daegu, South Korea, in 2007, and the M.S. degree in mechanical engineering from the Korea Advanced Institute of Science and Technology (KAIST), Daejeon, South Korea, in 2009. In 2019, he was a Senior Researcher with the Electrical Engineering Research and Development Center, Doosan Heavy Industries and Construction Company Ltd., where

he was involved with the Development of Advanced Turbine Control System. He is currently a Senior Engineer with the Plant Intelligent Team, Samsung SDS Company Ltd., Seoul, South Korea. His research interests include advanced utility control and data-driven optimization of manufacturing process.



YAFEI WANG (Member, IEEE) received the B.S. degree in internal combustion engine from Jilin University, Changchun, China, in 2005, the M.S. degree in vehicle engineering from Shanghai Jiao Tong University, Shanghai, China, in 2008, and the Ph.D. degree in electrical engineering from The University of Tokyo, Tokyo, Japan, in 2013.

From 2008 to 2010, he was with the Automotive Industry for a period of two years. From 2013 to 2016, he was a Postdoctoral Researcher with The

University of Tokyo. He is currently a Lecturer with the School of Mechanical Engineering, Shanghai Jiao Tong University. His research interests include state estimation and control for connected and automated vehicles.



KANGHYUN NAM (Member, IEEE) received the B.S. degree in mechanical engineering from Kyungpook National University, Daegu, South Korea, in 2007, the M.S. degree in mechanical engineering from the Korea Advanced Institute of Science and Technology, Daejeon, South Korea, in 2009, and the Ph.D. degree in electrical engineering from The University of Tokyo, Tokyo, Japan, in 2012.

From 2012 to 2015, he was a Senior Engineer with Samsung Electronics Company Ltd., Gyeonggi, South Korea. Since 2015, he has been an Assistant Professor with the School of Mechanical Engineering, Yeungnam University, Gyeongbuk, South Korea. His research interests include motion control, vehicle dynamics and control, and electric vehicles.

Dr. Nam is a member of the Korean Society of Automotive Engineers. He received the 2013 Best Paper Award from the IEEE TRANSACTIONS ON INDUSTRIAL ELECTRONICS in 2014.



SEIBUM CHOI (Member, IEEE) received the B.S. degree in mechanical engineering from Seoul National University, Seoul, South Korea, in 1985, the M.S. degree in mechanical engineering from the Korea Advanced Institute of Science and Technology (KAIST), Daejeon, South Korea, in 1987, and the Ph.D. degree in control from the University of California, Berkeley, CA, USA, in 1993.

From 1993 to 1997, he was involved with the Development of Automated Vehicle Control Sys-

tems, Institute of Transportation Studies, University of California. In 2006, he was with TRW, Warren, MI, USA, where he was involved with the Development of Advanced Vehicle Control Systems. Since 2006, he has been with the Department of Mechanical Engineering, KAIST, as a Faculty Member. His current research interests include fuel-saving technologies, vehicle dynamics and control, and active safety systems.