Integrated vehicle mass estimation for vehicle safety control using the recursive least-squares method and adaptation laws

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What is This?
Integrated vehicle mass estimation for vehicle safety control using the recursive least-squares method and adaptation laws

Daeil Kim, Seibum B. Choi and Mooryong Choi

Abstract
The purpose of this study is to estimate the vehicle mass for vehicle safety control. The vehicle mass is considered to be a constant parameter for some vehicle safety control systems, but it changes according to the number of the passengers or the load weight that the vehicle carries. This paper suggests an integrated vehicle mass estimation algorithm using the recursive least-squares method and adaptation laws. First, the vehicle mass is estimated from the longitudinal dynamics using the recursive least-squares method. Second, three kinds of estimation algorithm are suggested from the roll dynamics. Two of the algorithms are designed using the adaptation law from a Lyapunov stability analysis and the roll angle observer, and the last algorithm is designed using the recursive least-squares method. Finally, the multiple-observer synthesis integrates the estimated mass values calculated using the longitudinal dynamics and the roll dynamics. The proposed vehicle mass estimation algorithm is evaluated via simulation using CarSim and via experimentation using a test vehicle.

Keywords
Mass estimation, recursive least squares, adaptation law, longitudinal dynamics, roll dynamics

Introduction
Many people are killed by ground vehicle accidents every year. For this reason, drivers and passengers expect new safety enhancement systems such as electronic stability control, rollover stability control, active front steering, continuous damping control and hill hold control. Such systems have contributed significantly to a reduction in the number of fatal accidents.

According to an analysis by the National Highway Traffic Safety Administration, electronic stability control has helped to decrease the total number of accidents, but it has not affected the number of rollover accidents. Therefore, more research and development on rollover prevention control are required and are currently under way.6,2,3

One significant problem of vehicle dynamics controllers is that they are usually model based and, therefore, susceptible to vehicle parameters such as the mass of the vehicle and the height of the centre of gravity (CG) of the vehicle, but these critical values are often handled as constants for vehicle safety control. However, these parameters are variable factors which are affected by the number of passengers or the weight of the load that the vehicle carries, and the difference between the parameter values and the expected values causes the performance of vehicle safety controllers to deteriorate accordingly. Furthermore, the accuracy of the value of the vehicle mass is crucial for estimation of the value of the peak friction of a road surface. The peak friction information plays a crucial role in vehicle safety controllers.

For these reasons, numerous studies have been conducted to estimate the CG height of the vehicle4,5 and the vehicle mass precisely.6–11 For instance, Vahidi et al.6 used the recursive least-squares method with multiple forgetting factors in order to estimate the vehicle mass and the time-varying road grade.
simultaneously. Huh et al.\textsuperscript{7} suggested an integrated mass estimation algorithm based on the longitudinal dynamics and the lateral dynamics of the vehicle and the vertical dynamics of the suspension. However, those mass estimation methods have some limitations. For instance, the algorithm\textsuperscript{6} considers only the longitudinal dynamics, and so the estimated value of the vehicle mass is updated only when no steering condition is satisfied. Moreover, the longitudinal acceleration sensor usually measures the gravity component value; therefore, it is impossible to classify the road grade from the pure longitudinal acceleration. For the algorithm,\textsuperscript{7} there are some difficulties in estimating the vehicle mass because of unknown parameters such as the cornering stiffness, which varies significantly when the vehicle dynamics are in a non-linear region.

In order to overcome those limitations, this paper suggests an integrated vehicle mass estimation algorithm that is robust for diverse driving situations. The integrated mass estimation algorithm includes two sub-estimation algorithms based on the longitudinal dynamics and the roll dynamics.\textsuperscript{12} First, in the case of the pure longitudinal driving situation with no steering, the longitudinal mass estimation algorithm is applied using the recursive least-squares method with a single forgetting factor. However, when the vehicle turns, the roll dynamics of the vehicle are exploited to estimate the vehicle mass instead of using the lateral dynamics of the vehicle including unknown parameters, which are difficult to estimate accurately.

The adaptation law and the recursive least-squares method\textsuperscript{13} are applied to estimate the vehicle mass when the roll dynamics are used. Finally, two algorithms are integrated to estimate the vehicle mass quickly for all sorts of driving situations through multiple-observer synthesis\textsuperscript{14,15} with a dynamic weighting strategy.

The organization of this paper is as follows. In the second section, the vehicle mass estimation method from the longitudinal dynamics using the recursive least-squares method with a single forgetting factor is derived. The third section develops three different vehicle mass estimation methods using the roll dynamics: two adaptation algorithms and one recursive least-squares method. Finally, in the fourth section, an integrated vehicle mass estimation algorithm is introduced that uses multiple-observer synthesis to integrate two mass estimation algorithms for quick estimation of the vehicle parameters for all sorts of real driving situation.

**Mass estimation from the longitudinal dynamics**

**Longitudinal dynamics model**

This section presents a model-based method to estimate the vehicle mass using the longitudinal dynamics. This is valid only when a vehicle accelerates or decelerates with no steering manoeuvres. The longitudinal dynamics model is as shown in Figure 1.

From the longitudinal force balance

\[
 m\ddot{v} = F_x - F_b - F_{aero} - F_{grade}
\]

where \( m \) is the total mass of the vehicle, \( v \) is the longitudinal velocity of the vehicle and \( F_x \) is the total longitudinal tyre force transmitted from the engine torque at the flywheel to the tyre and is given by

\[
 F_x = \frac{T_e - J_e \omega}{r_g}
\]

\( T_e \) is the engine torque and it must be scaled down considering all the torque losses as in the torque converter. \( J_e \) is the powertrain inertia and \( r_g \) is the effective wheel radius divided by the gear ratio and the final drive ratio, as given by the equation

\[
 r_g = \frac{r_w}{g_d g_f}
\]

where \( r_w \) is the effective wheel radius, \( g_d \) is the gear ratio and \( g_f \) is the final drive ratio. \( F_b \) is the brake force generated by brake friction at the wheels. The aerodynamic drag force is derived as

\[
 F_{aero} = \frac{1}{2} \rho C_d A_F v_x^2
\]

where \( \rho \) is the air density, \( C_d \) is the aerodynamic drag coefficient and \( A_F \) is the vehicle’s frontal area. \( F_{grade} \) indicates the integrated force due to the rolling resistance of the road and the road grade and is defined as

\[
 F_{grade} = mg(\mu \cos \theta + \sin \theta)
\]

where \( g \) is the gravity constant and \( \mu \) is the rolling resistance coefficient.

![Figure 1. Longitudinal dynamics.](image-url)
Recursive least-squares method with a single forgetting factor

The unknown parameters of a mathematical model must be selected in such a way that the sum of the squares of the differences between the actually observed values and the computed values is minimal in the least-squares problem. The unknown parameter must be selected such that the least-squares loss function $V(\xi, t)$ is minimized according to

$$V(\xi, t) = \frac{1}{2} \sum_{i=1}^{t} [y(i) - \psi^T(i)\xi]^2$$

(6)

The parameter that minimizes the above loss function is defined as

$$\hat{\xi} = \left[ \sum_{i=1}^{t} \psi(i)\psi^T(i) \right]^{-1} \left[ \sum_{i=1}^{t} \psi(i)y(i) \right]$$

(7)

However, it is more appropriate to use a forgetting factor $\lambda$ for the loss function to give more weight to the latest values. Using the forgetting factor $\lambda$, the loss function is redefined as

$$V(\xi, t) = \frac{1}{2} \sum_{i=1}^{t} \lambda^{t-i} [y(i) - \psi^T(i)\xi(i)]^2$$

(8)

This concept is based on the fact that the old data need to be discarded gradually.

It is proper to make the computations recursive to alleviate the computational burden. Therefore, a recursive least-squares algorithm will be used in this paper to update the estimation of the unknown parameter $\xi(t)$ at the time $t$ using the results obtained at the time $t-1$ and the regression vector $\psi(t)$. The procedure of the recursive least-squares algorithm at each step $t$ is as follows.

**Step 1.** Measure the output $y(t)$ and calculate the regression vector $\psi(t)$.

**Step 2.** Calculate the update gain $K(t)$, which is called a weighting factor that shows how the correction and previous estimation should be united, according to

$$K(t) = P(t)\psi(t)$$

$$= P(t-1)\psi(t)[1 + \psi^T(t)P(t-1)\psi(t)]^{-1}$$

(9)

Then, calculate the covariance matrix derived as

$$P(t) = [I - K(t)\psi^T(t)]P(t-1)\frac{1}{\lambda}$$

(10)

**Step 3.** Update the unknown parameter vector $\hat{\theta}(t)$ as

$$\hat{\theta}(t) = \hat{\theta}(t-1) + K(t)[y(t) - \psi^T(t)\hat{\theta}(t-1)]$$

(11)

The correction term $[y(t) - \psi^T(t)\hat{\theta}(t-1)]$ is proportional to the difference between the measurement and the prediction of the former estimation. Now, the previous longitudinal dynamics equation (1) can be rearranged in the regression form $y(t) = \psi^T(t)\xi(t)$

(12)

where

$$y(t) = a_c + g\mu \cos \theta$$

$$\psi^T(t) = \frac{T_c - J_o \omega}{rg} - F_b - F_{aero}$$

$$\xi(t) = \frac{1}{m}$$

(13)

where $y(t)$ is the measured output, $\psi(t)$ is the known variable and $\xi(t)$ is the unknown parameter that must be estimated. Here, the $g\mu \cos \theta$ term is assumed to be a small constant compared with other terms, the sensor measurement $a_c$ includes the $g \sin \theta$ term, and a forgetting factor is used since the vehicle mass can vary.

Variable-forgetting-factor design

The forgetting factor plays a decisive role in the convergence of the parameters. For faster convergence, a smaller forgetting factor is needed, which gives more weight to recently measured values. However, one disadvantage of using a small forgetting factor is that the results are prone to be sensitive to the measurement noise.

To achieve fast estimation and robustness to the measurement noise at the same time, a variable forgetting factor is defined such that the magnitude of the factor is inversely proportional to the absolute value of the correction term according to

$$\lambda = \lambda_2 \lambda_3 [||y(t) - \psi^T(t)\hat{\theta}(t-1)||]$$

(14)

where $\lambda_2$ is a constant set in the interval $[0.9, 1]$, $\lambda_3$ is the variable factor limited in the interval $[0.8, 1]$ and $\lambda$ is a tunable constant parameter.

Finally, the forgetting factor $\lambda$ should be bounded so as not to make the covariance matrix $P$ too large, since too large a covariance matrix might make the gain matrix very large, rendering the system unstable.

Mass estimation by the roll dynamics

This section proposes three different mass estimation schemes from the roll dynamics.

Mass estimation using the adaptation law from the roll dynamics

This section deals with the mass estimation method from the roll dynamics using the measured lateral acceleration, the roll rate and the vehicle roll dynamics model. Figure 2 represents the vehicle roll dynamics model.\(^{4,12}\)

Considering the moment balance about the roll centre, the roll dynamics equation is written as

$$(I_{xx} + m_i h^2)\ddot{\phi} = m_i a_i \cos \phi + m_i g h \sin \phi - \frac{f_y}{2} (F_{z1} - F_{z2})$$

(15)

where $I_{xx}$ is the roll moment of inertia about the CG, $m_i$ is the sprung mass of the vehicle, $m_i a_i$ is the lateral
force on the CG of the vehicle, and \( F_{zl} \) and \( F_{zr} \) are the left and right suspension forces respectively.

Now, consider the suspension deflection of both sides due to vehicle roll which is given by

\[
x_l = -\frac{t_w}{2} \sin \phi \tag{16}
\]

\[
x_r = \frac{t_w}{2} \sin \phi \tag{17}
\]

Then, the suspension forces are derived as

\[
F_{zl} = \frac{m_s g}{2} + \frac{t_w}{2} k \sin \phi + \frac{t_w}{2} \dot{\phi} \cos \phi \tag{18}
\]

\[
F_{zr} = \frac{m_s g}{2} - \frac{t_w}{2} k \sin \phi - \frac{t_w}{2} \dot{\phi} \cos \phi \tag{19}
\]

\[
F_{zl} - F_{zr} = t_a k \sin \phi + t_w c \dot{\phi} \cos \phi \tag{20}
\]

Combining equations (20) and (15), we obtain

\[
\left( I_{xx} + m_s h^2 \right) \ddot{\phi} = m_s a_{ym} h \cos \phi
\]

\[
+ m_s a_{ym} h \sin \phi - \frac{t_w}{2} k \sin \phi - \frac{t_w}{2} \dot{\phi} \cos \phi \tag{21}
\]

Now, replace the above parameter using

\[
I_x \triangleq I_{xx} + m_s h^2 \tag{22}
\]

\[
k_t \triangleq \frac{t_w}{2} k \tag{23}
\]

\[
c_t \triangleq \frac{t_w}{2} c \tag{24}
\]

It should also be noted that

\[
m_s a_{ym} h = m_s a_t \cos \phi + m_s g \sin \phi \tag{25}
\]

Considering the above roll dynamics model, the roll dynamics can be simplified as a linear second-order model according to

\[
I_x \ddot{\phi} = m_s a_{ym} h - k_t \dot{\phi} - c_t \ddot{\phi} \tag{26}
\]

where the bouncing motion of the sprung mass is neglected. Here, \( m_s \) is the sprung mass, \( h \) is the CG height of the sprung mass from the roll centre, \( k_t \) is the roll spring coefficient of the suspension and \( c_t \) is the roll damping coefficient of the suspension. It is also assumed that the CG height is a known value. This simplified roll dynamics model does not consider some non-linear dynamics relating to roll motion, but this model is still sufficient for this study since an accurate understanding of the roll angle is not the purpose of this paper.

From the roll dynamics equation (26), a roll dynamics observer is derived using the measured roll rate \( \dot{\phi} \) and the estimated sprung mass \( \dot{\bar{m}}_s \) as

\[
I_x \ddot{\bar{m}}_s = \dot{\bar{m}}_s a_{ym} h - k_t \dot{\phi} - c_t \ddot{\phi} + k_o (\dot{\phi} - \ddot{\phi}) \tag{27}
\]

where \( \dot{\phi} \) is the estimated roll angle, \( a_{ym} \) is the measured lateral acceleration and \( k_o \) is the observer gain.

Defining the estimation errors of the roll angle and the vehicle’s sprung mass as

\[
\dot{\bar{\phi}} = \phi - \dot{\phi} \tag{28}
\]

\[
\dot{\bar{m}}_s = m_s - \dot{\bar{m}}_s \tag{29}
\]

respectively and then subtracting equation (27) from equation (26), the error dynamics of the observer are derived as

\[
I_x \ddot{\bar{\phi}} = \dot{\bar{m}}_s a_{ym} h - k_t \ddot{\phi} - c_t \dot{\phi} - k_o \dot{\phi} \tag{30}
\]

The stability of the above roll dynamics observer is proved through a Lyapunov stability analysis. Also, an adaptation law algorithm for the vehicle’s sprung mass is derived through the same analysis. Let a positive definite scalar function \( V \) be given as

\[
V = \frac{1}{2} (\dot{\phi} + \ddot{\phi})^2 + \frac{1}{2} \lambda \dot{\phi}^2 + \frac{1}{2} \frac{k_o}{\bar{m}_s} \dot{\phi}^2 > 0 \tag{31}
\]

where it should be noted that the observer gain \( k_o \) is defined to make \( V \) positive definite.

Taking the derivatives of the Lyapunov function (31) and combining it with equation (30), we obtain

\[
\dot{V} = \left( \dot{\phi} + \ddot{\phi} \right) \left( \ddot{\bar{\phi}} + \dddot{\phi} \right) + \lambda \dot{\phi} \dddot{\phi} - \frac{1}{k_o} \bar{m}_s \dot{\phi}^2
\]

\[
= \left( \dddot{\phi} + \dot{\phi} \right) \frac{\dot{\bar{m}}_s a_{ym} h - k_t \dot{\phi} - c_t \ddot{\phi} + k_o (\dot{\phi} - \ddot{\phi})}{I_x} \dot{\phi}
\]

\[
+ \lambda \dddot{\phi} \dot{\phi} - \frac{1}{k_o} \bar{m}_s \dot{\phi}^2
\]

\[
= - \bar{c}_t + \frac{k_o - I_x}{I_x} \dddot{\phi}^2 - \frac{k_t}{I_x} \dddot{\phi}^2
\]

\[
+ \lambda \dddot{\phi} \dot{\phi} + \frac{\dot{\bar{m}}_s a_{ym} h}{I_x} (\dot{\phi} + \dddot{\phi}) + \lambda \dddot{\phi} \dot{\phi} - \frac{1}{k_o} \bar{m}_s \dot{\phi}^2 \tag{32}
\]
Define $\lambda$ such that
\[
\lambda = \frac{k_t + c_t + k_o - I_x}{I_x}
\]
Then,
\[
\dot{V} = -c_t + k_o - I_x \frac{2}{I_x} \ddot{\phi}^2 - k_I \ddot{\phi}^2 + \frac{a_{ms} h}{I_x} (\phi + \dot{\phi}) - \frac{1}{k_a} m_t \ddot{m}_t
\]
To make equation (34) negative semidefinite, the third term on the right-hand side of the equation should be zero. Therefore, an adaptation law is derived as
\[
\dot{m}_t = \frac{k_o a_{ms} h}{I_x} (\ddot{\phi} + \dot{\phi})
\]
Now, equation (34) can be written as
\[
\dot{V} = -c_t + k_o - I_x \frac{2}{I_x} \ddot{\phi}^2 - k_I \ddot{\phi}^2
\]
Equation (36) is negative semidefinite for the arbitrary and positive observer gain $k_o$ defined such that $c_t + k_o > I_x$. From equation (36), it is confirmed that $\dot{\phi}$, $\ddot{\phi}$ and $\ddot{m}_t$ are bounded and so, from equation (30), $\dot{m}_t$ is bounded. Therefore, $\ddot{m}_t$ is bounded from equation (35) and, applying the Barbalat lemma, the system turns out to be asymptotically stable. Finally, it can be proved that the error of the roll angle estimate will converge to zero under the persistence of the excitation condition. In addition, the error dynamics equation (30) shows that the sprung mass estimation error converges to zero, as well.

**Mass estimation using adaptation law from the simplified error dynamics**

This section also deals with the mass estimation from the roll dynamics using the measured lateral acceleration, the roll rate and the vehicle roll dynamics model. However, knowing that the roll dynamics are very fast pace that the vehicle mass changes very slowly, the error dynamics model is simplified by neglecting the $\dot{\phi}$ term.

By neglecting the $I_x \ddot{\phi}$ term, equation (30) can be simplified as
\[
\ddot{m}_t a_{ms} h - k_t \ddot{\phi} - c_t \ddot{\phi} - k_o \ddot{\phi} = 0
\]
or equivalently as
\[
(c_t + k_o) \ddot{\phi} + k_t \ddot{\phi} = \ddot{m}_t a_{ms} h
\]
The stability of the roll dynamics observer is derived through a Lyapunov stability analysis, and an adaptation law algorithm for the vehicle’s sprung mass is derived through a similar analysis to that in the previous section. Let a positive definite scalar function $V$ candidate be given by
\[
V = \frac{1}{2} \ddot{\phi}^2 + \frac{1}{2} \frac{1}{k_a} \ddot{m}_t^2 > 0
\]
Taking the derivatives of the Lyapunov function (39) and combining them with equation (38), we obtain
\[
\dot{V} = \ddot{\phi} \dddot{\phi} + \frac{1}{k_a} \ddot{m}_t \dddot{m}_t
\]
\[
= \ddot{\phi} \dddot{\phi} - \frac{1}{k_a} \ddot{m}_t \dddot{m}_t
\]
\[
= - \frac{k_t}{c_t + k_o} \ddot{\phi}^2 + \left( \frac{\ddot{\phi} a_{ms} h}{c_t + k_o} - \ddot{m}_t \dddot{m}_t \right) \ddot{m}_t
\]
In order to make equation (40) negative semidefinite, the second term of the right-hand side of the equation should be zero. Therefore, we can derive the adaptation law as
\[
\dot{m}_t = \frac{k_o a_{ms} h}{c_t + k_o} \ddot{\phi}
\]
Now, equation (40) can be written as
\[
\dot{V} = - \frac{k_t}{c_t + k_o} \ddot{\phi}^2
\]
Equation (42) is negative semidefinite for $c_t + k_o > 0$. It is confirmed that $\ddot{\phi}$ and $\ddot{m}_t$ are bounded in equation (39), and so $\ddot{\phi}$ is bounded from equation (38). Also, $\ddot{m}_t$ is bounded from equation (41). By applying the Barbalat lemma, the system is guaranteed to be asymptotically stable. Therefore, it is proved that the error of the roll angle estimation will converge to zero under persistent excitation. Finally, the error dynamics equation (38) shows that the sprung mass estimation error converges to zero, as well.

**Mass estimation using the recursive least-squares method from the roll dynamics**

This section proposes a vehicle mass estimation scheme using the recursive least-squares method from the roll dynamics, as well as measurements of the roll rate and the lateral acceleration.

The roll dynamics equation (26) can be rearranged in a regression form as
\[
y(t) = a_{ms} h
\]
\[
\psi^T(t) = I_x \ddot{\phi} + c_t \ddot{\phi} + k_t \ddot{\phi}
\]
\[
\xi(t) = \frac{1}{m_s}
\]
where $y(t)$ is the measured output and $\psi(t)$ is the known variable and where $\xi(t)$ and the unknown parameter need to be estimated. Here, the vehicle attitude observer algorithm developed by Oh and Choi is used to estimate $\phi$. The algorithm includes the Euler angle observer which needs no information about inertia parameters to estimate the roll angle $\phi$.

Here, the same recursive least-squares algorithm as used in the previous section is employed, and the forgetting factor is utilized again since the vehicle mass can

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change with the number of the passengers or the load weight that the vehicle carries.

Integrated mass estimation using multiple-observer synthesis

The mass estimation using just the longitudinal dynamics is valid only when the steering input is near zero, and the mass estimation from the roll dynamics is available only when a sufficiently high steering input is applied to cause the vehicle to roll. Therefore, it is crucial to integrate both estimation methods efficiently. To integrate them, a proper weighting factor called a multiple-observer synthesis index (MOSI) is defined as

\[
\text{MOSI} = \max \left( \frac{\text{sat} \left( \frac{1}{2} \left( \text{roll angle} - \Gamma_{ra} + \varepsilon_{ra} \right) \right)}{\text{sat} \left( \frac{1}{2} \left( \text{roll rate} - \Gamma_{rr} + \varepsilon_{rr} \right) \right)} \right)
\]

where \(\Gamma\) and \(\varepsilon\) are constant tuning parameters.

Figure 3 illustrates how the index is formulated as a function of the roll angle and the roll rate values.

When the maximum values of the roll angle and the roll rate are small, mass estimation using the longitudinal dynamics is more accurate. Therefore, the weighting is mostly on the longitudinal-dynamics-based mass estimation. However, as the maximum values of the roll angle and the roll rate are sufficiently large, the mass estimation using the longitudinal dynamics starts to become inaccurate. Hence, in this case, it is more suitable to use instead the roll-dynamics-based mass estimation. Such shifting, thus, normally makes the estimated value less sensitive to the measurement noise for all sorts of driving scenario.

Incorporating the MOSI, the final mass estimation value is defined as

\[
\hat{m}_{\text{final}} = \text{MOSI} \hat{m}_{\text{roll}} + (1 - \text{MOSI}) \hat{m}_{\text{longitudinal}}
\]

Here, for the mass estimation from the roll dynamics, the recursive least-squares method is used since the recursive least-squares method shows the fastest convergence performance and is still sufficiently simple for online implementation.

Simulation results

In this section, simulations are conducted to evaluate the developed algorithms through the commercial vehicle simulation software CarSim and Simulink.

For the simulation model, the vehicle’s total mass is 1530 kg and the sprung mass is 1370 kg. In the first scenario, the vehicle moves in a straight line with a sinusoidal velocity profile between 65 km/h and 75 km/h without any steering input to evaluate the longitudinal mass estimation algorithm. The tuning parameters used are as follows: \(k_o = 6 \times 10^4\) and \(k_a = 3 \times 10^6\) for the full roll dynamics model; \(k_o = 2 \times 10^4\) and \(k_a = 3.5 \times 10^8\) for the simplified roll dynamics model; \(\varepsilon_{ra} = 0.004, \Gamma_{ra} = 0.02, \varepsilon_{rr} = 0.005\) and \(\Gamma_{rr} = 0.05\).

Figure 4 shows that the estimated mass value converges to an actual value, but slowly; the estimation accuracy deteriorates when the vehicle acceleration or deceleration is not sufficiently large.

In the second scenario, after sinusoidal steering by \(\pm 60^\circ\), the steering angle decreases gradually and the vehicle velocity is maintained at 80 km/h. Figure 5 shows the steering profile. The two adaptation algorithms and one recursive least-squares method from the roll dynamics model are compared.

Figure 6 shows that the simulation results of each roll dynamics observer are tracking the true roll angle. Initially, the observer shows some tracking error due to the difference between the estimated mass values and the true mass value. However, eventually, the adaptation mass values converge to the true value and each roll angle observer tracks the true roll angle well.

Figure 7 compares two adaptive mass estimation laws: one from the full roll dynamics and the other from the simplified roll dynamics. In both cases, the vehicle mass is estimated very accurately.
Figure 8 shows that the estimated mass value from the roll dynamics using the recursive least-squares method tracks the true mass value well. Using the variable forgetting factor appropriately, the convergence speed of the estimation can be maximized.

In the final driving scenario, the vehicle moves in a straight line with a sinusoidal velocity profile from 65 km/h to 75 km/h for 50 s and then a $\pm 60^\circ$ sinusoidal steering input is applied to the vehicle. This scenario demonstrates how this integrated mass estimation algorithm is beneficial.

Figure 9 shows that the estimated mass value considering only the longitudinal dynamics has some drifting issue after the steering input is applied and the roll-dynamics-based estimation method works only if the steering input is induced.

However, Figure 10 shows that the mass estimation by the proposed integrated mass estimation algorithm is efficient for all sorts of driving situations. This integrated method can estimate the vehicle mass very quickly with no drifting problem.

**Experimental results**

**Test environments**

Some experiments are conducted to demonstrate the performance of the proposed mass estimation
algorithm using a compact-size production sport utility vehicle. Table 1 shows the specifications of the test vehicle used for the experiments.

Here, an Analog Devices ADW22307 gyro sensor is used for the roll rate measurement and an Analog Devices ADXL103 sensor for the lateral acceleration. Also, a sensor zeroing algorithm is applied to compensate for the offset error of the six-dimensional (6D) inertia measurement unit (IMU). The 6D IMU is positioned at the CG of the vehicle. For verification purposes such as the accurate roll angle of the vehicle, the RT3100 model from the RT3000 family of Oxford Technical Solutions Ltd is also mounted on the vehicle.

**Test results and analysis**

In this section, the experimental results are analysed to evaluate the performance of the proposed mass estimation algorithms. In the experiment scenarios, first the vehicle moves in a straight line and then a severe sinusoidal steering input is added for a variable longitudinal velocity of the vehicle on a dry asphalt surface that has no severe road grade or bank angle. The tuning parameters used are as follows: $k_o = 2 \times 10^4$ and $k_a = 4 \times 10^5$ for the full roll dynamics model; $k_o = 2 \times 10^4$ and $k_a = 6 \times 10^7$ for the simplified roll dynamics model; $e_{ra} = 0.005$, $\Gamma_{ra} = 0.03$, $e_{rr} = 0.006$ and $\Gamma_{rr} = 0.07$.

In the first test run, the longitudinal velocity, the yaw rate and the steering angle of the vehicle are as shown in Figure 11, Figure 12 and Figure 13 respectively.

Figure 14 shows the performance of the roll angle observers using a simplified roll dynamics model and a full model. Figure 15 shows the performance of the mass estimation adaptation laws. The estimation results of both adaptation laws are quite accurate, but the initial convergence rate is not very fast when the steering input is added at 15 s. This initial rate can be

<table>
<thead>
<tr>
<th>Table 1. Test vehicle specifications (Tucsan ix 2WD gasoline Theta II 2.0 specifications).</th>
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</thead>
<tbody>
<tr>
<td><strong>Feature</strong></td>
</tr>
<tr>
<td><strong>Dimensions (mm)</strong></td>
</tr>
<tr>
<td>Wheelbase</td>
</tr>
<tr>
<td>Track</td>
</tr>
<tr>
<td>Overall length</td>
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<tr>
<td>Overall width</td>
</tr>
<tr>
<td>Height (unloaded)</td>
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<tr>
<td>Kerb mass (kgf)</td>
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</table>
accelerated by using a large adaptation gain, but this large gain may make the system unstable.

Figure 16 demonstrates how beneficial the recursive least-squares method is. The mass estimation using the recursive least-squares method has a much faster convergence rate than the adaptation laws do, and so it is more suitable for multiple-observer synthesis. Figure 17 shows the integrated mass estimation results using multiple-observer synthesis. The initial convergence rate is very fast, and the steady state drifting is minimized.

In the second test run, the longitudinal velocity and the steering angle of the vehicle are as shown in Figure 18 and Figure 19 respectively.

Figure 20 shows the result of the integrated mass estimation algorithm using multiple-observer synthesis. The mass estimation result between 20 s and 25 s is slightly inaccurate. The vehicle attitude observer algorithm used in this study requires no information about the inertia parameter values to calculate the roll angle $\phi$, and an inaccurate roll angle observation can
induce an inaccurate mass estimation result. Practically, such inaccuracy is not an issue since the vehicle mass changes very slowly, and it can be smoothed out easily by a low-pass filter.

Figure 21 and Figure 22 show the longitudinal velocity and the steering angle respectively of the vehicle in the final test run.

Figure 23 shows the result of the integrated mass estimation algorithm using the same synthesis. Also, during the time period when the vehicle attitude observer algorithm is not very accurate, the mass estimation result is not accurate. Eventually, however, the vehicle mass estimation result converges to the actual value. Considering the slowly varying characteristics of the vehicle mass, this might not be an issue.

Conclusion
Vehicle safety problems are crucial since many serious accidents have claimed the lives of many people. Information on the vehicle mass, which is an important parameter, plays a critical role in vehicle safety control. The vehicle mass is a variable parameter, but it is treated as a constant parameter for some vehicle safety control systems. This paper is focused on the development of a robust mass estimation algorithm for all kinds of real driving situation. In order to estimate the vehicle mass, the recursive least-squares method was applied for both the longitudinal dynamics and the roll dynamics, and adaptive observers were developed to observe the roll angle and to update the vehicle mass using the roll dynamics model. For the recursive least-squares method, a variable forgetting factor was used, which is based on the correction term for faster convergence. The stability and the performance of the adaptive observers were proven by a Lyapunov analysis. Finally, several mass estimation methods were synthesized to achieve the best transient and steady state performances.

The performance of the developed algorithm was investigated by simulations using CarSim and MATLAB/Simulink and experimentally using a test vehicle. The simulation and experimental results confirmed that the developed algorithm performs well for all kinds of real driving situation.

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Declaration of conflicting interest
The authors declare that there is no conflict of interest.

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