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RESEARCH ARTICLE

Extended Disturbance Observer for Nonlinear System with Time Varying Disturbance Gain†

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Summary

In control engineering, the disturbance is a major problem that deteriorates system robustness. To address this problem, a disturbance observer (DOB) has been proposed and developed. In particular, a general method of estimating high-order disturbance, expressed as a time series expansion, was studied (H-DOB). However, systems to which H-DOB are applicable are limited to those in which disturbance gain is constant. This constraint degrades the generality of H-DOB because applicable systems are limited. Therefore, by expanding to a system with a time-varying disturbance gain, we propose a more general H-DOB. The DOB proposed in this paper redesigns the dynamics of the observer based on the structure of the H-DOB. The proposed DOB is applicable even if the system is highly nonlinear. This paper verifies the convergence of the proposed DOB through proof. Using this proof, the same characteristic error dynamics can be obtained, so the same design method can be applied. The simulation for the verification of the proposed method performs disturbance estimation of the extension system. This simulation proves that, compared to H-DOB, the proposed DOB has good disturbance estimation performance.

KEYWORDS:
Disturbance observer, Nonlinear system, Disturbance estimation, High order disturbance observer, DOB

1 | INTRODUCTION

In the early 1970s, control engineering researchers reported that one weakness of modern optimal control techniques was a lack of robustness[1,2,3,4]. These reports reached the consensus that a robust controller should be designed for disturbance[1]. As a result, the disturbance estimation problem became one of the most actively studied topics in control engineering[4]. Soon, an unknown input observer (UIO) based on a robust state observer was proposed to estimate external disturbances[5]. However, UIO does not include model-plant mismatches. To cope with this, research was conducted on observers of disturbances, including plant uncertainty and external disturbances. The first disturbance observer (DOB) proposed by Ohnishi consisted of an inversion of a linear system and a low pass filter[6]. DOB has been widely used to cancel external disturbances and plant mismatches, in combination with a linear technique such as linear quadratic control or robust control synthesis such as loop-shaping control techniques[7,8,9,10,11]. It was also used in an algorithm that compensates for disturbance while estimating the parametric change of

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the system as adaptation [12,13]. Recently, DOB-based control methods using model predictive control [14,15?] and $\mathcal{H}_\infty$ synthesis [16] have been proposed as advanced robust control methods.

In addition, DOB was applied to various applications and used for disturbance estimation and rejection in such areas as motion control of servo systems [17], power electronics [18], fuel cell systems [19], unmanned aerial vehicles [20,21], automobiles [22], electric commuter trains [23], networks [24], missile seekers [25], aircraft [26], system identification [27] and fault diagnosis [28].

Recent research on DOB has been subdivided according to structures of disturbances, so it is no longer limited to unknown structures of disturbances. In addition, through several studies, DOB has been developed to apply to nonlinear systems via extension from linear systems. First, a nonlinear constant DOB was proposed [29]. However, if a constant DOB is used to estimate a fast-varying disturbance, the convergence rate of the estimation decreases. This algorithm has been used to estimate disturbances in nonlinear systems such as surgical device [12]. To address this, harmonic nonlinear DOB was proposed [30]. In some systems, the disturbance is referred to as a time series expansion. A high-order DOB (H-DOB) was proposed to estimate disturbance with a time series expansion [31]. Since H-DOB is expressed in generalized terms, it can be applied to various applications with nonlinear systems. In addition, a characteristic polynomial can be used to adjust the convergence rate of the disturbance estimation. The time-domain analysis is simplified by using the observer state, which is independent of the system state. However, since the gain of a disturbance is assumed to be a constant in the system structure, the generality of the target system can deteriorate.

Therefore, the contribution of this paper is to propose a method for extending the constraint of the system to which H-DOB can be applied. In addition, we propose a method for selecting the expected order by examining the relationship between the expected order of disturbance and the estimation error, which has not been addressed in the H-DOB study. Since the proposed method is inspired by H-DOB, it has almost the same structure. Specifically, in order not to lose the generality of the disturbance gain, the proposed method defines the disturbance gain as a time-varying matrix rather than as a constant. When H-DOB is applied to this extended system, the estimation error becomes unstable. To cope with this, we propose H-DOB modified observer state dynamics. Also, the relationship between the expected order of disturbance and the fluctuation of the estimation error is investigated. The dominant frequency of the estimation error varies according to the relationship between the actual disturbance order and the expected order. Considering these points, this paper proposes the necessary conditions when designing the proposed DOB.

This paper is organized as follows. Section 2 revisits the H-DOB and demonstrates stability when applied to an extended system. In Section 3, a disturbance observation algorithm that can be applied to the extended system is proposed. In Section 4, verification of the proposed algorithm through simulation is performed. Finally, this work is concluded in Section 5.

2 | MOTIVATION

For an understanding of this manuscript, the H-DOB in [31], designed to estimate disturbance in time series expansions, was revisited. H-DOB targets the following systems.

$$\dot{x} = f(x, u; t) + Fd(t) \tag{1}$$

$$F^{+}\dot{x} = F^{+}f(x, u; t) + d(t) \tag{2}$$

where $x \in \mathbb{R}^n, u \in \mathbb{R}^m, d \in \mathbb{R}^r$, and the nonlinear term of systems $f(\cdot)$, and the constant matrix F with $rank(F) = r$ are known. Eq. (2) is a reduced-order system of Eq. (1). Also, $(\cdot)^{+}$ denotes the Moore-Penrose pseudo-inverse of the argument matrix. The high-order disturbance information given here is as follows [31].

$$d(t) = d_0 + d_1 t + \cdots + d_q t^q = \sum_{k=0}^{q} d_i t^k \tag{3}$$

where d_i's ($i \in [0, q]$) are constant but unknown.

Theorem 1 (H-DOB [31]). Given matrices $\Gamma_k = diag\{\gamma_{k1}, \cdots, \gamma_{kr}\}, (k \in [0, q])$, suppose that γ_{ij}'s are chosen such that the polynomials $p_j(s)$ are Hurwitz stable, where

$$p_j(s) = s^{q+1} + \gamma_{0j}s^q + \gamma_{1j}s^{q-1} + \cdots + \gamma_{(q-1)j}s + \gamma_{qj} \tag{4}$$

for $j = 1, \cdots, r$. Then, a disturbance estimate given by

$$\hat{d}(t) = \sum_{k=0}^{q} \Gamma_k g_k(t) \tag{5}$$

is asymptotically convergent to the high-order disturbance d, where $g_k(t)$'s are defined by

$$g_k(t) = \begin{cases} F^+x - z, & (k = 0) \\ \int_0^t g_{k-1} d\tau, & (k \geq 1) \end{cases} \tag{6}$$

where the state variable z is defined by

$$\dot{z} = F^+ f(x, u; t) + \hat{d}. \tag{7}$$

Proof. To investigate the error dynamics of the disturbance observer, let us introduce a variable such that $e_d = \hat{d} - d$. Then, its time derivative is:

$$\dot{e}_d = \dot{d}(t) - \dot{\hat{d}} = \dot{d}(t) - \Gamma_0 \dot{g}_0(t) - \cdots - \Gamma_q \dot{g}_q(t) \tag{8}$$

Using the fact that $\dot{g}_k(t) = \begin{cases} e_d, & (k = 0) \\ g_{k-1}(t), & (k \geq 0) \end{cases}$, this can be rewritten as

$$\dot{e}_d = \dot{d}(t) - \Gamma_0 e_d - \Gamma_1 g_0(t) - \cdots - \Gamma_q g_{q-1}(t) \tag{9}$$

The $(q + 1)$-th time derivative of e_d is:

$$e_d^{(q+1)} + \Gamma_0 e_d^{(q)} + \cdots + \Gamma_q e_d = d^{(q+1)}(t) \tag{10}$$

where $(\cdot)^{(k)}$ denotes the k-th time derivative of the argument function. By the definition of disturbance, it is noted that $d^{(q+1)} = 0$. Then, the Laplace transform of j-th raw of Eq. (10) is:

$$E_{dj}(s)(s^{q+1} + \gamma_{0j} s^q + \cdots + \gamma_{(q-1)j} s + \gamma_{qj}) = 0 \tag{11}$$

If the poles in Eq. (11) are all in the left half plane, the characteristics of the error dynamics of the disturbance observer are asymptotically and exponentially stable for the initial error. That is, the H-DOB exactly estimates the high-order disturbance in the steady state. □

We extend the H-DOB to estimate the following system.

$$\dot{x} = f(x, u; t) + F(x, u; t)d(t) \tag{12}$$

$$F^+(x, u; t)\dot{x} = F^+(x, u; t)f(x, u; t) + d(t) \tag{13}$$

where the time varying matrix $F(x, u; t)$ with $rank\{F(x, u; t)\} = r$ for all x and $t > 0$ is known.

To investigate the error dynamics, let us rewrite Eq. (9) using the definition of $g_k(t)$ by Eq. (6), as

$$\dot{e}_d = \dot{d}(t) - \Gamma_0 \dot{F}^+(x, u; t)x - \Gamma_0 e_d - \Gamma_1 g_0(t) - \cdots - \Gamma_q g_{q-1}(t) \tag{14}$$

The $(q + 1)$-th time derivative of e_d is:

$$e_d^{(q+1)} + \Gamma_0 e_d^{(q)} + \cdots + \Gamma_q e_d = d^{(q+1)}(t) - \Gamma_0 \{F^+(x, u; t)x\}^{(q)} - \Gamma_1 \{F^+(x, u; t)x\}^{(q-1)} - \cdots - \Gamma_q F^+(x, u; t)x \tag{15}$$

where $\dot{F}^+(x, u; t)$ is defined as follows.

$$\dot{F}^+(x, u; t) = \frac{dF^+}{dt} = \frac{\partial F^+}{\partial x}\dot{x} + \frac{\partial F^+}{\partial u}\dot{u} \tag{16}$$

As shown in the $(q+1)$th time derivative of e_d in Eq. (15), the time derivative term of $F^+(x, u; t)$ remains, so the characteristics of the error dynamics from Eq. (11) are not derived. Therefore, the stability of the error dynamics cannot be guaranteed.

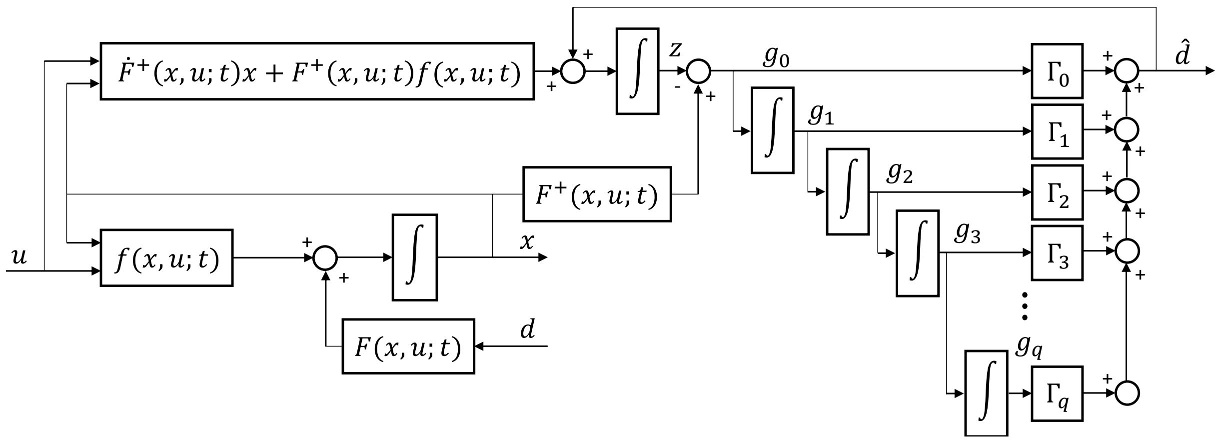

Figure 1 Generalized structure diagram of proposed EH-DOB for estimation of disturbance with q-th time series expansion.

Remark 1. If $F(x,u;t)$ and d are regarded as a time-varying disturbance and expressed as $D = F(x,u;t)d$, an extended system is constructed with $F'(x,u;t) = 1$ and time-varying disturbance D. Therefore, this method has the same effect as increasing the order of disturbance. However, since the q corresponding to the order of the disturbance is adjusted to d, the estimation accuracy is reduced. As a result, since the system information $F(x,u;t)$ is not used, estimation accuracy decreases. Additionally, as mentioned in[31], since the order of D is larger than that of d, the estimation performance may be reduced when D is not sufficiently slow with respect to the bandwidth of the H-DOB. In addition, when Eq. (15) is rewritten using $D = F(x,u;t)d$, the time derivative term of $\dot{F}^{+}(x,u;t)x$ remains. Therefore, it is not appropriate to apply H-DOB to estimate the integrated disturbance D in a system where $F(x,u;t)$ is a time-varying matrix.

3 | EXTENDED DISTURBANCE OBSERVER ALGORITHM

Based on inspiration from the H-DOB, the extended high order disturbance observer (EH-DOB) proposed for disturbance estimation in an extended system in Eqs. (12) and (13) is as follows.

Theorem 2 (EH-DOB)**.** Given matrices $\Gamma_k = diag\{\gamma_{k1}, \cdots, \gamma_{kr}\}, (k \in [0,q])$, suppose that γ_{ij}'s are chosen such that the polynomials $p_j(s)$ are Hurwitz stable for $j = 1, \cdots, r$. Then, the disturbance estimate given by

$$\begin{cases} \dot{z} = \dot{F}^{+}(x,u;t)x + F^{+}f(x,u;t) + \hat{d} \\ \hat{d}(t) = \sum_{k=0}^{q} \Gamma_k g_k(t) \end{cases} \tag{17}$$

is asymptotically convergent to the high-order disturbance d, where $g_k(t)$'s are defined by

$$g_k(t) = \begin{cases} F^{+}(x,u;t)x - z, & (k = 0) \\ \int_0^t g_{k-1} d\tau, & (k \geq 1) \end{cases} \tag{18}$$

Proof. To investigate the error dynamics of the disturbance observer, the error dynamics in Eq. (15) is rewritten as Eq. (10) using the fact that

$$e_d = \dot{F}^{+}(x,u,t)x + F^{+}(x,u;t)\dot{x} - \dot{z} \tag{19}$$

Then, its time derivative is:

$$\dot{e}_d = \dot{d} - \dot{\hat{d}} = \dot{d}(t) - \Gamma_0\left(\dot{F}^{+}(x,u,t)x + F^{+}(x,u;t)\dot{x} - \dot{z}\right) - \Gamma_1 g_0(t) - \cdots - \Gamma_q g_{q-1}(t) \tag{20}$$

$$\dot{e}_d = \dot{d}(t) - \Gamma_0 e_d - \Gamma_1 g_0(t) - \cdots - \Gamma_q g_{q-1}(t) \tag{21}$$

The $(q+1)$-th time derivative of e_d is:

$$e_d^{(q+1)} + \Gamma_0 e_d^{(q)} + \cdots + \Gamma_q e_d = d^{(q+1)}(t) \tag{22}$$

Table 1 $e_d(s)$ according to the p

Case	$e_d(s)$
$p < q$	$\dfrac{d_0 s^q + d_1 s^{q-1} + \cdots + q!d_q}{s^{q-p}(s^{p+1} + \Gamma_0 s^p + \cdots + \Gamma_p)}$
$p = q$	$\dfrac{d_0 s^q + d_1 s^{q-1} + \cdots + q!d_q}{s^{p+1} + \Gamma_0 s^p + \cdots + \Gamma_p}$
$p > q$	$\dfrac{(d_0 s^q + d_1 s^{q-1} + \cdots + q!d_q)s^{p-q}}{s^{p+1} + \Gamma_0 s^p + \cdots + \Gamma_p}$

That is, the time derivative term of $F^+(x, u; t)x$ of the error dynamics in Eq. (15) disappears. Therefore, the characteristics of the error dynamics in Eq. (11) are derived equally, and the EH-DOB is asymptotically and exponentially stable for the initial error and will exactly estimate the disturbance in the steady state. □

Fig. 1 shows the architecture of the proposed disturbance observer. q can be determined according to the given disturbance information; the number of zero Γ_k is determined according to q. Also, the figure shows how the extended system incorporates EH-DOB. Here, when the given disturbance information and the order of the EH-DOB are different, a steady-state error or vibration occurs in the convergence of the error. This phenomenon can also be expressed as a formula. First, if Eq. (17) is summarized for $\dot{\hat{d}}$ as follows.

$$\begin{aligned}\dot{\hat{d}} &= \Gamma_0 \dot{g}_0(t) + \Gamma_1 g_0(t) + \cdots + \Gamma_p g_{p-1}(t) \\ &= \Gamma_0(\dot{F}^+(t)x + F^+(t)x - \dot{z}) + \Gamma_1 g_0(t) + \cdots + \Gamma_p g_{p-1}(t) \\ &= \Gamma_0 e_d + \Gamma_1 \int e_d + \Gamma_2 \iint e_d + \cdots + \Gamma_p \underbrace{\int \cdots \int}_{p} e_d\end{aligned} \tag{23}$$

Here, p is the expected order of disturbance used in the design of the EH-DOB, and $\int \cdots \int_p$ represents the multiple integrations of p times. The Laplace transform of Eq. (23) is as follows.

$$s^{p+1}\hat{d}(s) = (\Gamma_p + \Gamma_{p-1}s + \cdots + \Gamma_0 s^p)e_d(s) \tag{24}$$

Summarizing the above expression for $e_d(s) = \hat{d}(s) - d(s)$ as as follows.

$$e_d(s) = \frac{s^{p+1}}{s^{p+1} + \Gamma_0 s^p + \cdots + \Gamma_p} d(s) \tag{25}$$

The disturbance information of Eq. (3) is expressed as Laplace transform as follows.

$$d(s) = \frac{d_0 s^q + d_1 s^{q-1} + \cdots + q!d_q}{s^{q+1}} \tag{26}$$

By combining Eqs. (26) and (25), the change in $e_d(s)$ according to the value of p can be obtained. Table 1 shows cases where p is less than, equal to, and greater than q. Referring to this table, when p is less than q, a pole with a value of 0 is generated, and when p is greater than q, a pole with a value of 0 is generated. That is, if the expected order (p) in EH-DOB is lower than the actual (q), convergence may decrease, and if it is high, vibration may increase. This phenomenon can be seen in Fig. 2.

In (a) of the Fig. 2, e_d that changes according to the value of p is shown. When p is greater than q, the frequency of e_d increases. This can be seen through the frequency composition analysis of e_d shown in (b) of Fig. 2. Referring to the frequency composition analysis of e_d, it can be seen that as p increases, the high-frequency composition relatively increases. These oscillations can cause divergence if they match the resonant frequency of the system. Therefore, when selecting the order of EH-DOB, p should be selected so that the dominant frequency of e_d avoids the resonance frequency of the system.

Remark 2. The EH-DOB presented in *Theorem* 2 has clear advantages compared to the previous study H-DOB[31]. Unlike H-DOB, EH-DOB adds a derivative term of the disturbance coefficient term ($\dot{F}$) to the disturbance observer's internal dynamics (Eq. 18). Through this, it was possible to guarantee the convergence of the estimation error of the disturbance even in a system

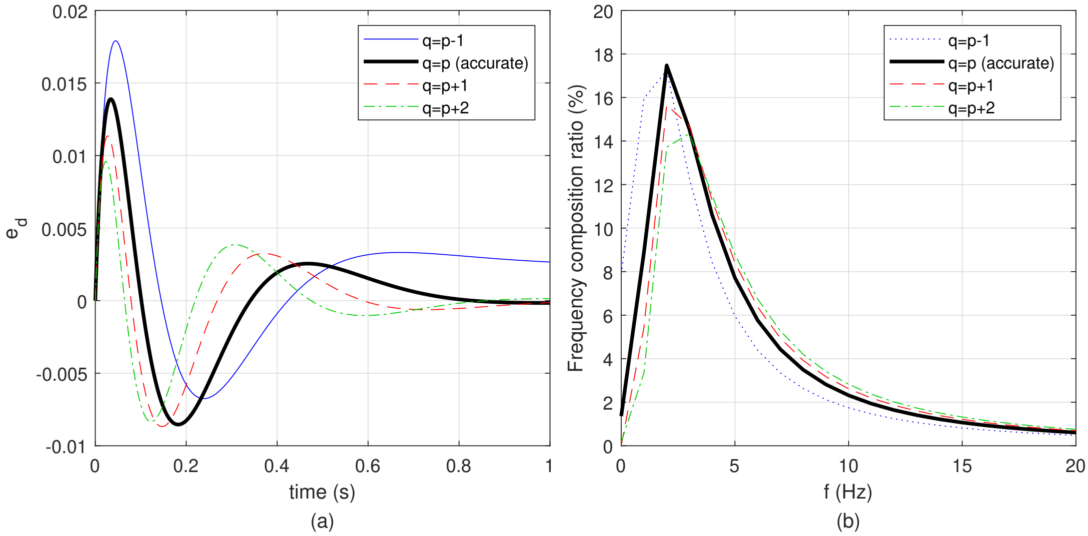

Figure 2 Error fluctuations are caused by the difference between the order of EH-DOB (p) and the order of disturbance (q).

in which the disturbance coefficient term is time-varying. That is, in H-DOB, disturbance estimation is possible only in a system in which the coefficient term of disturbance is constant, but in EH-DOB, disturbance estimation is possible in a system in which the coefficient term of disturbance is time-varying. Therefore, EH-DOB can be used for estimating disturbance in a more general system than H-DOB.

4 | SIMULATION

In this section, simulations were performed to verify the proposed EH-DOB. The nonlinear system with time varying disturbance gain for simulation is given by

$$\dot{x} = -2x^2 + (x-1)d(t) \tag{27}$$

To design the proposed EH-DOB, the system given in Eq. (27) is represented by $f(x,u;t) = -2x^2$ and $F(x,u;t) = x - 1$. In addition, $F^+(x,u;t)$ and its time derivatives, used in the design of DOB, are given as follows.

$$F^+(x,u;t) = \frac{1}{x-1} \tag{28}$$

$$\dot{F}^+(x,u;t) = -\frac{\dot{x}}{(x-1)^2} \tag{29}$$

The disturbance information is as follows.

$$d(t) = t - t^2 \tag{30}$$

Time series order of disturbance is defined as $q = 2$ according to Eq. (30).

Therefore, the proposed EH-DOB algorithm in Eq. (17) can be designed using the system information in Eq. (27), $F^+(x,u;t)$ and $\dot{F}^+(x,u;t)$ to estimate the second order disturbance.

$$\hat{d} = \gamma_0\left(\frac{x}{x-1} - z\right) + \gamma_1\int_0^t \left(\frac{x}{x-1} - z\right)d\tau + \gamma_2\int_0^t\int_0^\tau \left(\frac{x}{x-1} - z\right)d\tau dt \tag{31}$$

where the state variable z is defined by

$$\dot{z} = -\frac{\dot{x}x}{(x-1)^2} - \frac{2x^2}{x-1} + \hat{d} \tag{32}$$

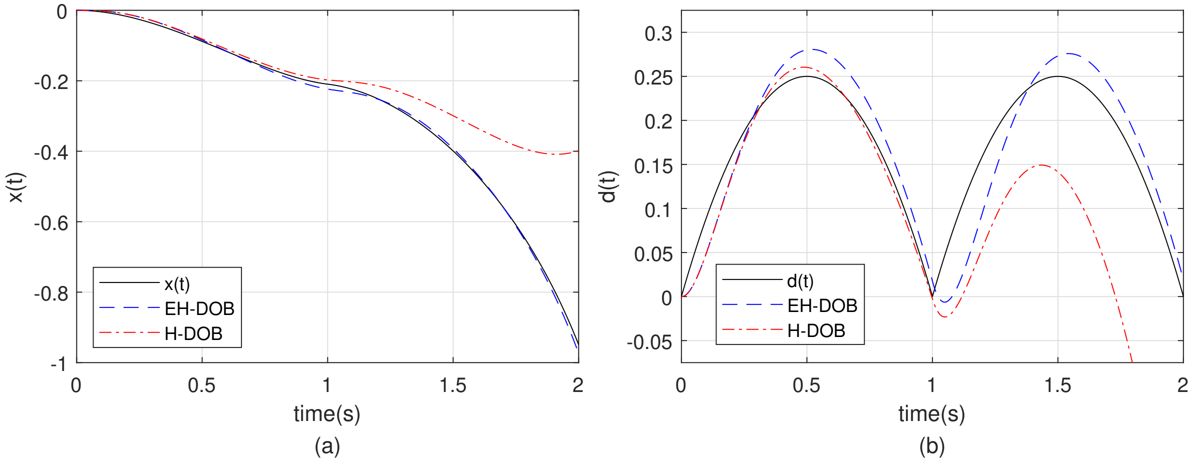

Figure 3 Simulation results for verification example. (a) state related (b) disturbance related.

For comparison, H-DOB was designed to estimate the disturbance of the target system, in which H-DOB was designed by applying a method using the basic formulation in Eqs. (5) and (7). $\hat{d}$ of the method using H-DOB is expressed in the same way as in the proposed method. However, the state variable z is expressed as follows.

$$\dot{z} = -\frac{2x^2}{x-1} + \hat{d} \tag{33}$$

Note that the difference from EH-DOB in Eq. (32) is that there is no $\dot{F}^{+}(x, u; t)x$ term.

For comparison, we selected the same two sets of parameters $(\gamma_0, \gamma_1, \gamma_2)$. That is, poles representing the same convergence rate were selected, in which the selected parameters are 15, 75, and 125. These parameters mean that all poles of the characteristic equation, Eq. (11), are assigned to $s = -5$. The simulation results are shown in Fig. 3.

When analyzing the simulation results, it can be seen that the proposed EH-DOB has better estimation performance. Specifically, EH-DOB is asymptotic and exponentially stable, but H-DOB is seen as unstable. In addition, it can be seen that H-DOB has a steady state error even when it is stable.

5 | CONCLUSION AND FUTURE WORK

In this paper, an extended high-order disturbance observer based on the high-order disturbance observer in[31] was proposed to estimate disturbances in the time series expansion of a system composed of time-varying disturbance gain. Through the proof, the proposed algorithm was confirmed to have the same characteristics of error dynamics as H-DOB. That is, the convergence rate can be adjusted using the poles of the characteristic equation, and the same method of selecting parameters to design the desired poles can be applied to H-DOB. Additionally, the estimation error caused by the difference between the expected disturbance order and the actual disturbance order was analyzed. As a result, the fluctuation of the error increased as the expected order was higher than the actual order. Therefore, the predicted order should be selected so that the dominant frequency of the estimation error avoids the resonance frequency of the system. The simulation was performed to verify the proposed algorithm, with results showing that the proposed algorithm has excellent estimation performance of disturbance of a system having a time-varying disturbance gain.

The simulation was used to validate the EH-DOB proposed in this paper. However, it is necessary to investigate whether there are practical problems when applied to an actual system or whether it is affected by system limitations such as sampling time. Therefore, as a follow-up study, we plan to apply EH-DOB to a system with time-varying disturbance gain.

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