Optimal Tongue Weight Selection Criteria for Light Vehicle-Trailer Combinations

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ARTICLE HISTORY
Compiled September 26, 2024

ABSTRACT
When towing a trailer, proper weight distribution between the vehicle and trailer axles is crucial for system stability and manoeuvrability. Therefore, drivers are strongly recommended to keep the tongue weight, the downward force applied by the trailer at the hitch, within 10-15% of the trailer's gross weight. Despite extensive searches, the current academic and technical literature appears to lack quantitative analyses justifying the standard tongue weight recommendations. This paper presents comprehensive analyses of the lateral behaviour characteristics of light vehicle-trailer combinations influenced by variations in tongue weight. Subsequently, it proposes two novel selection criteria for determining the optimal tongue weight from the perspectives of stability and consistency. The optimal tongue weights for the stability and consistency of the nominal vehicle-trailer system are identified as 22.61% and 48.61% of the trailer weight respectively. Finally, the effectiveness of the proposed criteria is verified through CarSim simulation experiments, assessing 3% settling time and Root-Mean-Square error of vehicle yaw rate.

Abbreviations: VT = Vehicle-Trailer, TW = Tongue Weight

KEYWORDS
vehicle-trailer; towing; optimal design; tongue weight; stability; handling performance

1. Introduction

A vehicle-trailer or articulated vehicle is inherently more unstable and unpredictable in terms of lateral behaviour due to the presence of an articulation point, commonly referred to as a hitch. Due to the additional degree of freedom in the lateral dynamics, a vehicle-trailer system may experience trailer sway or jackknifing, which are not observed in a single-unit vehicle system. Also, for someone accustomed to operating a vehicle without a trailer, the change in the dynamic characteristics of the system may be detrimental for one's comfortable driving [1]. Such unique physical characteristics of the vehicle-trailer systems are well addressed in [1–3] and can be summarised as follows:

- Reduced stability and inferior manoeuvrability

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- Unique lateral behaviours including trailer sway, jackknife, trailer swing
- Phase lag between vehicle and trailer motions
- Rearward amplification (RWA)
- Driver's perception limited to the tow vehicle
- Unstable backward motion

Lateral instabilities in light vehicle-trailer systems result from various causes, including speeding, overloading, improper weight distribution, strong crosswinds, and excessive steering input. While each and every one is equally important, drivers are often required to pay extra attention to check if the weight distribution is done properly or not. There are several reasons: firstly, the other issues are easily avoidable by the drivers or beyond their control; secondly, a vehicle-trailer with improper weight distribution can be highly unstable or uncontrollable even under mild driving conditions; lastly, once the vehicle is in motion, adjusting weight distribution becomes impossible. Despite the importance of proper weight distribution, existing explanations on how and why it should be achieved are insufficient and tend to rely merely on empirical evidence.

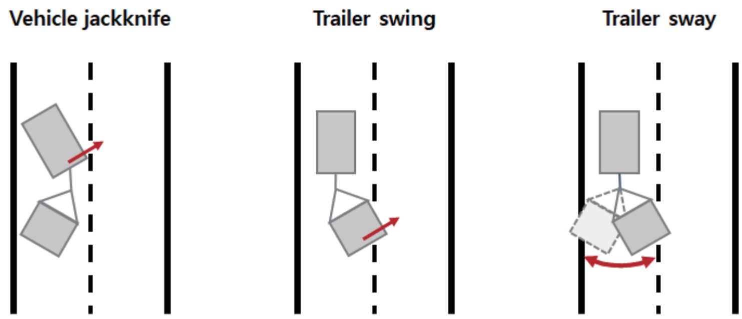


Figure 1. Three types of yaw instabilities in vehicle-trailer systems

The weight distribution for a typical passenger car pulling a travel trailer is determined by tongue weight, also called hitch load, and its value is typically recommended to be about 10-15% of the gross trailer weight as a rule of thumb [4–6]. It is generally said that both too much tongue weight and too little tongue weight should be avoided. Excessive tongue weight can bring about problems such as overloading at the vehicle's rear tyres and suspension, incorrect headlight aim, decreased steering responsiveness [7–9]. On the other hand, insufficient tongue weight greatly increases the risk of vehicle jackknife or trailer sway. For these reasons, the drivers towing trailers are advised to maintain the tongue weight within the acceptable range by distributing the trailer's payload properly.

Despite extensive searches, however, the current academic and technical literature appears to lack quantitative analyses justifying the standard tongue weight recommendations. Also, the recommendations often suggest a uniform tongue weight without considering the specific system parameters, neglecting the impact of vehicle and trailer's inertia and length parameters on the system's dynamic characteristics. These oversights imply that many vehicle-trailer systems on the road could benefit from more precise tongue weight selection.

Consequently, this paper provides the comprehensive analyses on vehicle-trailer combinations and the cost function-based criteria for selecting the optimal tongue weight for yaw stability and consistent handling quality.

1.1. Literature survey

It has widely been understood that the longitudinal position of the centre of gravity in vehicle-trailer systems critically affects stability and performance. Literature from as early as the 1960s addresses this, with W. Korn of Airstream Inc. noting that hitch weight below 12% causes "tailwagging," which means the trailer sway, and with T.J. Reese of Reese Products Inc. stating that well-designed trailers should have at least 10% of their weight on the hitch [4,5]. This understanding evolved into the 10-15% tongue weight guideline and the 60/40 rule, which recommends putting the 60% of the trailer weight in front of the trailer axle and 40% behind. However, no documents have provided the mathematical basis or logical development for the 10-15% recommendation despite the author's best efforts to find such evidence.

There have been some parametric studies on the longitudinal centre of gravity position in vehicle-trailer systems. Still, most studies only performed the eigenvalue analysis and did not attempt to provide criteria for selecting the optimal tongue weight.

The study by R.T. Bundorf qualitatively analysed vehicle-trailer parameters based on the approximate trailer damping ratio and examined the system responses to different centre of gravity positions through mathematical model experiments [10]. Like other studies, it noted that high trailer mass and a rearward trailer centre of gravity worsen system stability. A series of reports made by National Highway Traffic Safety Administration (NHTSA) suggested that, for desirable characteristics in vehicle-trailer systems, the trailer's damping ratio should be at least 0.2, and the system should exhibit understeer characteristics when the vehicle's lateral acceleration reaches 0.3g during steady-state cornering [7–9,11,12]. Nevertheless, the logical derivation of these suggestions was not described in the reports. The study by W. Deng and X. Kang analysed the effects of attaching a trailer to a vehicle and categorised the poles of the vehicle-trailer system into vehicle-associated pole pairs and trailer-associated pole pairs based on the root locus [13]. It conducted a basic eigenvalue analysis to examine system characteristic changes with respect to speed, trailer mass, and centre of gravity position. However, it performed no other analyses than the eigenvalue analysis and made a partly wrong conclusion that a lower trailer mass and lower tongue weight are preferable. Similarly, A. Hac noted that low tongue weight experimentally resulted in lower stability and should be avoided, but did not provide further discussion or guidelines on selecting tongue weight [6]. Ultimately, none of the above studies have proposed the criteria for selecting optimal tongue weight.

To determine the optimal tongue weight, the desirable characteristics of the vehicle-trailer system should first be discussed. These desirable characteristics include stability and consistency. The stability primarily refers to the yaw-plane stability, and the consistency refers to how closely the handling performance of the vehicle-trailer system matches that of the original tow vehicle. Excluding heavy tractor-trailer systems, consistent handling performance is desirable for light vehicle-trailer systems since drivers are typically accustomed to driving without a trailer. [1,2,10,13,14]

1.2. Open-loop system design in vehicle-trailer combinations

Vehicle motion control includes two types of feedback control. One is the feedback control by drivers, and the other is the feedback control by vehicle control modules. The feedback control by the driver primarily aims to navigate the vehicle along a desired path, while feedback control by the control module mainly aims to improve the stability and performance of path-following control based on driver inputs. The stability and performance of a vehicle-trailer system are usually ensured through these two types of feedback control.

However, the vehicle system is constrained by environmental factors such as road conditions, meaning that feedback control cannot be a magic solution. When designing a PID controller for lateral motion control using the pole placement method, for example, ideal pole placement is impossible in practice due to strong input constraints. If the speed is too high or the road is too slippery, a vehicle can lose control and slide, regardless of how well it is controlled by a driver or control modules. Thus, the stability margin of an open-loop system without feedback control should be large enough, highlighting the importance of thoughtful open-loop system design in vehicle-trailer systems.

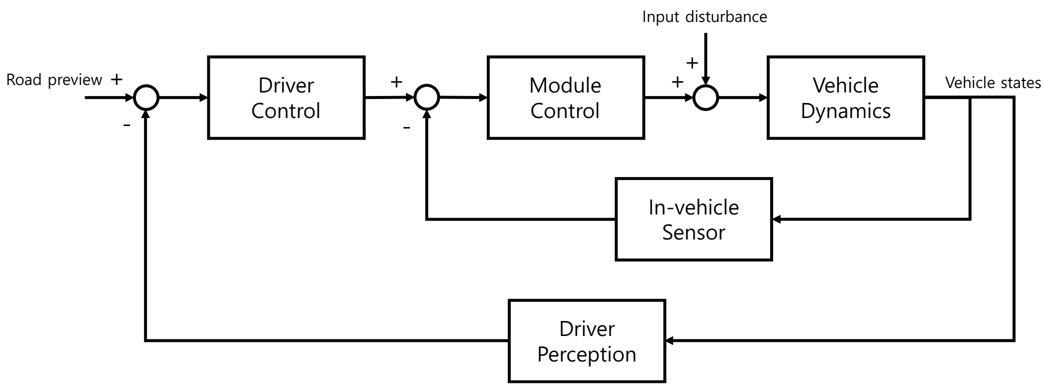


Figure 2. Feedback control loop for vehicle systems

For single-unit vehicles, most of the open-loop system design is completed during the manufacturing phase. Ordinary drivers use and enjoy vehicles with a well-balanced combination of parameters that have been optimised over decades. However, the dynamic characteristics of a vehicle changes as a trailer is attached due to the changes in weight distribution and hitch interaction. Thus, integrating tow vehicles and trailers and distributing the payload on the trailer can be effectively considered an open-loop system design determined by drivers.

1.3. Paper structure

The paper is organised as follows. Section 2 addresses the modelling of vehicle-trailer systems for lateral behaviour analysis. Section 3 conducts open-loop system analyses without considering the driver's influence, including pole-zero analysis, Bode analysis, and transient response analysis. Subsequently, Section 4 proposes two novel selection criteria for determining the optimal tongue weight, which are verified through CarSim simulation experiments in Section 5. Section 6 briefly provides an interpretation of the conventional tongue weight recommendation, which is 10-15% of the trailer weight.

The paper concludes with a summary and further considerations that can be made in the future study in Section 7.

2. System modelling

2.1. Lateral dynamics model

A 4-DOF single-track model is adopted for the vehicle-trailer system [15–17]. Notations for related parameters and variables can be found in Table 1 and Table 2. Several standard assumptions allow the approximation of nonlinear vehicle-trailer dynamics to a linear state-space model. The assumptions used for the modelling are as follows:

- The effects of road grade, road bank, wind and other external forces are neglected.
- The system is a combination of a double-axle vehicle and a single-axle trailer.
- The effects of roll, pitch, heave motions of the system are negligible.
- The longitudinal speeds of the vehicle and trailer change slowly.
- The steering angles and the hitch angle are small enough.

With the above assumptions, following equations for the lateral dynamics of vehicle-trailers can be obtained:

$$m_1 a_{y1} = F_{y1} + F_{y2} - F_{yh} \tag{1a}$$

$$I_{zz1}\dot{r}_1 = l_{f1}F_{y1} - l_{r1}F_{y2} + (l_{r1} + P)F_{yh} \tag{1b}$$

$$m_2 a_{y2} = F_{yh} + F_{y3} \tag{1c}$$

$$I_{zz2}\dot{r}_2 = l_{f2}F_{yh} - l_{r2}F_{y3} \tag{1d}$$

By combining the above assumptions with the kinematic constraints at the hitch, the following kinematic relationships are derived:

$$a_{y1} = \dot{V}_{y1} + r_1 V_{x1} \tag{2a}$$

$$a_{y2} = \dot{V}_{y2} + r_2 V_{x2} \tag{2b}$$

$$\dot{\psi}_h = r_h = r_1 - r_2 \tag{2c}$$

$$\beta_2 = \beta_1 - \frac{l_{r1} + P}{V_x} r_1 - \frac{l_{f2}}{V_x} r_2 + \psi_h \tag{2d}$$

Finally, the state-space representation of the system lateral dynamics can be expressed as follows:

$$\mathbf{M}\dot{\mathbf{x}} = \mathbf{D}\mathbf{x} + \mathbf{F}\mathbf{u} \tag{3}$$

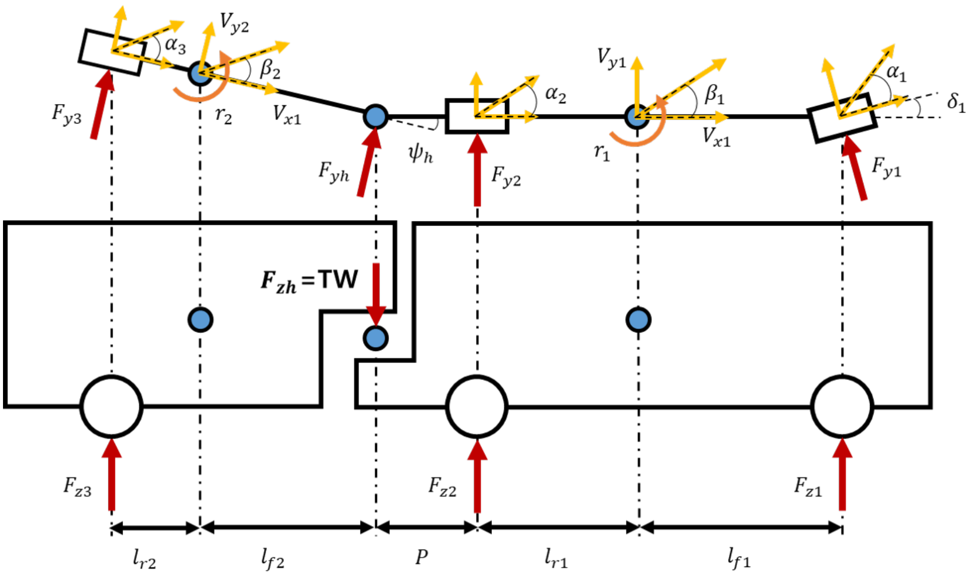


Figure 3. A vehicle-trailer system model

Table 1. Notation: parameters in a vehicle-trailer system

Notation	Definition	Unit	Value
m_1	mass of front unit	kg	2057.71
m_2	mass of rear unit	kg	1053
I_{zz1}	Yaw moment of inertia of front unit	$kg \cdot m^2$	5192.28
I_{zz2}	Yaw moment of inertia of rear unit	$kg \cdot m^2$	3696.54
l_{f1}	Distance between 1st axle and c. g. of front unit	m	1.373
l_{r1}	Distance between c. g. of front unit and 2nd axle	m	1.888
L_1	Distance between 1st axle and 2nd axle	m	3.261
P	Distance between 2nd axle and hitch	m	1.039
l_{f2}	Distance between hitch and c. g. of rear unit	m	2.542
l_{r2}	Distance between c. g. of rear unit and 3rd axle	m	0.458
L_2	Distance between hitch and 3rd axle	m	3
C_{y1}	Tyre cornering stiffness of 1st axle	N/rad	$\propto F_{z1}$
C_{y2}	Tyre cornering stiffness of 2nd axle	N/rad	$\propto F_{z2}$
C_{y3}	Tyre cornering stiffness of 3rd axle	N/rad	$\propto F_{z3}$

Table 2. Notation: states in a vehicle-trailer system

Notation	Definition	Unit
a_{yi}	Lateral acceleration at c.g. of ith unit	m/s^2
β_i	Body sideslip angle at c.g. of ith unit	rad
r_i	Yaw rate at c.g. of ith unit	rad/s
α_i	Tyre sideslip angle of ith axle	rad
δ_i	Road steer angle of ith axle	rad
ψ_h	Hitch angle	rad
r_h	Hitch angle rate	rad/s
V_{xi}	Longitudinal velocity at c.g. of ith unit	m/s
V_{yi}	Lateral velocity at c.g. of ith unit	m/s
F_{xi}	Longitudinal force at ith axle or hitch	N
F_{yi}	Lateral force at ith axle or hitch	N
F_{zi}	Vertical load at ith axle or hitch	N

where

$$\mathbf{M} = \begin{bmatrix} 1+\frac{l_{r2}}{L_2}\frac{m_2}{m_1} & \frac{I_{zz2}-m_2 l_{r2}(l_{r1}+P+l_{f2})}{m_1 L_2 V_x} & -\frac{I_{zz2}-m_2 l_{f2} l_{r2}}{m_1 L_2 V_x} & 0 \\ -\frac{l_{r2}(l_{r1}+P)}{L_2}\frac{m_2 V_x}{I_{zz1}} & 1-\frac{l_{r1}+P}{L_2}\frac{I_{zz2}-m_2 l_{r2}(l_{r1}+P+l_{f2})}{I_{zz1}} & \frac{l_{r1}+P}{L_2}\frac{I_{zz2}-m_2 l_{f2} l_{r2}}{I_{zz1}} & 0 \\ -\frac{m_2 l_{f2} l_{r2} V_x}{I_{zz2} L_2} & \frac{l_{r2}}{L_2}\frac{I_{zz2}+m_2 l_{f2}(l_{r1}+P+l_{f2})}{I_{zz2}} & -\frac{l_{r2}}{L_2}\frac{I_{zz2}+m_2 l_{f2}^2}{I_{zz2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\mathbf{D} = \begin{bmatrix} -\frac{C_{y1}+C_{y2}}{m_1 V_x} & -(1+\frac{l_{r2}}{L_2}\frac{m_2}{m_1}+\frac{l_{f1}C_{y1}-l_{r1}C_{y2}}{m_1 V_x^2}) & 0 & 0 \\ -\frac{l_{f1}C_{y1}-l_{r1}C_{y2}}{I_{zz1}} & \frac{m_2 l_{r2}(l_{r1}+P)V_x}{I_{zz1}L_2} - \frac{l_{f1}^2 C_{y1}+l_{r1}^2 C_{y2}}{I_{zz1}V_x} & 0 & 0 \\ \frac{l_{r2}C_{y3}}{I_{zz2}} & \frac{m_2 l_{f2} l_{r2} V_x}{I_{zz2}L_2} - \frac{l_{r2}(l_{r1}+P+L_2)C_{y3}}{I_{zz2}V_x} & \frac{l_{r2}L_2 C_{y3}}{I_{zz2}V_x} & \frac{l_{r2}C_{y3}}{I_{zz2}} \\ 0 & 0 & 1 & 0 \end{bmatrix}, \mathbf{F} = \begin{bmatrix} \frac{C_{y1}}{m_1 V_x} \\ \frac{l_{f1}C_{y1}}{I_{zz1}} \\ 0 \\ 0 \end{bmatrix},$$

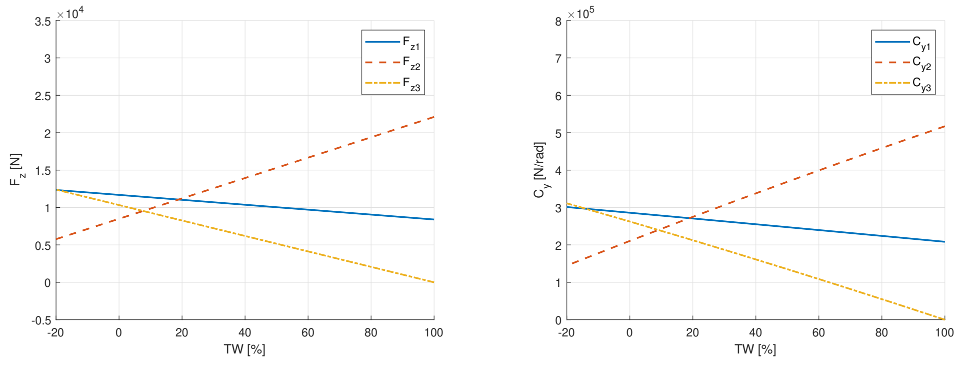
$$\mathbf{x} = \begin{bmatrix}\beta_1 & r_1 & r_h & \psi_h\end{bmatrix}^T, \quad r_h = \dot{\psi}_h, \quad \mathbf{u} = \delta_1$$

Equation (3) accurately describes the lateral behaviour of the vehicle-trailer combination unless the system enters highly nonlinear region, particularly with extreme roll or articulation angles [3,6,15,16,18].

It is worth noting that $\mathbf{M}$ is only dependent to the system's inertia and length parameter and remains unchanged during operation. However, $\mathbf{D}$ and $\mathbf{F}$, which are related to tyre friction forces, may vary a lot during operation due to their dependence on tyre cornering stiffness and longitudinal speed. The expression for system matrices $\mathbf{M}, \mathbf{D}, \mathbf{F}$ can be different depending on the selection of state variables. For example, $\mathbf{x} = \begin{bmatrix}V_{y1} & r_1 & r_2 & \psi_h\end{bmatrix}^T$ can be chosen as state variables.

3. Open-Loop System Analyses

In this section, the impacts of tongue weight on system's open-loop characteristics are discussed with various diagrams in classic control theory. As the trailer's centre of gravity moves fore and aft, the tongue weight and axle loads vary. Because tyre cornering stiffness is dependent on its vertical load, it changes in relation to tongue weight variation as seen in Figure 4.



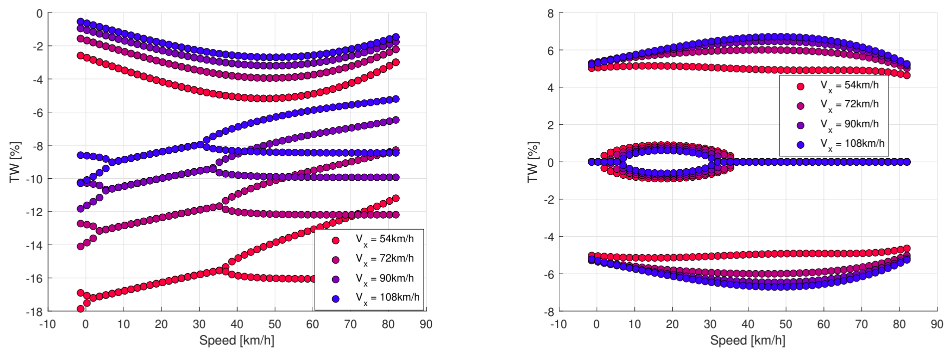
(a) Axle load F_z

(b) Axle cornering stiffness C_y

Figure 4. Effects of TW variation

3.1. Pole-Zero Analysis

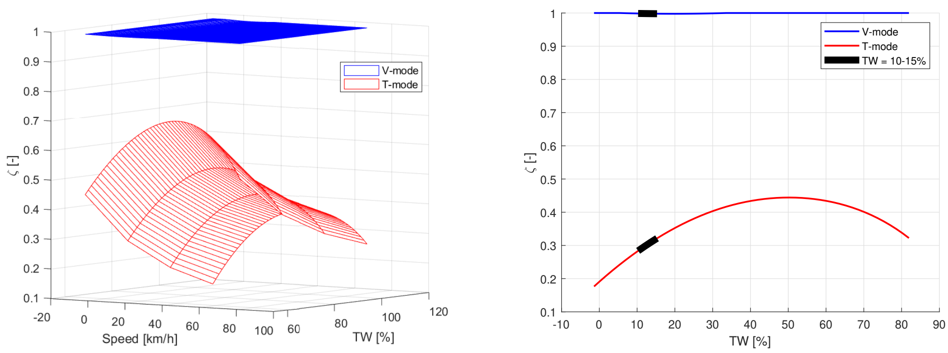
Poles and zeros are key parameters representing the dynamic characteristics of a system. Specifically, the locations of the poles, or eigenvalues, determine the system's modes and dictate its stability, making them extremely important for stability analysis. As depicted in Figure 5, the real parts of the poles move closer to the imaginary axis as speed increases while the imaginary parts remain almost unchanged. These changes lead to the reduction in the damping ratio as seen in Figure 6. Therefore, it can be inferred that higher speeds increase the likelihood of system instability and lengthen the time for oscillations to dissipate. Although speed significantly impacts system stability, the overall trend of the system remains consistent regardless of the speed. Thus, for simplicity, speed is fixed at 90km/h in most of the figures throughout the paper.



(a) Real parts of

(b) Imaginary parts

Figure 5. Pole locations with respect to speed and TW variation



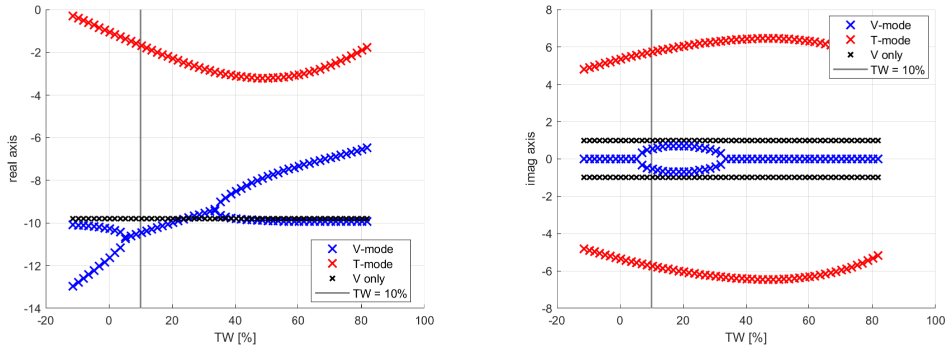
(a) Damping ratio ζ

(b) Damping ratio ζ at V_x=90km/h

Figure 6. Damping ratio with respect to speed and TW variation

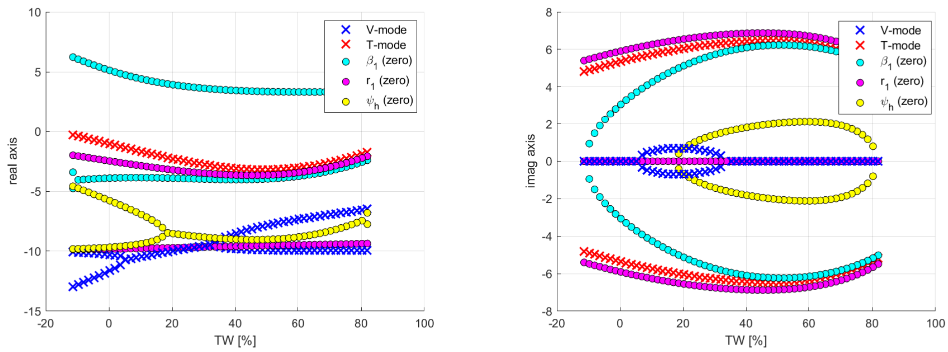
Figure 7 plots the eigenvalues of a system with nominal values, shifting only the trailer's centre of gravity. The black x-shaped points represent a pole pair of the original tow vehicle without a trailer and the gray solid line denotes the conventional tongue weight of the vehicle-trailer combination, which is 10% here. The pole pairs of vehicle-trailers can be divided into a vehicle-mode associated pole pair and a trailer-mode associated pole pair. The vehicle-mode associated poles, marked with the blue x-shaped points, are closely located to the poles of the original tow vehicle, while the trailer-mode associated poles, marked with the red x-shaped points, are closer to the imaginary axis in s-plane than the vehicle-mode associated poles. Such nomenclature was introduced in the previous studies [6–9,13].

The trailer-mode associated poles, which are the dominant poles of the system, have the smallest real parts at approximately 50% in tongue weight. It means that the more they deviate from this value, the more unstable the system becomes. It is consistent with the fact that if the tongue weight becomes excessively low, the vehicle's rear axle load decreases, degrading the system's stability. Conversely, if the tongue weight increases beyond 50%, the trailer axle's load may not be sufficient, also compromising the system's stability.



(a) Real parts (b) Imaginary parts

Figure 7. Pole locations with respect to TW variation at V_x=90km/h



(a) Real parts (b) Imaginary parts

Figure 8. Pole and zero locations with respect to TW variation at V_x=90km/h

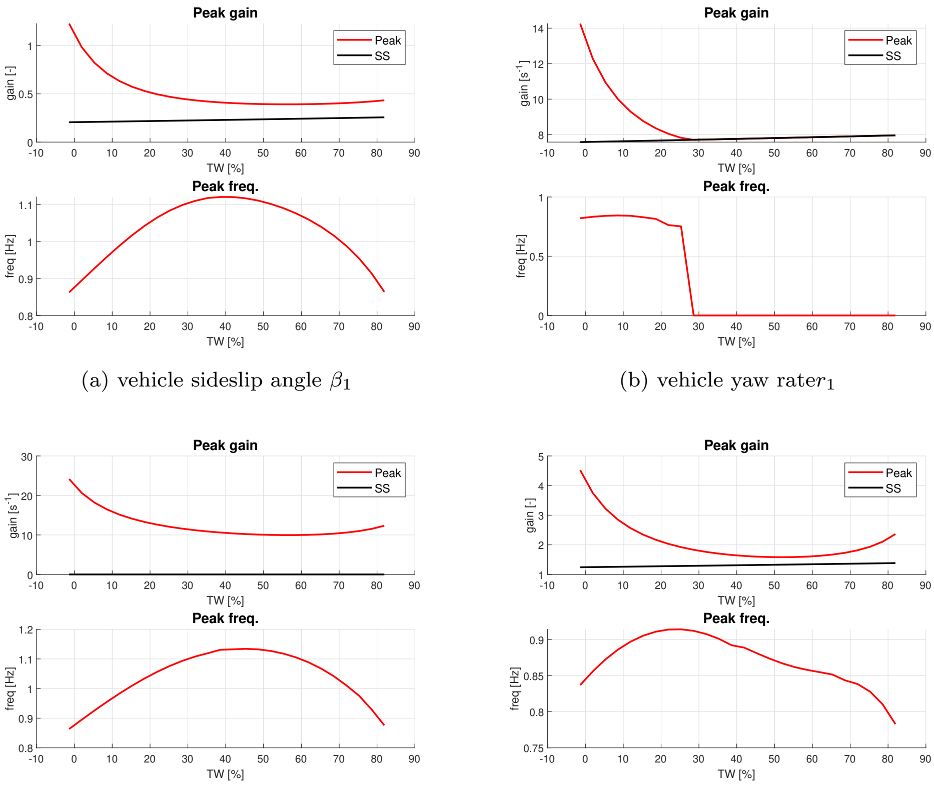
While the poles entirely determine the modes of a system, the zeros affect the actual response to inputs. In an ideal system, for example, pole-zero cancellation occurs and the corresponding mode disappears when a pole and a zero coincide completely. In Figure 8, the cyan, magenta, and yellow circles represent the zeros of vehicle sideslip angle β_1, vehicle yaw rate r_1, and hitch angle ψ_h, respectively. The zeros for β_1 and r_1 approach very close to the system's dominant poles as the tongue weight becomes exceptionally high. This implies that at high tongue weight, the vehicle's sideslip angle and yaw rate exhibit much simpler and more stable behaviours than the hitch angular rate and hitch angle due to the approximate pole-zero cancellation.

3.2. Bode Analysis

A Bode plot is a common tool for depicting a system's frequency response, encompassing both magnitude and phase. In this paper, the linear scale is used instead of the conventional decibel scale for magnitude plots to enhance visibility within the range of interest. Figure 10 demonstrates the system's frequency responses at a forward speed of 90km/h with variations in tongue weight. The corresponding peak frequencies and gains are plotted in Figure 9. It should be noted that the vehicle's sideslip angle gain is actually negative, but its magnitude is presented for simplicity.

First, the steady-state gain of the vehicle state variables, indicated by the black solid line, is barely affected by tongue weight as shown in Figure 9. This contradicts R.T. Bundorf's claim that the lateral acceleration gain is largely affected by the hitch load, as he ignored that the tyre cornering stiffness is proportional to the tyre's vertical load [10]. Next, all four state variables exhibit a peak frequency near 1Hz, which overlaps with the driver's steering bandwidth. This necessitates careful design around this frequency region to improve the entire system, including the lateral driver model.

In addition, it is observed in both Figures 9 and 10 that the peak gains for all four variables decrease with the increase in tongue weight until it surpasses approximately 50%, after which it starts to slightly increase again. The magnitude amplification near the peak frequency is induced by a low damping ratio, which should be avoided to ensure stability because it causes significant overshoot and oscillations in response to inputs near the peak frequency. This can be easily understood by referring to the example of a second-order mass-damper-spring system in Figure 11. Particularly for



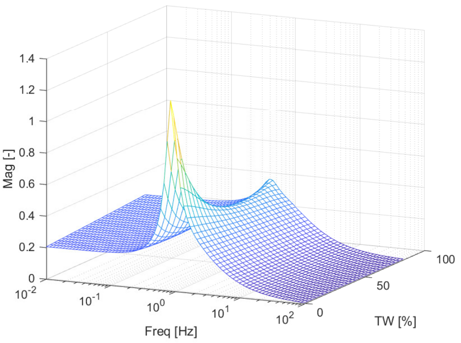
(a) vehicle sideslip angle β_1

(b) vehicle yaw rater_1

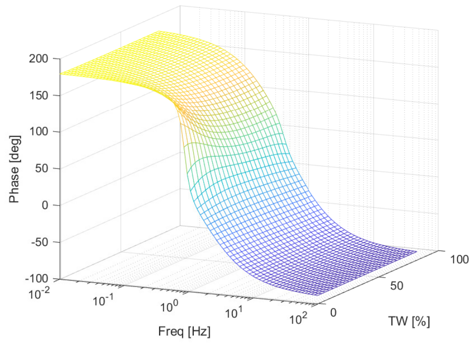
(c) hitch angular rate r_h

(d) hitch angle ψ_h

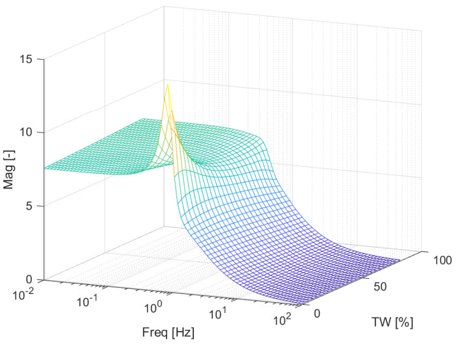
Figure 9. Peak gains and peak frequencies in Bode plots for a vehicle-trailer at V_x=90km/h



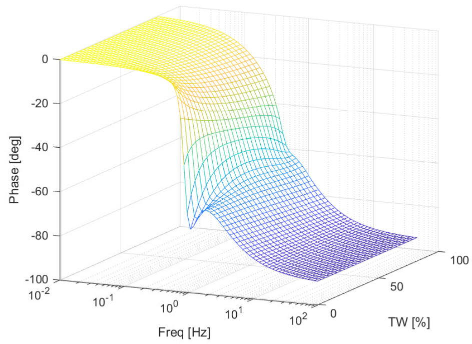
(a) Magnitude plot of β_1



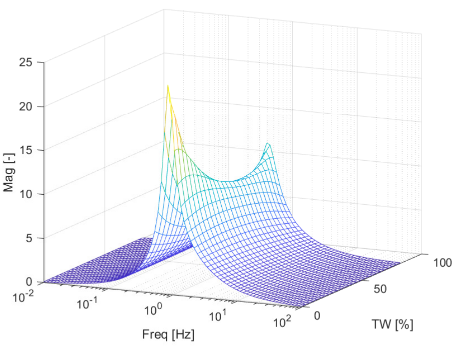
(b) Phase plot of β_1



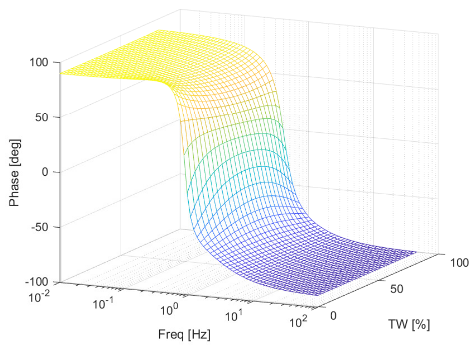
(c) Magnitude plot of r_1



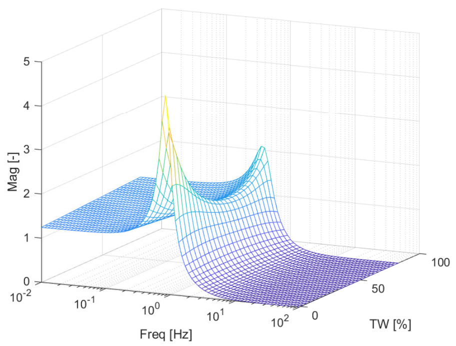
(d) Phase plot of r_1



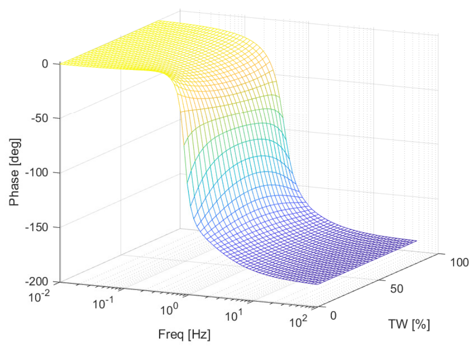
(e) Magnitude plot of r_h



(f) Phase plot of r_h



(g) Magnitude plot of ψ_h

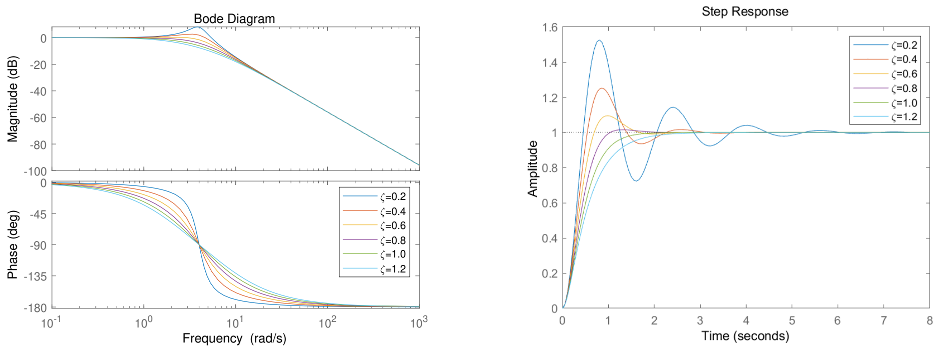


(h) Phase plot of ψ_h

Figure 10. Bode plots with respect to TW variation for a vehicle-trailer at V_x=90km/h

vehicle yaw rate, there is a significant phase distortion in the same region, which can negatively impact the system's transient response characteristics. Consequently, Bode analysis indicates that low tongue weight should be avoided to maintain system stability.

On the other hand, high tongue weight values also lead to magnitude amplification for trailer-associated state variables around the peak frequency, but this effect is markedly less pronounced for vehicle-associated state variables. This can be interpreted as the suppression of the trailer mode due to the approximate pole-zero cancellation explained in the earlier section.



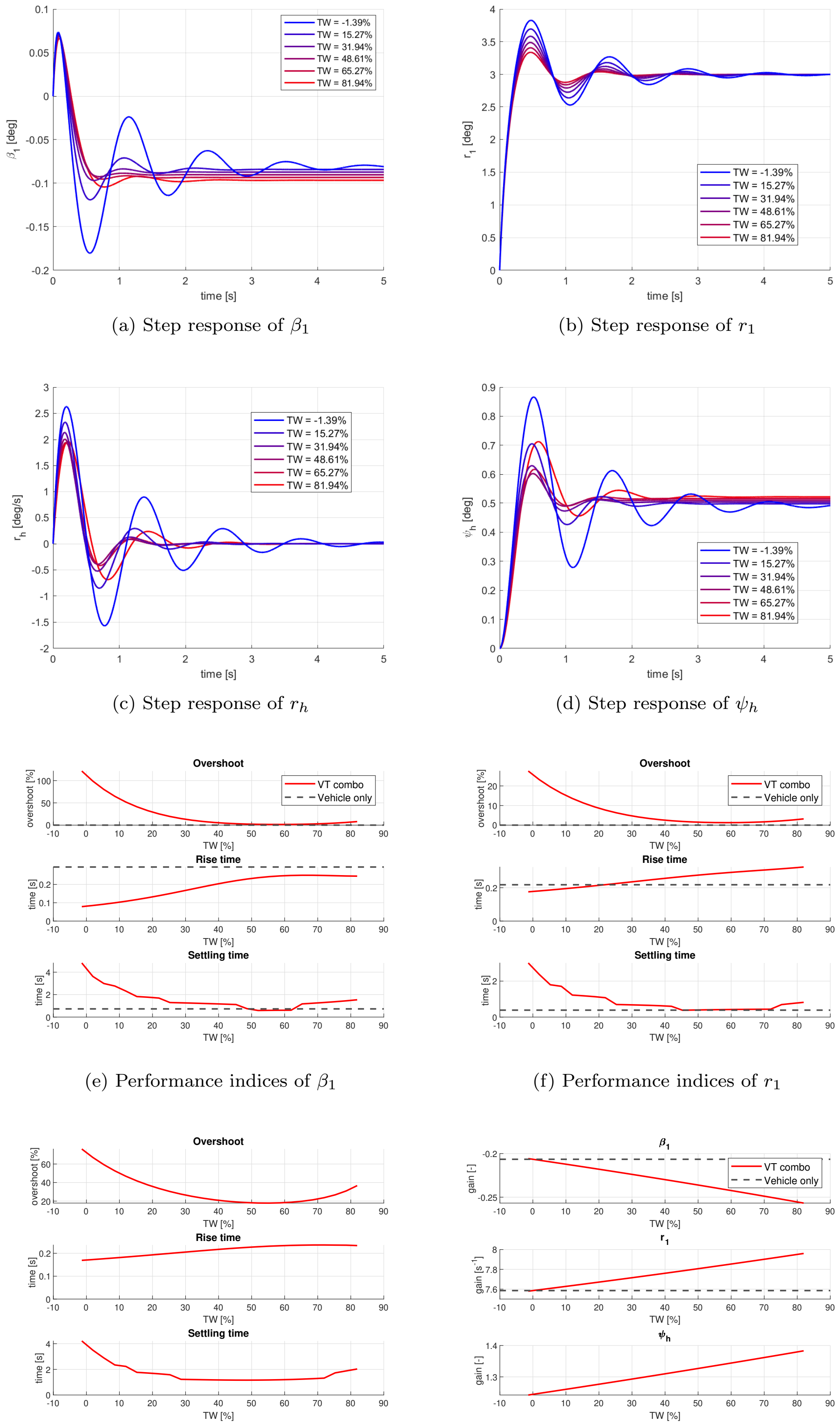
(a) Bode plots (b) Step responses

Figure 11. Example: the second-order systems with different damping ratios

3.3. Step Response Analysis

System analysis and design can be conducted in both the time and frequency domains. Each domain has its own advantages, but the time domain analysis is more intuitive and tangible to drivers. Additionally, it is necessary to examine the time response to investigate the system through practical design parameters such as overshoot, rise time, and settling time. Figure 12 shows the step responses and the related time-domain performance indices of a pure mathematical model at a forward speed 90km/h. The step input gain was here adjusted to achieve a constant steady-state gain of the vehicle's yaw rate $r_1 = 3$deg/s regardless of tongue weight.

The fact that the steady-state gains have no significant changes in relation to tongue weight variation is reaffirmed. As predicted by the Bode analysis, systems with low tongue weight exhibit large oscillations and overshoots due to low damping ratios, but as the tongue weight increases, system stability improves until it peaks around 50%, after which oscillatory behaviours and overshoots begin to increase again. However, while the hitch angular rate and hitch angle show large oscillations at high tongue weights, the vehicle sideslip angle and vehicle yaw rate display relatively stable responses. These results can be regarded as the effect of the approximate pole-zero cancellation mentioned earlier. Additionally, the vehicle sideslip angle exhibits a non-minimum phase behaviour, confirming the observation that it has right half plane zeros and an initial phase of 180 degrees in the phase plot. Furthermore, it can be observed that the system's rise time increases as the tongue weight increases. Although this can be interpreted as a result of increased system stability, it may be perceived by drivers as the reduced steering responsiveness.



(a) Step response of β_1

(b) Step response of r_1

(c) Step response of r_h

(d) Step response of ψ_h

(e) Performance indices of β_1

(f) Performance indices of r_1

(g) Performance indices of ψ_h

(h) Steady-state gains

Figure 12. Step responses and performance indices with respect to TW variation for a vehicle-trailer at V_x=90km/h

4. TW Selection Criteria

The importance of proper weight distribution in a vehicle-trailer system is well recognised by both drivers and control engineers. However, the specific conditions necessary for optimal handling performance and stability have not been rigorously analysed and are often based only on empirical evidence recommending a tongue weight of 10–15% of the gross trailer weight. Therefore, this section newly propose optimal criteria for determining tongue weight from the perspectives of stability and consistency in vehicle-trailer systems.

4.1. Stability

Although the driver has a dominant influence on the lateral stability of a vehicle-trailer, it is universally agreed that the system with improper weight distribution has a low stability margin. Since the lateral stability is a non-negotiable part in vehicle-trailer combinations, it should be one of the primary considerations in the tongue weight adjustment. To maximise the system's lateral stability margin, making the dominant poles have the minimum real parts can be chosen as an initial approach. This can be achieved with the following cost function:

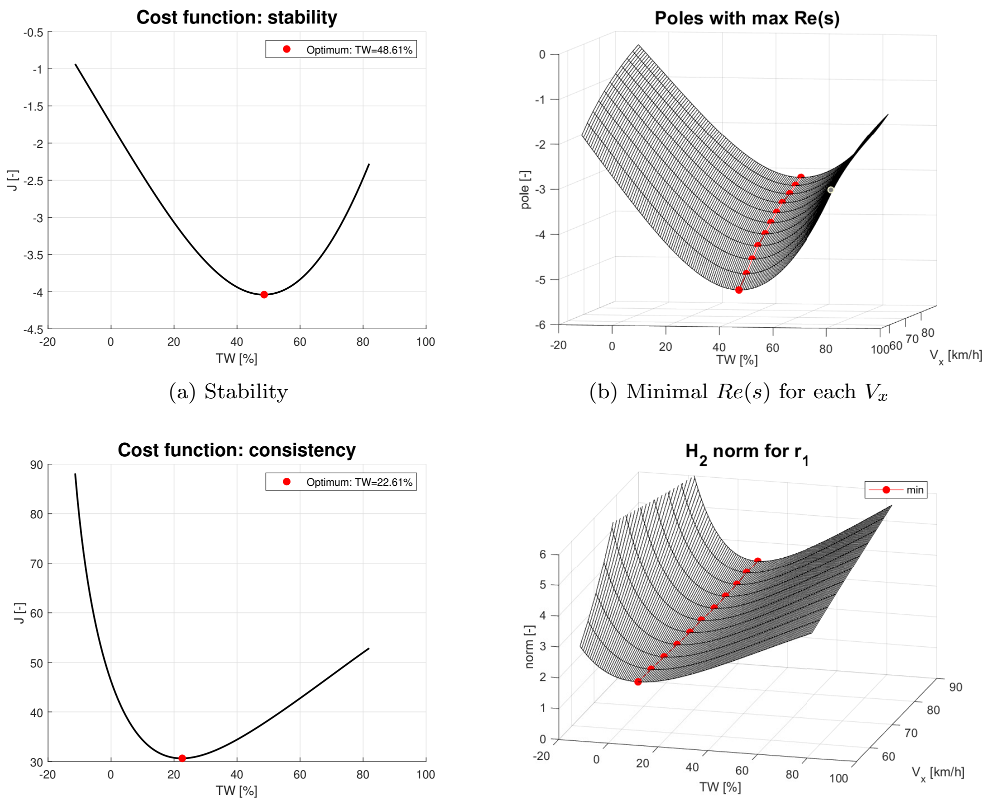
$$\min_{\mathrm{TW}} \left[J = \int_{V_{x,lb}}^{V_{x,ub}} \left(\max_{\substack{s \\ \mathrm{s.t.}\, P(s)=0}} Re(s) \right) dV_x \right] \tag{4}$$

Here, TW determines the tongue weight, and lb, ub, and $P(s)$ represent the lower and upper bounds of the longitudinal speed and the system's characteristic equation, respectively. As the tongue weight of a vehicle-trailer cannot be adjusted during a trip, the cost function is set to cover the entire speed range of interest. The lower speed limit is set at 15m/s, below which the system is less prone to instability, and the upper limit is set at 25m/s, which is generally the maximum recommended speed for vehicle-trailers. Applying Equation (4) to the nominal vehicle-trailer system yields Figure 13 (a), and the resultant optimal tongue weight for system stability is determined as 48.61% of the trailer weight. The black meshed surface in Figure 13 (b) represents the real parts of the dominant pole obtained for each combination of speed and tongue weight and the red circles denote the optimal tongue weights for a vehicle-trailer at each speed.

4.2. Consistency

The virtue of a vehicle system in a trade-off relationship with stability is responsiveness or maneuverability. This concept can be better understood by considering the design goals of different aircraft: passenger planes are designed to be as stable as possible, while fighter jets prioritize rapid maneuverability over stability. As discussed in the previous section, higher tongue weight is desirable for enhancing stability. However, as mentioned at the end of Section 3.3, higher tongue weight can lead to a decrease in the system's rise time, which may be perceived by the driver as reduced transient responsiveness or, in other words, decreased maneuverability.

Additionally, drivers of light vehicle-trailer combinations are accustomed to operating without a trailer, so the system is not only required to maintain stability against



(a) Stability

(b) Minimal $Re(s)$ for each V_x

(c) Consistency

(d) Minimal $\|G_o - G_{VT}\|_2$ for each V_x

Figure 13. Tongue weight selection criteria: for stability(upper) and consistency(lower)

inputs and disturbances, but also to preserve the handling performance of the original tow vehicle [1,2,10,13,14]. From this perspective, lateral control of the vehicle-trailer system has often been conducted to follow the steady state values of the original vehicle as a reference model. However, this approach could potentially compromise the system's stability at high speeds [1,2].

Two representative suggestions can be made to ensure the steering responsiveness and consistent handling performance. The first is to position the vehicle-associated pole pair in the s-plane as close as possible to the original tow vehicle's pole pair. The second is to minimise the difference in the frequency response of the vehicle-trailer and the tow vehicle in the frequency domain in the sense of $\mathcal{H}_2$ or $\mathcal{H}_\infty$. The first method requires the least effort in structuring the cost function and finding the corresponding solution, but it cannot guarantee consistency in all situations since the trailer-associated poles are neglected. The second method can ensure the optimality or robustness of the system with various design techniques in the frequency domain although it requires more efforts than the first method.

$$r_{1o}(t) = A_{1o}e^{p_{1o}t} + A_{2o}e^{p_{2o}t} \tag{5}$$

$$r_1(t) = A_1e^{p_1t} + A_2e^{p_2t} + A_3e^{p_3t} + A_4e^{p_4t} \tag{6}$$

$$\min_{\mathrm{TW}} \left[J = \int_{V_{x,lb}}^{V_{x,ub}} \left((p_{1o} - p_1)^2 + (p_{2o} - p_2)^2 \right) dV_x \right] \tag{7}$$

$$\min_{\mathrm{TW}} \left[J = \int_{V_{x,lb}}^{V_{x,ub}} \left(\int_0^{\infty} |r_{1o}(t) - r_1(t)|^2 dt \right) dV_x \right] \tag{8}$$

The two methods can be intuitively compared with the above equations. When examining the vehicle's yaw rate, the original vehicle without a trailer has a time response as shown in Equation (5), while the vehicle-trailer system has a time response as shown in Equation (6). Here, the subscript o denotes the original vehicle and the subscripts $1, 2$ and $3, 4$ represent the vehicle-associated modes and trailer-associated modes, respectively. As the first method utilises the cost function in Equations (7), it is apparent that the trailer-associated modes are not taken into account. On the other hand, the second method does not neglect any mode and performs cost minimisation considering the overall response. Therefore, this paper proposes the consistency criteria using the second method rather than the first.

Since the vehicle-trailer system exhibits four distinct responses for each state variable in frequency and time domains, the target state variable or output matrix must be determined first. This paper utilises the tow vehicle's yaw rate because drivers primarily use it as the main feedback information; The contribution of the paper lies in providing the novel criteria for selecting optimal tongue weight based on theoretical evidence. Detailed discussions and comparisons of each method should be pursued in future work. The cost function for ensuring the consistent handling performance is chosen as follows:

$$\min_{\mathrm{TW}} \left[J = \int_{V_{x,lb}}^{V_{x,ub}} \|G_o(s) - G_{VT}(s)\|_2 dV_x \right] \tag{9}$$

The speed range is maintained as in the previous case, with $G_o(s)$ representing the lateral dynamics transfer function of the original vehicle, and $G_{VT}(s)$ denoting the transfer function when the original vehicle is towing a trailer. The tongue weight selection criterion can be seen in Figure 13 (c), suggesting an optimal tongue weight of 22.61% of the trailer weight for consistent handling performance of the nominal system. The black meshed surface in Figure 13 (d) represents the $\mathcal{H}_2$ norm of the vehicle's yaw rate error between a vehicle-trailer and the original tow vehicle, and the red circles denote the optimal tongue weights for a vehicle-trailer at constant speeds.

4.3. Changes in Optimal TW depending on trailer weight ratio

One of the key parameters that critically affects the stability and performance of the vehicle-trailer system is the vehicle-to-trailer weight ratio (m_2/m_1). Some may wonder how significantly this ratio influences the optimal value of the tongue weight. The results is shown in Table 3, indicating that both of the proposed strategies recommend slightly higher tongue weight values as the trailer's weight relative to the vehicle increases. However, it is also notable that the changes are not substantial. Here, the yaw moment of inertia of the trailer I_{zz2} varies proportionally with changes in the trailer mass m_2.

Trailer Weight Ratio = $\frac{m_2}{m_1}$	0.5	0.75	1.0	1.25	1.5
Stability	48.61%	49.94%	51.94%	53.27%	55.94%
Consistency	22.61%	23.94%	23.94%	24.61%	24.61%

Table 3. Changes in optimal tongue weight depending on vehicle-to-trailer weight ratio

5. Simulation results

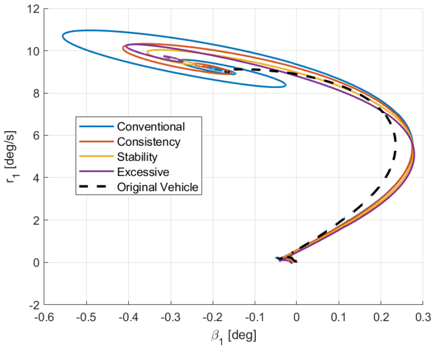
Simulation experiments were conducted for the nominal system with a 2-ton mid-size pickup truck with a 1-ton travel trailer using the CarSim vehicle simulation program at constant speeds of 60km/h and 90km/h to assess the proposed tongue weight selection criteria. Step steer and pulse steer tests were used to validate the proposed criteria. Both tests involve open-loop steering control; step steer for steady-state response and pulse steer for transient response [7–9]. In the step steer scenario, the steering wheel angle was changed to 45 degrees after driving straight for 10 seconds. For pulse steer, the steering wheel angle was rapidly increased to a maximum of 90 degrees and then returned to 0 degrees after driving straight for 10 seconds. The system parameters were set according to the nominal system values listed in Table 1. However, the changes in tongue weight were permitted by changing l_{f2} and l_{r2}. The experiments were conducted for tongue weights at 10%, 22.61%, 48.61%, and 70% of the trailer weight, representing the conventional recommendation, the proposed stability and consistency criteria, and excessive tongue weight, respectively. Additionally, experiments were conducted on the vehicle operating without the trailer for comparison.

The simulation results are depicted in Figures 14 and 15. Figure 14 shows the phase portraits of the vehicle state variables, the phase portraits of the hitch state variables, and the time responses of each variable for the experiments conducted at 60 km/h. Similarly, Figure 15 presents the same information for the experiments conducted at 90 km/h. For convenience, the four test manoeuvres determined by speed and steering scenarios are hereafter referred to in short as SS60, PS60, SS90, and PS90.

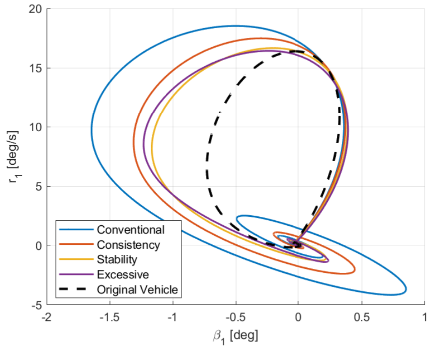
The phase trajectories in Figures 14 and 15 clearly show that the system with the conventional tongue weight has the largest overshoot and oscillations, followed by the consistency criteria-based system. Particularly, the conventional system fails to converge to a steady state in the SS90 scenario, and both systems based on the conventional criteria and consistency criteria becomes completely unstable in PS90 scenario. On the other hand, the stability criteria-based system has the fastest convergence and the smallest deviation. On the other hand, the stability criteria-based system demonstrates faster convergence and smaller deviations compared to the previous two systems, proving the effectiveness of the proposed stability criteria. Although the overall performances of the stability criteria-based system and the system with excessive tongue weight are comparative, a closer look at the time responses in Figure 15 reveals that the oscillations disappear more quickly in the stability criteria-based system.

The 3% settling time and Root-Mean-Square (RMS) error of the vehicle yaw rate were plotted for quantitative comparison between the systems in Figure 16. For the step steer scenario, the 3% settling time is the time required for the vehicle yaw rate to converge within ±3% of its final steady-state value. In the pulse steer scenario, the 3% settling time is the time required for the vehicle yaw rate to settle within ±3% of its peak value after reaching zero. The RMS error was calculated using the yaw rate error between the vehicle-trailer system and the original tow vehicle.

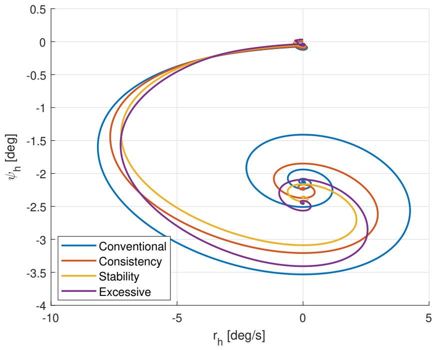
As shown in Figure 16 (a), the tongue weight determined by the stability criteria



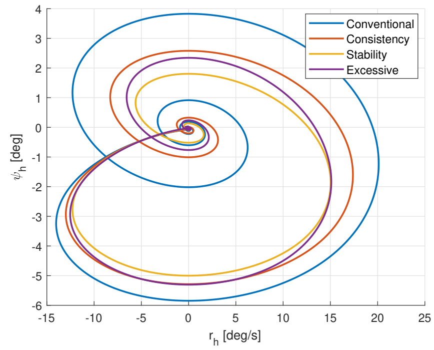
(a) SS60) unit 1 phase portrait



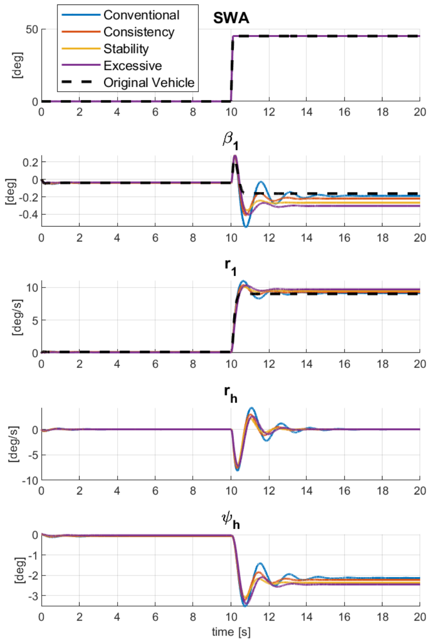
(b) PS60) unit 1 phase portrait



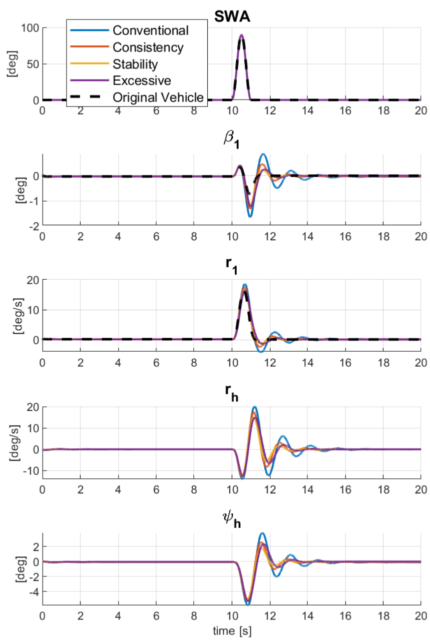
(c) SS60) hitch phase portrait



(d) PS60) hitch phase portrait

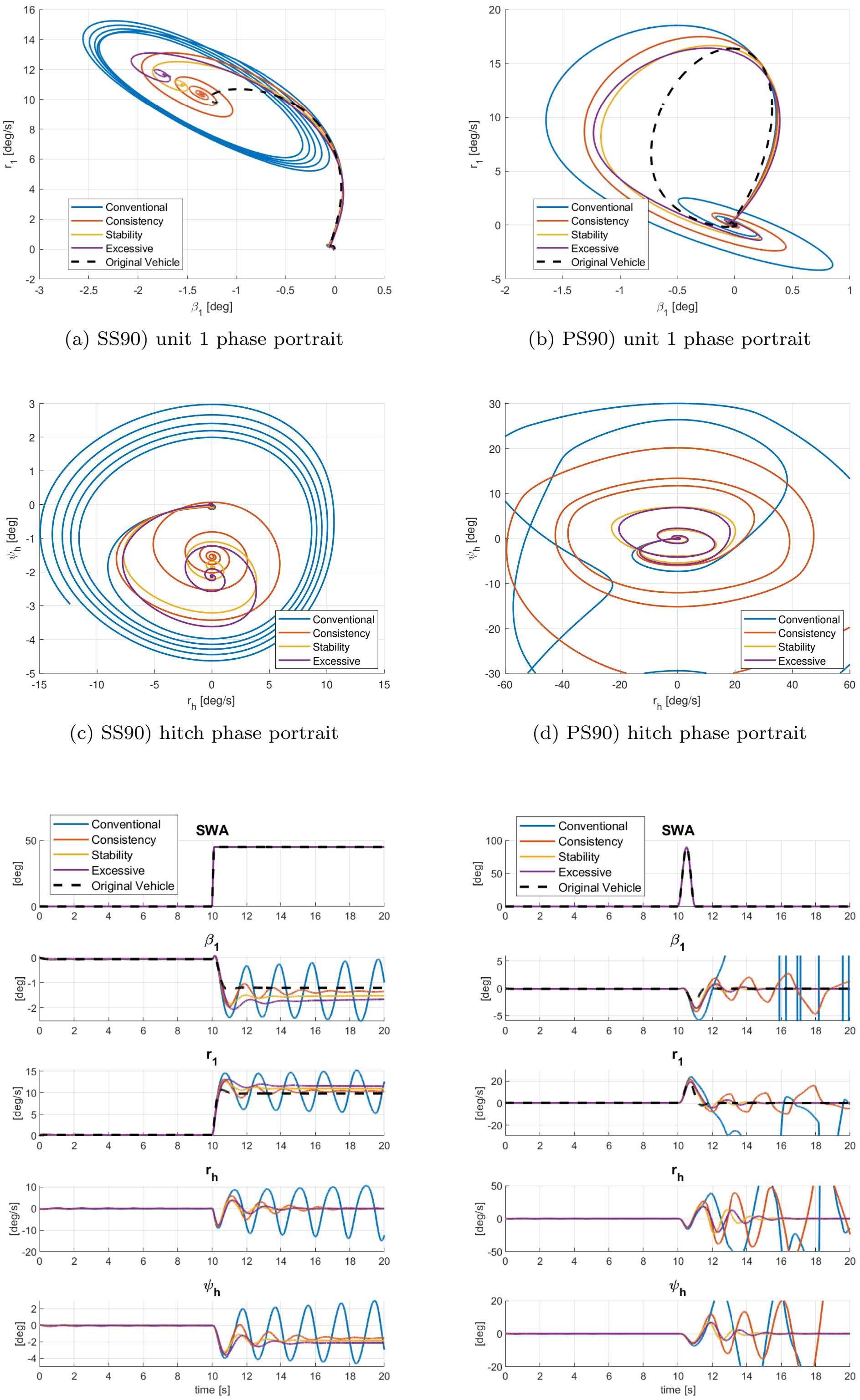


(e) SS60) time responses



(f) PS60) time responses

Figure 14. CarSim simulation results at $V_x = 60$km/h: (a) $\beta_1 - r_1$ phase plane for step steer, (b) $r_h - \psi_h$ phase plane for pulse steer, (c) $\beta_1 - r_1$ phase plane for step steer, (d) $r_h - \psi_h$ phase plane for pulse steer, (e) time responses for step steer and (f) time responses for pulse steer



(a) SS90) unit 1 phase portrait

(b) PS90) unit 1 phase portrait

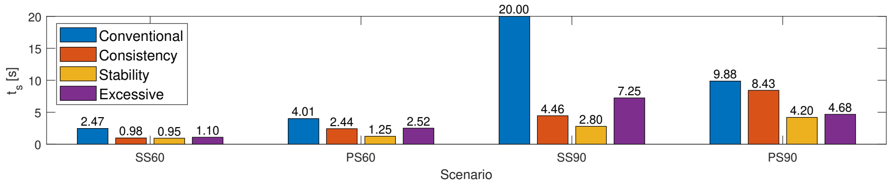
(c) SS90) hitch phase portrait

(d) PS90) hitch phase portrait

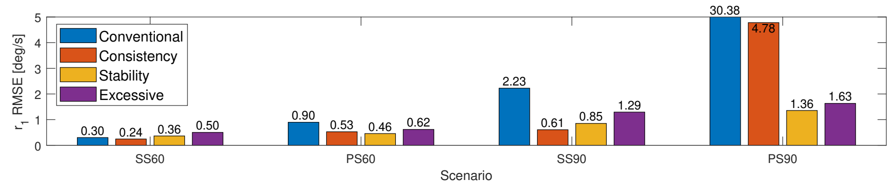
(e) SS90) time responses

(f) PS90) time responses

Figure 15. CarSim simulation results at $V_x = 90$km/h: (a) $\beta_1 - r_1$ phase plane for step steer, (b) $r_h - \psi_h$ phase plane for pulse steer, (c) $\beta_1 - r_1$ phase plane for step steer, (d) $r_h - \psi_h$ phase plane for pulse steer, (e) time responses for step steer and (f) time responses for pulse steer



(a) 3% Settling time for the stability comparison



(b) RMS error for the consistency comparison

Figure 16. CarSim simulation results: (a) 3% settling time for vehicle yaw rate r_1 and (b) RMS error for vehicle yaw rate r_1

achieves the fastest settling time for all scenarios, demonstrating the effectiveness of the proposed stability criteria. Regarding the RMS error in Figure 16 (b), the tongue weight determined by the consistency criteria resulted in the smallest vehicle yaw rate error compared to the original vehicle in the step steer scenarios. However, the stability criteria in the pulse steer scenarios revealed better consistency, as shown in Figure 16 (b). This is because, as clearly seen in Figure 15 (f), the vehicle-trailer system failed to maintain the stable lateral behaviour, resulting in significant handling performance discrepancies from the stable original vehicle. Nevertheless, it is agreeable with that the proposed consistency criteria effectively ensure the minimum discrepancy between the vehicle-trailer system and the original tow vehicle under ordinary driving conditions, from the results for SS60 and PS60 scenarios in Figure 16 (b).

6. Interpretation on the conventional tongue weight recommendation

The recommended tongue weight, traditionally set at 10-15% of the total trailer weight, lacks quantitative justification. However, the discussions made through this paper suggest it is the minimum necessary to ensure stability for light vehicle-trailer combinations.

As demonstrated in Figure 6 (b), lateral dynamics maintain a minimum damping ratio of about 0.3 within the linear slip region near the trailer's maximum legal speed, 90km/h (55mph). This demonstrates that the widely used standard tongue weight recommendation already meets the minimum condition proposed by D. E. Johnston et al., which requires the system's damping ratio to be at least 0.2 [7–9]. The conventional tongue weight recommendation also ensures that the vehicle yaw rate overshoot remains below 15%, with rise and settling times that do not exceed three times that of the original configuration, as shown in Figure 12 (f). Figure 4 (a) indicates that this tongue weight helps maintain even static loads between the front and rear axles.

However, given that optimal tongue weight for system stability is around 49% and for consistent handling performance is about 23%, it is evident that higher tongue weight is beneficial for stability and manoeuvrability compared to the traditionally recommended values. This conclusion is also supported by the results of the CarSim experiment, where a 10% tongue weight resulted in the most unstable behaviours. Nevertheless, considering that the severe driving scenarios used in the experiments are very rare in real life, a tongue weight of 10-15% can be deemed sufficient for ordinary driving conditions.

Moreover, the durability issue cannot be ignored from a practical standpoint, provided that the proposed optimal tongue weights are significantly larger than that of the conventional tongue weight recommendation. Fortunately, SAE standard J684 related to hitch assemblies specifies that commonly used ball hitches and hitch receivers must withstand loads up to 10 times the recommended tongue weight as a minimum test load requirement [19]. Despite such a safety margin, prolonged overloading of the hitch can lead to issues such as vehicle frame deformation or ball hitch warping. Therefore, it is crucial to pay extra attention when applying the proposed optimal tongue weights to on-road vehicles.

Consequently, the conventional tongue weight range can be regarded as the minimum requirements for the stability of light vehicle-trailers, satisfying the practical requirements such as durability, tyre wear, load equalisation.

7. Conclusion

Weight distribution in a vehicle-trailer system significantly affects system stability and manoeuvrability. Nonetheless, the appropriate tongue weight has been recommended merely based on the empirical evidence, not on the analytical evidence. Therefore, this paper conducted the pole-zero analysis, Bode analysis, and step response analysis for the lateral dynamics of vehicle-trailer combinations as influenced by tongue weight variation.

Subsequently, it proposed optimal tongue weight selection criteria to ensure system stability and consistent handling performance. For stability, a cost function was designed to minimise the real part of the system's dominant pole, identifying 48.61% of the trailer weight as the optimal tongue weight for the nominal system. For consistency, a cost function aimed to minimise the discrepancy between the system's frequency response and the original tow vehicle, resulting in an optimal tongue weight of 22.61% for the nominal system. The effectiveness of these optimal values was qualitatively validated through CarSim simulation results, confirming the validity of the proposed tongue weight selection criteria. The traditionally recommended tongue weight range was practically interpreted in comparison to the tongue weights chosen by the proposed criteria.

Finally, the proposed criteria were validated through CarSim simulation results, demonstrating their superiority over other options in terms of stability and consistency by evaluating the vehicle yaw rate's 3% settling time and RMS error.

The key contributions of this paper are as follows:

- The paper underscores that adjustments in the trailer's centre of gravity contribute to the open-loop system design of vehicle-trailer combinations. It provides comprehensive analyses, including pole-zero, Bode, and step response analysis, with variations in tongue weight.
- The paper introduces new criteria for selecting optimal tongue weight in light vehicle-trailer systems, focusing on improving stability and consistency. The effectiveness of the proposed criteria are confirmed with the 3% settling time and RMS error of vehicle yaw rate via CarSim simulation experiments.

The paper indicates the need for further research in the following areas:

- Further studies on the influence of drivers are required. The driver can be considered a path-tracking controller within the open loop, performing path following with feedforward control and feedback control against disturbances. Particularly, the stability analysis of the vehicle-trailer system with driver feedback is expected to provide a more reliable basis for setting optimal tongue weight.
- The brief sensitivity analysis of the vehicle-to-trailer weight ratio (m_2/m_1) was introduced in Section 4.3. In addition to m_2, further analysis with other parameters such as L_2, P is necessary. For example, analysing systems with a 5th-wheel hitch, where the distance P from the vehicle's rear axle to the hitch is negative and allows roll moment exchange between the vehicle and trailer, might be valuable.
- The paper only addresses steering inputs, but real-world instability of vehicle-trailer systems can also arise from disturbances, notably crosswinds. Stability analysis and strategies considering such disturbances need additional research.

In conclusion, this study proposed the optimal tongue weight selection criteria for the stability and consistency of light vehicle-trailer systems and provided a comprehensive analysis. While the authors regard the findings in this study as novel and intriguing, it is essential to approach the conclusion cautiously, given the significant deviation from existing guidelines. The authors hope that this study serves as a foundational resource for future studies, encouraging not only further exploration into the two proposed criteria but also constructive discussions and approaches from various perspectives.

Disclosure statement

No potential conflict of interest was reported by the authors.

Funding

This research was supported in part by the BK21 FOUR Program of the National Research Foundation Korea(NRF) grant funded by the Ministry of Education(MOE); the Technology Innovation Program (20010263, Development of innovative design for UX environment improvement and commercialization model of wheelchair electric motorization kit with enhanced portability and convenience) funded by the Ministry of Trade, Industry & Energy(MOTIE, Korea); the Technology Innovation Program (20014983, Development of autonomous chassis platform for a modular vehicle) funded by the Korean government(MOTIE); Autonomous Driving Technology Development Innovation Program (20018181, Development of Lv. 4+ autonomous driving vehicle platform based on point-to-point driving to logistic center for heavy trucks) funded by the Korean government(MOTIE) and Korea Evaluation Institute of Industrial Technology(KEIT); KEIT grant funded by the Korean government(MOTIE) (No. 20023815, Development of Traction and Steering Integrated Plug and Drive Module)

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