Observer-based iterative learning control for a high relative degree nonlinear system and its application to a vehicular wet-clutch system

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ABSTRACT

In this paper, an observer-based iterative learning control with an output feedback control scheme is proposed for a high relative degree nonlinear system with an unmeasurable control state. Multiple differentiation of measurements can make the control result sensitive to measurement noise when controlling such systems. A convergence analysis was established that can use only lower-order differentiation regardless of the highest-order differentiation, based on the robustness condition to measurement noise and initial estimation error. The application to the vehicular wet-clutch system is presented to illustrate the effectiveness of the proposed method and its learning gain selection. These contributions are verified through theoretical analysis and simulation of the wet-clutch system. The simulation results show the effectiveness of the proposed approach for a class of nonlinear systems with a high relative degree.

1. Introduction

Iterative learning control (ILC) is an efficient method that offers significant improvement in the performance of systems that operate in an iterative or repetitive fashion over a fixed time interval. It is particularly effective in the presence of unmodeled dynamics, parametric uncertainties, and disturbances because better control performance is expected as the iteration progresses. ILCs are applied to dynamic systems working on the same task repetitively over a fixed operation cycle. The idea is to get one step closer to the desired control result by updating the control input of the next iteration based on model information about the system and previous control results. Control problems that are difficult to solve on the time axis might be solved in the iteration domain. For example, a wet-clutch system needs to be controlled without a sensor in a short time, but the system and control reference is iteration invariant.

For ILCs including linear and nonlinear systems, refer the review article[1, 2]. A pseudoinverse-based method [3] and adjoint-type method [4] have been developed for nonlinear nonminimum phase system. The robustness and monotonic convergence (MC) of ILC, which are very useful in the presence of disturbance and model uncertainty, have been studied [5]. For nonlinear systems, ILC has been studied extensively [6].

Research on the relative degree and the structure of ILC is not yet extensive. ILC is a method of updating the system input of the next iteration with an error in the system output. Consequently, the relative degree closely related to the relationship between system input and output should also be considered in the structure of ILC. In this paper, a high relative degree means greater than 2. ILC designed in most of the previous efforts for a high relative degree system has error terms differentiated by the relative degree [7, 8, 9, 10, 11]. A PD^{α}-type ILC has been studied for fractional-order nonlinear systems [12]. However, in actual practice, it is not generally desirable to use measured values differentiated more than twice. The signal-to-noise ratio (SNR) of laboratory-level equipment may be high enough, but sensors of commercially available products may have lower SNR. According to conventional methods, the ILC of a system with a high relative degree should include a measurement data differentiated by multiple times. Therefore, the main results of this paper propose an ILC that converges regardless of the highest-order differentiation.

Moreover, in many cases, systems with a high relative degree cannot measure all the states needed for control and require an observer; a wet-clutch system of a vehicle is a typical mechanical system of this type. It has a mechanical structure in which the hydraulic pressure from the solenoid valve pushes a piston and compresses the clutch to generate friction torque. As a result, the high relative degree from the system input current to the clutch torque is high. The

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clutch torque, the system output, can be measured on a vehicle test bench, but the position of the clutch piston inside the transmission is difficult to measure due to its complex structure.

To solve this problem, there have been many studies on observer-based ILC schemes for a class of time-varying nonlinear systems with a relative degree of one [13, 14]. Previous works can be applied to a system with a relative degree of zero or one. However, for systems with a relative degree of 2 or more, measured values differentiated by the relative degree were used based on what has been studied so far.

Therefore, this paper proposes a structure using lower-order differentiation in an observer-based output feedback control scheme for ILC of a nonlinear system with a high relative degree. The main contribution of this paper is ILC convergence, which includes not only highest-order differentiation but also lower-order differentiation for a class of nonlinear systems. Unlike with the proposed method, it is possible for variable gain ILCs using an observer to be sensitive to noise when the system singular values fluctuate. In nonlinear systems with a high relative degree, ILC convergence may not be guaranteed due to measurement noise due to large learning gains and multiple error differentiation. The proposed method is robust to measurement noise by excluding highest-order error differentiation and using an observer. To ensure satisfactory control performance, robustness with respect to measurement noise and initial estimation errors is guaranteed.

A wet-clutch system has complex dynamics, and its model parameters change significantly even with small parameter changes. Since the wet-clutch system has many difficulties to control, model-based control methods are being studied. While control of filling phase is the research scope in this paper, one related study solved wet clutch control in the inertia phase of gear shift using ILC [15]. This paper deals with the control of the filling phase, which is the initial stage of operation of the wet-clutch. When modeling such systems for control, it should be assumed that unmodeled dynamics, parametric uncertainties, and disturbances exist, i.e., robustness should be concerned. There was research on the robust control of the filling phase on the time axis [16]. There have been studies on learning control of the hydraulic actuator using the relationship between the normal force and the current, which has a relative degree of 1 [17, 18]. Wang et al. [19] studied the filling phase control of a wet-clutch with an adaptive fuzzy iterative control strategy. However, this research did not analyze the convergence of ILC and had no proof of convergence. This paper provides the convergence of ILC on the system with a high relative degree, such as the wet-clutch system.

One more significant consideration in wet-clutch system control is that the operating sequence has a mode change in which the clutch torque occurs after the piston makes contact. A heuristic approach for controlling an operating sequence with a mode change may result in control discontinuity. The piston's travel and ramp-up of the clutch torque should be controlled in a unified way, which requires output feedback control. Pinte et al. [20] showed that ILC could be a good alternative for time-consuming calibration procedures. A position-based control system, rather than a pressure-based one, was presented. However, in practice, the piston position is almost impossible to measure, and measurements are only made on specially designed test benches. In addition to the piston position, the clutch torque should also be targeted as a control reference to enable quick and smooth clutch operation. The proposed ILC method was applied and verified with the wet-clutch system simulation. The simulation was compared with the actual experimental data to validate its accuracy.

The contents are organized as follows. Section 2 first describes the preliminaries and definitions to be used. Section 3 details the main results, including the ILC update scheme for a nonlinear system with a high relative degree and its convergence proof. In Section 4, the application for the algorithm developed in Section 3 is discussed, as well as the simulation results obtained by using the proposed ILC. By testing the performance of the proposed ILC structure, the convergence in the presence of model uncertainty and measurement noise was verified.

2. Preliminaries

Consider a class of nonlinear affine system described by

$$\begin{aligned} \dot{x}(t) &= f(x(t)) + g(x(t))u(t) \\ y(t) &= h(x(t)) + w(t) \\ z(t) &= Cx(t), \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$, $y \in \mathbb{R}$, $w \in \mathbb{R}$, $u \in \mathbb{R}$, and $z \in \mathbb{R}^l$ represent the state vector, measurable system output, additive measurement noise, the control input, and control state, respectively. The nonlinear functions $f(\cdot) \in \mathbb{R}^n$, $g(\cdot) \in \mathbb{R}^n$, and $h(\cdot) \in \mathbb{R}$ are smooth in and Lipschitz on their domain of definition. Suppose that (1) operates iteratively over a finite-time interval $t \in [0, T]$. In the sequel, the time argument t will be omitted where there is no matter to any confusion.

For a given realizable trajectory y_d, the objective of this work was to find the input profile that allows the system output to follow the desired trajectory. If the system functions are Lipschitz, there exists a unique desired input u_d s.t.

$$\begin{aligned} \dot{x}_d &= f(x_d) + g(x_d)u_d \\ y_d &= h(x_d) \\ z_d &= Cx_d, \end{aligned} \tag{2}$$

where x_d is the resultant state.

2.1. Relative degree

The definition of relative degree and Lie derivative L from [21] is used. A system having the structure of (1) and having a relative degree μ and μ_x for measurements y and states z has the following properties.

$$L_g L^r_f h(x) = 0, \quad \text{for } r = 1, ..., \mu - 1,$$

and

$$\begin{aligned} y &= h(x) + w \\ \overset{(r)}{y} &= L^r_f h(x) + \overset{(r)}{w}, \quad \text{for } r = 1, ..., \mu - 1 \\ \overset{(\mu)}{y} &= L^\mu_f h(x) + L_g L^{\mu-1}_f h(x)u + \overset{(\mu)}{w} \\ \overset{(r)}{z} &= CL^{r-1}_f f(x), \quad \text{for } r = 1, ..., \mu_x - 1 \\ \overset{(\mu_x)}{z} &= CL^{\mu_x-1}_f f(x) + CL_g L^{\mu_x-2}_f f(x)u, \end{aligned}$$

when μ and μ_x are 2 or greater.

The iterative learning law is closely related to the input-to-output relation, and the relative degree is the minimum number of derivatives of the system output for which the system input appears in the equation. In order to use this relation, in previous studies, the system output error differentiated by the relative degree was fed back for input updating. The convergence of the ILC is also highly dependent on $L_g L^{\mu-1}_f h(x)$ (in a linear system, $CA^{\mu-1}B$) which is $\partial \overset{(\mu)}{y} / \partial u$.

The use of lower-order derivatives for ILCs with high relative degree systems might effectively handle measurement noise. The standard deviation of the rth order derivative of a Gaussian random variable can be calculated by the usual rules for error propagation, and is $\sqrt{\sum_{j=0}^{r} \left(\frac{r!}{(r-j)!j!} \right)^2}$ times larger than the original noise. Although noncausal filters are available in ILC, it is generally undesirable to differentiate measured data more than twice from a practical point of view in automotive control applications. In the following content, lower-order differentiations gains were also included in the ILC convergence criteria so that engineers could select them.

Assumption 1. *Assume the system functions f, g, h, and their Lie derivative functions are continuous on the closed interval $[0, T]$, differentiable on the open interval $(0, T)$, and Lipschitz in x on their domain of definition as*

$$\begin{aligned} \left\| L^r_f f(x_1) - L^r_f f(x_2) \right\| &\le f_{(r)} \left\| x_1 - x_2 \right\| \\ \left\| L_g L^r_f f(x_1) - L_g L^r_f f(x_2) \right\| &\le \eta_{(r)} \left\| x_1 - x_2 \right\| \\ \left\| L^r_f g(x_1) - L^r_f g(x_2) \right\| &\le g_{(r)} \left\| x_1 - x_2 \right\| \\ \left\| h(x_1) - h(x_2) \right\| &\le l_h \left\| x_1 - x_2 \right\| \\ \left\| L_g L^r_f h(x_1) - L_g L^r_f h(x_2) \right\| &\le h_{(r)} \left\| x_1 - x_2 \right\|, \end{aligned} \tag{3}$$

for $r = 0, ..., \mu - 1$, and positive Lipschitz constants $f_{(r)}$, $\eta_{(r)}$, $g_{(r)}$, l_h and $h_{(r)}$.

2.2. State observer

According to [22, Theorem 2], a system is uniformly observable if it is diffeomorphic to a system of the normal form [23]. A state observer was designed using a linearized observability matrix-based method [24].

$$\begin{aligned} \dot{\hat{x}}_i &= f(\hat{x}_i) + g(\hat{x}_i)u_i + K(\hat{x}_i)\left(y_i - h(\hat{x}_i)\right) \\ \hat{z}_i &= C\hat{x}_i, \end{aligned} \tag{4}$$

where $K(\hat{x}_i(t))$ is the estimation gain and subscript i is iteration number. For other methods for nonlinear systems with a high relative degree, refer to [22, 25, 26].

Assumption 2. *A system (1) is uniformly observable, and a state observer (4) can achieve desired estimation error convergence.*

The function ϕ is defined to substitute the derivative of $\hat{x}_i$ in the following content, and the estimation error and control errors are defined as:

$$\begin{aligned} \phi(\hat{x}_i, u_i) &= f(\hat{x}_i) + g(\hat{x}_i)u_i \\ e_i &= x_i - \hat{x}_i \\ \delta x_i &= x_d - x_i \\ \delta \hat{x}_i &= x_d - \hat{x}_i \\ \delta u_i &= u_d - u_i \\ \delta y_i &= y_d - h(x_i). \end{aligned}$$

2.3. Grönwal-Bellman lemma

From (2) and (4),

$$\begin{aligned} \delta\hat{x}_i(t) =& \delta\hat{x}_i(0) + \int_0^t \left(\dot{x}_d(\tau) - \dot{\hat{x}}_i(\tau)\right) d\tau \\ =& \delta\hat{x}_i(0) + \int_0^t \left[f(x_d(\tau)) + g(x_d(\tau))u_d(\tau) - f(\hat{x}_i(\tau)) - g(\hat{x}_i(\tau))u_i(\tau) - K(\hat{x}_i(\tau))\left(h(x_i(\tau)) - h(\hat{x}_i(\tau)) + w(\tau)\right)\right] d\tau. \end{aligned}$$

If the sampling time is short enough, the integration of the zero mean Gaussian random variable over finite time is small enough to be negligible.

$$\begin{aligned} \left\|\delta\hat{x}_i(t)\right\| \le& \left\|\delta\hat{x}_i(0)\right\| + \int_0^t \left[f_0 \left\|\delta\hat{x}_i(\tau)\right\| + g_0 \left\|\delta\hat{x}_i(\tau)\right\| \cdot \left\|u_d(\tau)\right\|_\infty + \left\|g(\hat{x}_i(\tau))\right\|_\infty \left\|\delta u_i(\tau)\right\| + \|K(\tau)\|_\infty h_0 \left\|e_i\right\|\right] d\tau \\ =& \left\|\delta\hat{x}_i(0)\right\| + \int_0^t \left[\alpha_0 \left\|\delta\hat{x}_i(\tau)\right\| + \beta_1 \left\|\delta u_i(\tau)\right\| + \beta_2 \left\|e_i(\tau)\right\|\right] d\tau, \end{aligned}$$

where $\alpha_0 = f_0 + g_0 \left\|u_d(\tau)\right\|_\infty$, $\beta_1 = \left\|g(\hat{x}_i(\tau))\right\|_\infty$, and $\beta_2 = \|K(\tau)\|_\infty h_0$.

Applying the Grönwal-Bellman lemma, we obtain

$$\left\|\delta\hat{x}_i(t)\right\| \le \left\|\delta\hat{x}_i(0)\right\| e^{\alpha_0 t} + \int_0^t \left[\beta_1 \left\|\delta u_i(\tau)\right\| + \beta_2 \left\|e_i(\tau)\right\|\right] e^{\alpha_0(t-\tau)} d\tau. \tag{5}$$

2.4. λ-norm

The supremum norm denoted by $\|*(t)\|_\infty = \sup_{t\in[0,T]} \|*(t)\|$ and the λ-norm denoted by $\|*(t)\|_\lambda = \sup_{t\in[0,T]} \left[e^{-\lambda t} \|*(t)\|\right]$ will be used.

3. Main results

ILCs using only $L_g L_f^{\mu-1} h(x)$ or $CA^{\mu-1}B$ in their contraction mapping-based convergence [9, 27] require μ times derivative of the measurement with measurement noise, and this is undesirable in terms of robustness against noise. We propose an extension to determine the learning gain of the lower-order differentiation of error using their Jacobian supremum in the output feedback scheme. Before derivation, the following assumptions are made:

Assumption 3. *The system has a relative degree of 2 or more and n or less, i.e.,* $2 \le \mu \le n$.

Assumption 4. *The additive measurement noise is white and has a truncated zero mean Gaussian distribution.*

Assumption 5. *The desired input, observation error, initial state error, and initial observation error are bounded.*

Assumption 6. *The desired output trajectory is* μ *times differentiable and the resultant state is* μ_x *times differentiable on the open interval* $(0, T)$.

Consider an observer-based ILC with an output feedback control scheme.

$$u_{i+1} = u_i + \sum_{r=0}^{\mu} \Gamma_{r,y} \left(\overset{(r)}{y}_d - \overset{(r)}{y}_i \right) + \Gamma_{0,x} \left(z_d - \hat{z}_i \right) + \sum_{r=1}^{\mu_x} \Gamma_{r,x} \left(\overset{(r)}{z}_d - C \overset{(r-1)}{\phi_i} \right), \tag{6}$$

where $\Gamma_{r,y}$, $\Gamma_{0,x}$, and $\Gamma_{r,x}$ are non-negative iterative learning gains. The superscript (r) denotes the r-th derivative, and the subscript i denotes the iteration number.

Using mean value theorem, there exists some $\xi_r = x_i + \Sigma\delta x_i$, where $\Sigma = \mathrm{diag}(\sigma_1, ..., \sigma_n)$, $\sigma_k \in [0, 1]$, for $k = 1, ..., n$,

$$L_f^r h(x_d) - L_f^r h(x_i) = \frac{d}{du} L_f^r h(\xi_r)\delta u_i, \quad \text{for } r = 0, ..., \mu.$$

Then,

$$\begin{aligned} u_{i+1} =& u_i + \sum_{r=0}^{\mu} \Gamma_{r,y} \frac{d}{du} L_f^r h(\xi_r)\delta u_i + \Gamma_{\mu,y} \left(L_g L_f^{\mu-1} h(x_d) u_d - L_g L_f^{\mu-1} h(x_i) u_i \right) - \sum_{r=0}^{\mu} \Gamma_{r,y} \overset{(r)}{w_i} \\ &+ \Gamma_{0,x} C \delta\hat{x}_i + \sum_{r=1}^{\mu_x} \Gamma_{r,x} C \left(L_f^{r-1} f(x_d) - L_f^{r-1} f(\hat{x}_i) \right) + \Gamma_{\mu_x,x} C \left(L_g L_f^{\mu_x-2} f(x_d) u_d - L_g L_f^{\mu_x-2} f(\hat{x}_i) u_i \right), \end{aligned}$$

$$\begin{aligned} \delta u_{i+1} =& \left(1 - \sum_{r=0}^{\mu} \Gamma_{r,y} \frac{d}{du} L_f^r h(\xi_r) - \Gamma_{\mu,y} L_g L_f^{\mu-1} h(x_i) - \Gamma_{\mu_x,x} C L_g L_f^{\mu_x-2} f(\hat{x}_i) \right) \delta u_i \\ &- \Gamma_{\mu,y} \left(L_g L_f^{\mu-1} h(x_d) - L_g L_f^{\mu-1} h(x_i) \right) u_d + \sum_{r=0}^{\mu} \Gamma_{r,y} \overset{(r)}{w_i} \\ &- \Gamma_{0,x} C \delta\hat{x}_i - \sum_{r=1}^{\mu_x} \Gamma_{r,x} C \left(L_f^{r-1} f(x_d) - L_f^{r-1} f(\hat{x}_i) \right) - \Gamma_{\mu_x,x} C \left(L_g L_f^{\mu_x-2} f(x_d) - L_g L_f^{\mu_x-2} f(\hat{x}_i) \right) u_d. \end{aligned}$$

Define $\rho \triangleq \left\| 1 - \sum_{r=0}^{\mu} \Gamma_{r,y} \frac{d}{du} L_f^r h(\xi_r) - \Gamma_{\mu,y} L_g L_f^{\mu-1} h(x_i) - \Gamma_{\mu_x,x} C L_g L_f^{\mu_x-2} f(\hat{x}_i) \right\|_\infty$. Then according to Assumption 1, we obtain

$$\begin{aligned} \|\delta u_{i+1}\| \le& \rho \|\delta u_i\| + \left[\|\Gamma_{0,x} C\| + \sum_{r=1}^{\mu_x} \|\Gamma_{r,x} C\| f_{(r-1)} \right] \|\delta\hat{x}_i\| + \left(\left\|\Gamma_{\mu,y}\right\| h_{(\mu-1)} + \left\|\Gamma_{\mu_x,x} C\right\| \eta_{(\mu_x-2)} \right) \|u_d\| \cdot \|\delta x_i\| \\ &+ \sum_{r=0}^{\mu} \left\|\Gamma_{r,y}\right\| \left\|\overset{(r)}{w_i}\right\|. \end{aligned}$$

The state control error include estimation error as

$$\|\delta x_i\| = \|\delta\hat{x}_i - e_i\| \le \|\delta\hat{x}_i\| + \|e_i\|,$$

which leads to

$$\begin{aligned} \|\delta u_{i+1}\| \le& \rho \|\delta u_i\| + \left[\|\Gamma_{0,x} C\| + \sum_{r=1}^{\mu_x} \|\Gamma_{r,x} C\| f_{(r-1)} + \left(\left\|\Gamma_{\mu,y}\right\| h_{(\mu-1)} + \left\|\Gamma_{\mu_x,x} C\right\| \eta_{(\mu_x-2)} \right) \|u_d\| \right] \|\delta\hat{x}_i\| \\ &+ \left(\left\|\Gamma_{\mu,y}\right\| h_{(\mu-1)} + \left\|\Gamma_{\mu_x,x} C\right\| \eta_{(\mu_x-2)} \right) \|u_d\| \cdot \|e_i\| + \sum_{r=0}^{\mu} \left\|\Gamma_{r,y}\right\| \left\|\overset{(r)}{w_i}\right\|. \end{aligned}$$

Define $\alpha_{(\mu)} \triangleq \left\|\Gamma_{0,x}C\right\| + \sum_{r=1}^{\mu_x} \left\|\Gamma_{r,x}C\right\| f_{(r-1)} + \left(\left\|\Gamma_{\mu,y}\right\| h_{(\mu-1)} + \left\|\Gamma_{\mu_x,x}C\right\| \eta_{(\mu_x-2)}\right) \left\|u_d\right\|_\infty$. Then,

$$\left\|\delta u_{i+1}\right\| \leq \rho \left\|\delta u_i\right\| + \alpha_{(\mu)} \left\|\delta \hat{x}_i\right\| + \left(\left\|\Gamma_{\mu,y}\right\| h_{(\mu-1)} + \left\|\Gamma_{\mu_x,x}C\right\| \eta_{(\mu_x-2)}\right) \left\|u_d\right\| \cdot \left\|e_i\right\| + \sum_{r=0}^{\mu} \left\|\Gamma_{r,y}\right\| \left\|\overset{(r)}{\omega}_i\right\|.$$

Applying the Grönwal-Bellman lemma,

$$\begin{aligned}\left\|\delta u_{i+1}\right\| \leq& \rho \left\|\delta u_i\right\| + \alpha_{(\mu)} \left[\left\|\delta \hat{x}_i(0)\right\| e^{\alpha_0 t} + \int_0^t \left[\beta_1 \left\|\delta u_i(\tau)\right\| + \beta_2 \left\|e_i(\tau)\right\|\right] e^{\alpha_0(t-\tau)} d\tau\right] \\ &+ \left(\left\|\Gamma_{\mu,y}\right\| h_{(\mu-1)} + \left\|\Gamma_{\mu_x,x}C\right\| \eta_{(\mu_x-2)}\right) \left\|u_d\right\| \cdot \left\|e_i\right\| + \sum_{r=0}^{\mu} \left\|\Gamma_{r,y}\right\| \left\|\overset{(r)}{\omega}_i\right\|.\end{aligned}$$

Multiplying the previous inequality by $e^{-\lambda t}$, $\lambda > \alpha_0$, and applying the λ-norm, we obtain

$$\begin{aligned}\left\|\delta u_{i+1}\right\|_\lambda \leq& \rho \left\|\delta u_i\right\|_\lambda + \alpha_{(\mu)} \sup_{t\in[0,T]} \left[\left\|\delta \hat{x}_i(0)\right\| e^{(\alpha_0-\lambda)t}\right] \\ &+ \alpha_{(\mu)} \sup_{t\in[0,T]} \left[\int_0^t e^{-\lambda\tau} \left[\beta_1 \left\|\delta u_i(\tau)\right\| + \beta_2 \left\|e_i(\tau)\right\|\right] e^{(\alpha_0-\lambda)(t-\tau)} d\tau\right] \\ &+ \sup_{t\in[0,T]} \left[\left(\left\|\Gamma_{\mu,y}\right\| h_{(\mu-1)} + \left\|\Gamma_{\mu_x,x}C\right\| \eta_{(\mu_x-2)}\right) \left\|u_d\right\| \cdot \left\|e_i\right\| e^{-\lambda t}\right] \\ &+ \sup_{t\in[0,T]} \sum_{r=0}^{\mu} \left\|\Gamma_{r,y}\right\| \left\|\overset{(r)}{\omega}_i e^{-\lambda t}\right\| \\ \leq& \rho \left\|\delta u_i\right\|_\lambda + \alpha_{(\mu)} \left\|\delta \hat{x}_i(0)\right\| \\ &+ \alpha_{(\mu)}\beta_1 \sup_{t\in[0,T]} \left[\int_0^t e^{-\lambda\tau} \left\|\delta u_i(\tau)\right\| e^{(\alpha_0-\lambda)(t-\tau)} d\tau\right] \\ &+ \alpha_{(\mu)}\beta_2 \sup_{t\in[0,T]} \left[\int_0^t e^{-\lambda\tau} \left\|e_i(\tau)\right\| e^{(\alpha_0-\tau)(t-\lambda)} d\tau\right] \\ &+ \left(\left\|\Gamma_{\mu,y}\right\| h_{(\mu-1)} + \left\|\Gamma_{\mu_x,x}C\right\| \eta_{(\mu_x-2)}\right) \left\|u_d\right\|_\infty \cdot \left\|e_i\right\|_\lambda \\ &+ \sum_{r=0}^{\mu} \left\|\Gamma_{r,y}\right\| \left\|\overset{(r)}{\omega}_i\right\|_\lambda \\ \leq& \rho \left\|\delta u_i\right\|_\lambda + \alpha_{(\mu)} \left\|\delta \hat{x}_i(0)\right\| + \alpha_{(\mu)}\beta_1 \frac{1-e^{(\alpha_0-\lambda)T}}{\lambda-\alpha_0} \left\|\delta u_i\right\|_\lambda + \alpha_{(\mu)}\beta_2 \frac{1-e^{(\alpha_0-\lambda)T}}{\lambda-\alpha_0} \left\|e_i\right\|_\lambda \\ &+ \left(\left\|\Gamma_{\mu,y}\right\| h_{(\mu-1)} + \left\|\Gamma_{\mu_x,x}C\right\| \eta_{(\mu_x-2)}\right) \left\|u_d\right\|_\infty \cdot \left\|e_i\right\|_\lambda + \sum_{r=0}^{\mu} \left\|\Gamma_{r,y}\right\| \left\|\overset{(r)}{\omega}_i\right\|_\lambda,\end{aligned}$$

which leads to

$$\left\|\delta u_{i+1}\right\|_\lambda \leq (\rho+\gamma) \left\|\delta u_i\right\|_\lambda + \epsilon, \tag{7}$$

where

$$\begin{aligned}\gamma =& \alpha_{(\mu)}\beta_1 \frac{1-e^{(\alpha_0-\lambda)T}}{\lambda-\alpha_0} \\ \epsilon =& \alpha_{(\mu)} \left\|\delta \hat{x}_i(0)\right\| + \left[\alpha_{(\mu)}\beta_2 \frac{1-e^{(\alpha_0-\lambda)T}}{\lambda-\alpha_0} + \left(\left\|\Gamma_{\mu,y}\right\| h_{(\mu-1)} + \left\|\Gamma_{\mu_x,x}C\right\| \eta_{(\mu_x-2)}\right) \left\|u_d\right\|_\infty\right] \left\|e_i\right\|_\lambda + \sum_{r=0}^{\mu} \left\|\Gamma_{r,y}\right\| \left\|\overset{(r)}{\omega}_i\right\|_\lambda.\end{aligned}$$

There exists a sufficiently large λ making γ arbitrarily small. Thus, if $\rho < 1$, there exists λ such that $\rho + \gamma < 1$. Hence the contraction mapping is

$$\lim_{i\to\infty} \left\|\delta u_i\right\|_\lambda \leq \frac{\epsilon}{1-(\rho+\gamma)}.$$

Now, we can summarize the previous development in the following theorem.

Theorem 1. *Consider a class of nonlinear system (1) satisfying Assumptions 1-6 with the iterative learning controller (6), where $\hat{x}$ is given by the state observer (4). Then for a given realizable desired trajectory $y_d(t)$, $t \in [0, T]$, the system output error is stable in terms of bounded-input, bounded-output (BIBO) stability if $\rho < 1$ as $i \to \infty$.*

Proof. Again using Grönwal-Bellman lemma simmilar to (5), we obtain

$$\left\|\delta x_i(t)\right\| \leq \left\|\delta x_i(0)\right\| e^{\alpha_0 t} + \int_0^t \beta_1 \left\|\delta u_i(\tau)\right\| e^{\alpha_0 (t-\tau)} d\tau.$$

Multiplying by $e^{-\lambda t}$, $\lambda > \alpha_0$, and applying the λ-norm, we have

$$\begin{aligned}\left\|\delta x_i(t)\right\|_\lambda &\leq \left\|\delta x_i(0)\right\| + \beta_1 \int_0^t \left\|\delta u_i(\tau)\right\|_\lambda e^{(\alpha_0-\lambda)(t-\tau)} d\tau \\ &\leq \left\|\delta x_i(0)\right\| + \beta_1 \frac{1-e^{(\alpha_0-\lambda)T}}{\lambda-\alpha_0} \left\|\delta u_i(\tau)\right\|_\lambda .\end{aligned}$$

By the Lipschitzness,

$$\left\|\delta y_i(t)\right\|_\lambda \leq l_h \left\|\delta x_i(t)\right\|_\lambda .$$

The system output error is bounded and stable if (7) holds. □

Remark 2. Taking into account the term $\sum_{r=0}^{\mu} \left\|\Gamma_{r,y}\right\| \left\|\overset{(r)}{w_i}\right\|_\lambda$ of ϵ in (7), the noise from higher-order error derivatives can be accumulated to the upper bound of the error. As the derivative of noise increases, its norm becomes excessive. In this aspect, the effectiveness of using a lower-order differentiation in a system with a high relative degree can be confirmed.

3.1. Choosing learning gain Γ

To meet the convergence condition, the learning gains should be selected such that $\rho < 1$.

$$\begin{aligned}\Gamma_{r,y} &< \left\|\frac{d}{du} L_f^r h(\xi_r)\right\|_\infty^{-1}, \quad \text{for } r = 0, ..., \mu \\ \Gamma_{\mu,y} &< \left\|\left(\frac{d}{du} L_f^\mu h(\xi_\mu) + L_g L_f^{\mu-1} h(x_i)\right)\right\|_\infty^{-1} \\ \Gamma_{\mu_x,x} &< \left\|C L_g L_f^{\mu_x-2} f(\hat{x}_i)\right\|_\infty^{-1}\end{aligned}$$

Example. For example, for a nonlinear affine system below,

$$\begin{aligned}\dot{x}_1 &= f_{x1}(x_2) \\ \dot{x}_2 &= f_{x2}(x_2) + g_{x2}(x_2)u \\ y &= h(x_1)\end{aligned}$$

The system is order 2 and has a relative degree μ and μ_x of 2 and $C = (1, 0)$. Assume zero initial conditions at each iteration.

For $\Gamma_{0,y}$, $\Gamma_{1,y}$, and $\Gamma_{1,x}$, by the total derivative and Leibniz rule,

$$\begin{aligned}
\left\|\frac{d}{du}h(\xi_0)\right\|_\infty &= \left\|\frac{\partial h(x_1)}{\partial x_1}\frac{\partial x_1}{\partial x_2}\frac{dx_2}{du}\right\|_\infty \\
&\le \left\|\frac{\partial h(x_1)}{\partial x_1}\right\|_\infty \cdot \left\|\frac{\partial x_1}{\partial x_2}\right\|_\infty \cdot \left\|\frac{dx_2}{du}\right\|_\infty \\
&= \left\|\frac{\partial h(x_1)}{\partial x_1}\right\|_\infty \cdot \left\|\frac{\partial}{\partial x_2}\int_0^t f_{x1}(x_2(\tau))d\tau\right\|_\infty \cdot \left\|\frac{d}{du}\int_0^t \left(f_{x2}(x_2(\tau)) + g_{x2}(x_2(\tau))u(\tau)\right)d\tau\right\|_\infty \\
&= \left\|\frac{\partial h(x_1)}{\partial x_1}\right\|_\infty \cdot \left\|\int_0^t \frac{\partial}{\partial x_2} f_{x1}(x_2(\tau))d\tau\right\|_\infty \cdot \left\|\int_0^t \frac{\partial}{\partial u}\left(f_{x2}(x_2(\tau)) + g_{x2}(x_2(\tau))u(\tau)\right)d\tau\right\|_\infty \\
&= \left\|\frac{\partial h(x_1)}{\partial x_1}\right\|_\infty \cdot \left\|\int_0^t \frac{\partial}{\partial x_2} f_{x1}(x_2(\tau))d\tau\right\|_\infty \cdot \overline{g}_{x2}T.
\end{aligned}$$

where $\overline{g}_{x2}$ is the supremum of the function $g_{x2}(x_2)$. Similarly,

$$\begin{aligned}
\left\|\frac{d}{du}L_f h(\xi_1)\right\|_\infty &= \left\|\left(\frac{\partial^2 h}{\partial x_1^2} f_{x1}\frac{\partial x_1}{\partial x_2} + \frac{\partial h(x_1)}{\partial x_1}\frac{\partial f_{x1}(x_2)}{\partial x_2}\right)\frac{dx_2}{du}\right\|_\infty \\
&\le \left(\left\|\frac{\partial^2 h}{\partial x_1^2}\right\|_\infty \cdot \overline{f}_{x1} \cdot \left\|\frac{\partial x_1}{\partial x_2}\right\|_\infty + \left\|\frac{\partial h(x_1)}{\partial x_1}\right\|_\infty \cdot \left\|\frac{\partial f_{x1}(x_2)}{\partial x_2}\right\|_\infty\right)\overline{g}_{x2}T.
\end{aligned}$$

$$\begin{aligned}
\left\|CL_g L_f^{\mu_x-2} f(\hat{x}_i)\right\|_\infty &= \left\|\frac{\partial f_{x1}(x_2)}{\partial x_2} g_{x2}\right\|_\infty \\
&\le \left\|\frac{\partial f_{x1}(x_2)}{\partial x_2}\right\|_\infty \cdot \overline{g}_{x2},
\end{aligned}$$

where $\overline{f}_{x1}$ is the supremum of the function $f_{x1}(x_2)$.

While satisfying (7), we can choose learning gain as follows:

$$\begin{aligned}
\Gamma_{0,y} &= \left[\left\|\frac{\partial h(x_1)}{\partial x_1}\right\|_\infty \cdot \left\|\int_0^t \frac{\partial}{\partial x_2} f_{x1}(x_2(\tau))d\tau\right\|_\infty \cdot \overline{g}_{x2}T\right]^{-1} \\
\Gamma_{1,y} &= \left[\left(\left\|\frac{\partial^2 h}{\partial x_1^2}\right\|_\infty \cdot \overline{f}_{x1} \cdot \left\|\frac{\partial x_1}{\partial x_2}\right\|_\infty + \left\|\frac{\partial h(x_1)}{\partial x_1}\right\|_\infty \cdot \left\|\frac{\partial f_{x1}(x_2)}{\partial x_2}\right\|_\infty\right)\overline{g}_{x2}T\right]^{-1} \\
\Gamma_{2,y} &= 0 \\
\Gamma_{0,x} &= 0 \\
\Gamma_{1,x} &= 0 \\
\Gamma_{2,x} &= \left[\left\|\frac{\partial f_{x1}(x_2)}{\partial x_2}\right\|_\infty \cdot \overline{g}_{x2}\right]^{-1}.
\end{aligned}$$

4. Application to the vehicular wet-clutch system

The wet multi-plate clutch system operates with a hydraulic actuator, and the cross-section schematic diagram is drawn in the Fig. 1a. Working fluid from the variable force solenoid (VFS) enters the pressure chamber and pushes the piston to bring the multi-plate clutches into contact, and eventually, friction occurs. The higher the pressure in the chamber, the harder the piston squeezes the clutches and the greater the friction torque. A simplified schematic diagram is drawn in Fig. 1b. Spring puts the piston back in place when pressure is relieved in the pressure chamber. The inner and outer shafts are illustrated, and the role of the clutch is to transmit torque between the two shafts.

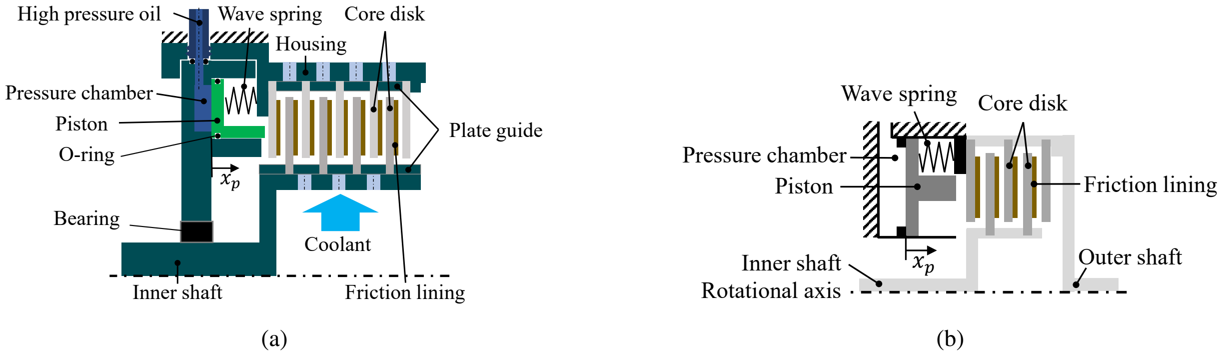


Figure 1: (a) Schematic diagram of a multi-plate clutch (brake) system. (b) Simplified schematic diagram of a multi-plate clutch (brake) system.

4.1. Control-oriented modeling

The wet-clutch system consists of a solenoid valve, a piston and a clutch pack. First, oil is supplied from the solenoid valve, then the fluid pushes the piston to create pressure, and finally, clutch torque is generated by the clutch friction. The system input is the current applied to the solenoid, and the major states are the position of the solenoid and the position of the piston. The measurable system output is the clutch torque.The overall objective of the system control is to obtain control input that follows a given clutch torque reference.

4.1.1. Variable force solenoid

The hydraulic actuator is a spool valve operated by a VFS. When an electric current is applied to it, the solenoid pushes the spool by electromagnetic force. It adjusts the opening between the supply port and the control port to control the flow rate and pressure.

$$F_{mag} = K_{mag}(I - I_0),$$

where F_{mag} is the magnetic force generated by the solenoid, K_{mag} is solenoid gain, I is the current input, and I_0 is a current offset.

The area of the opening is determined by the position of the spool x_s and the flow rate is changed. An important variable in spool dynamics is the position of the spool and its force balance equation is as follows.

$$m_s\ddot{x}_s + c_s\dot{x}_s + k_s(x_s + x_{s0}) = F_{mag} - P_p \cdot A_s, \tag{8}$$

where m_s, c_s, and k_s is the mass of the spool, damping friction coefficient, and the elastic modulus of the return spring, respectively. The term x_{s0} is the pre-compressed length of the spool return spring, P_p is the fluid pressure in the feedback chamber, and A_s is the feedback chamber pressure area.

The volume flowrate of a fluid Q is related to the open area and pressure difference, and is represented as follows:

$$Q = c_q A_q \operatorname{sgn}(P_s - P_p)\sqrt{\frac{2}{\rho}\left|P_s - P_p\right|}, \tag{9}$$

where c_q is a volume flowrate coefficient, A_q is the spool opening area, ρ is the fluid density and P_s is the supply pressure.

Assume that the spool notch angle is small. Then, the spool opening area is a function of the spool position as follows:

$$A_q = 2\sqrt{2r_n} \cdot x_s^{\frac{3}{2}},$$

where r_n is the notch radius. Then the flowrate equation (9) for the supply is

$$Q = 4c_q\sqrt{\frac{r_n}{\rho}}(P_s - P_p)^{\frac{1}{2}} x_s^{\frac{3}{2}} = C_Q(P_s - P_p)^{\frac{1}{2}} x_s^{\frac{3}{2}}, \tag{10}$$

where $C_Q = 4c_q\sqrt{\frac{r_n}{\rho}}$ is a flowrate coefficient for simplification.

4.1.2. Piston

When the spool valve is opened, the fluid flows into the piston chamber and moves the piston. The volume flow rate of the oil from the spool valve is also equal to the volume change of the piston chamber.

$$Q = A_p \dot{x}_p, \tag{11}$$

where A_p is the oil chamber area of the piston, and x_p is the piston position.

4.1.3. Wet clutch torque

The torque effective radius and mean slip speed of the clutch are given by:

$$R_{eff} = \frac{2}{3}\frac{R_o^3 - R_i^3}{R_o^2 - R_i^2},$$

$$v = R_{eff}\Delta\omega,$$

where R_{eff} is a torque effective radius of the clutch, R_o and R_i are the outer and inner radii of the clutch disk, and $\Delta\omega$ is the clutch rotational slip speed.

Neglecting the mass of the piston, the pressure generated by the hydraulic system P_p is in balance with the return spring force F_s, the contact force F_c, and the fluid squeeze pressure between the clutches F_{sq} as

$$A_p P_p = F_s + F_c + F_{sq}.$$

The wet-clutch torque τ, consisting of the asperity contact torque and viscous torque, is described as

$$\tau = R_{eff} N_c \big(\mu_f F_c + \mu_v A_f \frac{v}{\bar{\zeta}}\big), \tag{12}$$

where N_c is the number of clutches, and A_f is the nominal contact area of the lining material. The term μ_f and μ_v is the asperity friction coefficient and dynamic viscosity of the oil, respectively. The average fluid film thickness $\bar{\zeta}$ has the following relation [28] with the nominal separation of the clutches ζ:

$$\bar{\zeta} = \frac{\zeta}{2}\left[1 + \mathrm{erf}\left(\frac{\zeta}{\sqrt{2}\sigma}\right)\right] + \frac{\sigma}{\sqrt{2\pi}} e^{-\left(\frac{\zeta}{\sqrt{2}\sigma}\right)^2}. \tag{13}$$

According to [29, 30], the contact force for Gaussian distribution of asperity heights is modeled by a function of the nominal separation as

$$F_c = A_f p_c = \sqrt{\frac{\pi}{2}} E N \beta \sigma \cdot \left[e^{-\left(\frac{\zeta}{\sqrt{2}\sigma}\right)^2} - \frac{\zeta}{\sigma}\sqrt{\frac{\pi}{2}}\,\mathrm{erfc}\left(\frac{\zeta}{\sqrt{2}\sigma}\right)\right] \cdot A_f, \tag{14}$$

where E, N, β, σ, and erfc is, respectively, Young's modulus of the friction lining material, asperity density, asperity tip radius, RMS roughness, and complementary error function.

The nominal separation ζ and the piston position x_p have a static relationship as follows:

$$\zeta = \frac{x_{p,\zeta 0} - x_p}{N_c}, \tag{15}$$

where $x_{p,\zeta 0}$ is the position of the piston when the nominal separation is 0.

Finally, the wet-clutch torque is modeled as a nonlinear function of the piston position from equations (12), (13), (14), and (15) as

$$\tau = h(x_p). \tag{16}$$

Table 1
System parameters.

Solenoid valve gain, K_{mag}	1.745×10^{-2} N/mA	Asperity friction coefficient, μ_f	0.1192
Damping friction coefficient, c_s	27.77 N s/m	Dynamic viscosity, μ_v	30.8 mPa s
Spring constant, k_s	600.0 N/m	Number of clutches, N_c	8
Volume flowrate coefficient, c_q	0.7	Clutch effective radius, R_{eff}	0.07758 m
Notch radius, r_n	4.5×10^{-3} m	Clutch outer radius, R_o	0.08315 m
Fluid density, ρ	827 kg/m^3	Clutch inner radius, R_i	0.07174 m
Supply pressure, P_s	16.7×10^5 Pa	Young's modulus for friction lining, E	10×10^6 Pa
Piston pressure area, A_p	4981 mm^2	Asperity density, N	3×10^7 m^{-2}
Friction lining area, A_f	4053 mm^2	Asperity tip radius, β	5×10^{-4} m
Return spring force, F_s	593.3 N	RMS roughness, σ	6×10^{-6} m

4.1.4. *State-space form*

Set the current applied to the solenoid to the system input $u = I$, positions of the solenoid and piston to state variables $x_1 = x_p$ and $x_2 = x_s$, and clutch torque to the measurable system output $y = \tau$. Among the system states, x_1 is one of the control targets (i.e., $C = (1, 0)$). Then, from equations (8), (10), (11), and (16), a nonlinear affine system (1) is formulated:

$$\begin{aligned}
\dot{x}_1(t) &= f_{x1}(x_2(t)) \\
\dot{x}_2(t) &= f_{x2}(x_2(t)) + g_{x2}u(t) \\
y(t) &= h(x_1(t)) \\
z(t) &= Cx(t),
\end{aligned}$$

where $f_{x1}(x_2(t)) = \frac{C_Q}{A_p}(P_s - P_p)^{\frac{1}{2}} x_2(t)^{\frac{3}{2}}$, $f_{x2}(x_2(t)) = -\left(k_s(x_2(t) + x_{s0}) + P_p A_s + K_{mag} I_0\right)$, and $g_{x2} = \frac{K_{mag}}{c_s}$. This system has the same structure as the example in Section 3.1, and the learning gains are obtained by the proposed method.

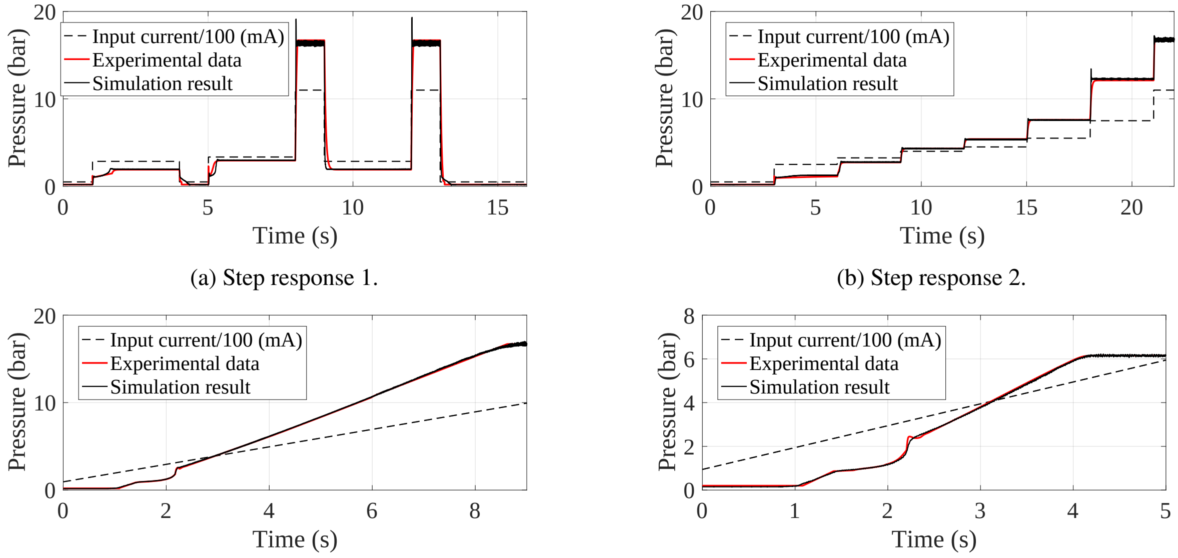
Before using the proposed method, assumptions should be checked. The functions used in the model are differentiable and continuous. The system is uniformly observable, and both the system order n and the relative order μ are 2. The system input is physically bound as a solenoid current. A constant gain observer (4) satisfying assumption A3.1-A3.3 in [24] gives a bound observation error. Similarly, since we can assume bounded initial observation errors in this system, we can assume that observation errors are also bounded. Finally, the control reference should be a function differentiable by the relative order of 2.

4.2. Simulation results

The simulation was developed using Matlab/Simulink. It included complex physical phenomena not represented in the control-oriented modeling in Chapter 4.1, such as the squeezing of the oil film between the clutches, the oil accumulator in the hydraulic circuit and the jet force on the spool. Furthermore, the simulation was verified by comparison with the actual plant. A 500 Hz zero-phase filter was applied to all of the measurements as a Q-filter.

4.2.1. *Simulation verification*

Real plants may have additional unmodeled complex physical dynamics that are not taken into account in the model. The simulation is not the same as the real plant, but important physical phenomena such as friction and damping of each part, oil squeezing, and contact deformation on the clutch are considered. The system output from experimental data and simulation results for the same system input were plotted and compared in Fig. 2. The oil pressures of real plants and simulations were compared for various types of system inputs. Step input was applied in Fig. 2a and Fig. 2b, and ramp input was applied in Fig. 2c. For comparison under different conditions, the system output for the ramp input at different supply pressures (6.2 bar) is also demonstrated and plotted in Fig. 2d. In most cases, the two outputs are exactly match, but inconsistencies in the transient state occur, such as 1.6 sec, 5 sec, and 9.1 sec in Fig. 2a and 2.2 sec in Fig. 2d. Fortunately, system inputs that cause such a sudden change and model error are generally discouraged and rarely used in practice. Except for these instantaneous situations, the simulation provides an extremely similar output overall for the same inputs.



(a) Step response 1.

(b) Step response 2.

(c) Ramp response.

(d) Ramp response with lower supply pressure.

Figure 2: Comparative verification of the hydraulic system simulation for experimental data.

In all cases in Fig. 2, when the oil pressure exceeds a certain value (about 1 bar), there is a lagging phenomenon (i.e., not increasing in proportion to the input). This is called the filling phase of the hydraulic system, and it is the period in which the clutch moves from the initial state to the operating state. During this phase, the pressure does not increase, so the feedback circuit in the VFS does not operate, and then the pressure increases sharply when the travel ends with contact of the clutch faces.

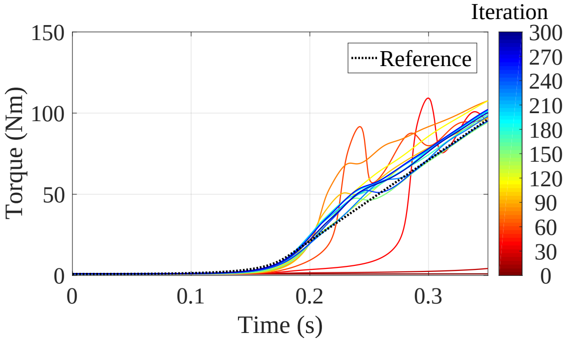
Control in this filling phase is the point at which the proposed method is specifically needed. It is a multi-reference problem: the position of the piston before the clutches contact, and the clutch torque after contact. The position of the piston is an unmeasurable variable in the complex structure of the clutch system. The systems and dynamics are complicated, but each operation performs the same action.

4.2.2. Control reference

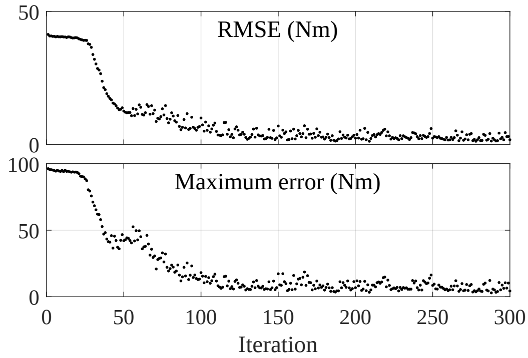
Considering the described operating process including the filling phase, it is reasonable to set the control target of the system to be the contact of the clutches first and then the ramp-up of the friction torque afterward. Consequently, the control problem is to follow the state reference and then the system output reference sequentially.

Control reference was divided into before and after clutch contact. Prior to clutch contact, the piston position reference is a third-order polynomial with respect to time, allowing for smooth contact such that the critical points are zero and maximum position. After contact, the clutch torque reference ramps up on a constant slope. In order to understand the shape of the clutch torque reference indicated by the dotted line in Fig. 3 and Fig. 4, it is necessary to explain the clutch-to-clutch shift of the automatic transmission. In order to shift gears, the clutch must be disconnected and the other clutch must be engaged at the same time, and this is called clutch-to-clutch shift. Since the driving torque must be continuously transmitted, the torque of the clutched clutch must be instantaneously transmitted to the next clutch. Since the torque of the on-coming clutch needs to be ramped up quickly but increased smoothly, consequently a linear ramp-up torque reference was used.

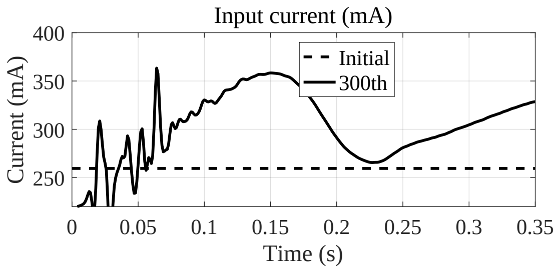
The clutch torque reference before contact and the piston position reference after contact use the model value using the other value, i.e., $y_d = h(x_d)$ before contact and $x_d = h^{-1}(y_d)$ after contact, and they are depicted in Fig. 3a and Fig. 4a. In the figures, the contact time of the reference is 0.185 seconds.



(a) Clutch torque control results through iteration.



(b) Root mean square error and maximum error versus the number of iterations.



(c) Initial input and system input after 300 iterations.

Figure 3: ILC results using conventional methods.

4.2.3. Conventional method

According to Ahn et al. [9], the following ILC makes the output error converge uniformly to zero for $t \in [0, T]$ as $i \to \infty$:

$$u_{i+1}(t) = u_i(t) + \sum_{r=0}^{\mu} s_{i,r}(t) \left(\overset{(r)}{y_d} - \overset{(r)}{y_i} \right),$$

and if the learning gain function $s_{i,r}(t)$ satisfies the relation,

$$\left\| 1 - s_{i,\mu}(t) L_g L_f^{\mu-1} h(x_i(t)) \right\| < 1, t \in [0, T].$$

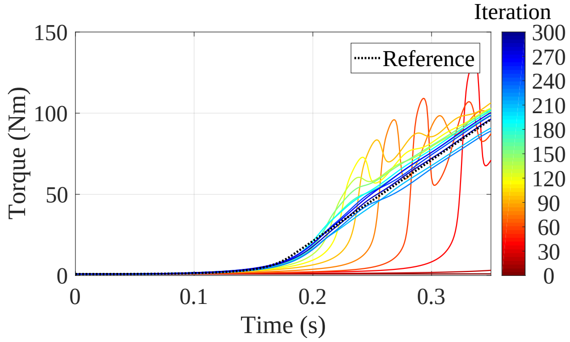
Then, using the observed states, choose the learning gain as,

$$s_{i,\mu}(t) = L_g L_f^{\mu-1} h(\hat{x}_i(t))^{-1},$$

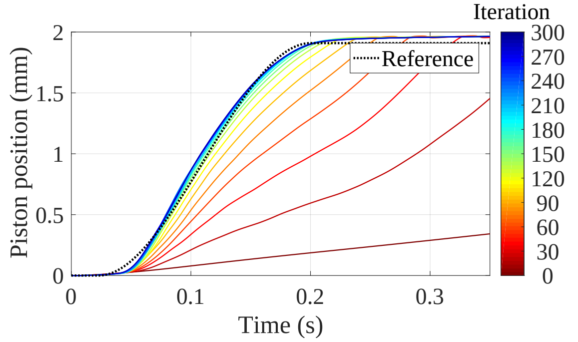
which is a model inverse-based time-varying gain.

4.2.4. Results and discussion

Using the conventional method introduced in Section 4.2.3, the variable gain is likely to be sensitive to noise. However, the proposed method reduces the influence of noise through the observer. In the converged control input of Fig. 3c, the unsettled input is shown before 0.1 sec due to noise. This is a result of high-gain to measurement noise. In the initial operation of the wet-clutch, the system output is mainly oil viscous torque, which depends on the position of the piston. Before contact, the viscous torque is small, and additional large contact torque is generated after contact. As a result, the singular value of the system changes significantly before and after contact. In such a system in which the singular value changes significantly, controlling the system output with a variable gain may cause a decrease in control performance due to the measurement noise.

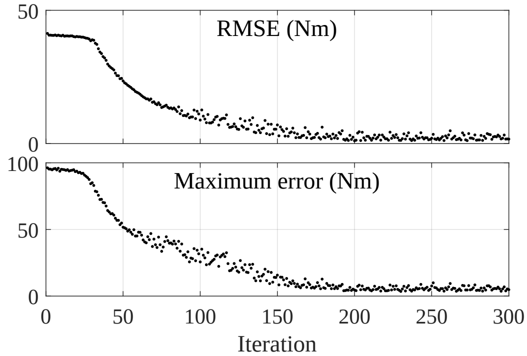


(a) Clutch torque control results through iteration.

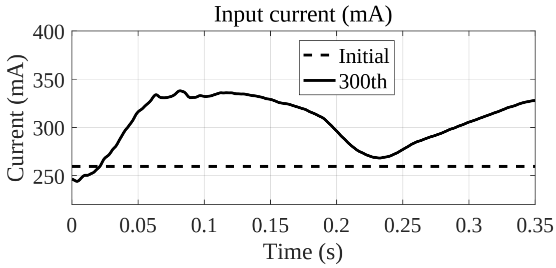


(b) Piston position control results through iteration.

Figure 4: ILC result through iteration using the proposed method.



(a) Root mean square error and maximum error versus number of iterations.



(b) Initial input and system input after 300 iterations.

Figure 5: ILC convergence through iteration using the proposed method.

In observer-based ILC using lower-order differentiation, the sensitivity to measurement noise in such a system can be minimized by using the estimated state error as a control error, together with the system output error. Comparing Fig. 3c and Fig. 5b, we can verify the effect of measurement noise for each method. In the variable gain method, fluctuations originating from noise exist in the initial period (filling-phase before 0.1 sec) and results in failure to converge to a unique u_d. However, in the proposed controller based on an the output feedback scheme, the converged input is stable even in the initial period.

This result can also be obtained from the convergence of the root mean square error (RMSE) and the maximum error through iteration. In Fig. (3b), the conventional method is still stable, but it is not monotonically convergent (MC). The error converges toward zero, and shows stability within a certain bound, but there is a temporary increase in the error at about 50 iterations. The proposed method is more robust to noise, resulting in MC. Fig. (5a) shows monotonically decreasing error, and eventual convergence within a certain bound. The averages of the maximum errors of the last 10 iterations of both methods are 9.42 Nm and 4.84 Nm, respectively. The difference shows how sensitive the converged error is to the measurement noise. The convergence speed in iteration axis is relatively slow because the proposed method has robustness characteristics. However, in mechanical systems, robustness against measurement noise may be treated as more important than convergence speed, especially when considering mechanical failures. Even in wet-clutch systems, sudden peaks in hydraulic pressure can damage a system.

5. Conclusion

In this paper, a novel ILC structure using estimated states and lower-order differentiation was proposed for a class of nonlinear systems with a high relative degree to enable robust control over measurement noise. The convergence result of the learning scheme was established, and a method for choosing the learning gain was derived. The latter includes not only highest-order differentiation but also lower-order differentiation. The convergence also includes an observer structure for systems with unmeasurable control states considering a high relative degree system. Under a sufficient condition, the robustness with respect to initial estimation errors and measurement noise is guaranteed. These original contributions were thoroughly verified through theoretical convergence analysis and simulation. The simulation results show the effectiveness of the proposed approach for the control of a wet-clutch system. With application of the proposed work to an actual wet-clutch system, a reduction of the tuning time can be anticipated.

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