Estimation of Four Individual Tire Forces Using Limited Sensor Signals

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***Abstract*— This paper suggests two methods of the simultaneous estimation of vehicle lateral and longitudinal tire forces. The two estimation methods based on Kalman filter are presented: The first method utilizes all the sensor signals available in commercial vehicles, including the brake and shaft torque signals, to estimate the vehicle tire forces. The second method uses only the lateral and longitudinal accelerations, and the yaw rate signals with the steering input to extended the usage of the tire force estimator to conventional vehicles with limited sensor signals. The advantages and disadvantages of the two methods are investigated by the various simulation scenarios on different surfaces using Carsim vehicle dynamics software.**

***Index Terms*—estimation, longitudinal/lateral tire-force estimator, Kalman filter**

NOMENCLATURE

$F_{x,i}$: longitudinal tire force of i th wheel
$F_{z,i}$: vertical tire force of i th wheel
δ_i : steer angle of i th wheel
r : vehicle yaw rate
β : vehicle side slip angle
l_f : CG-front axle distance
l_r : CG-rear axle distance
I_z : vehicle yaw moment of inertia
I_ω : wheel moment of inertia
m : vehicle mass
V_x : vehicle longitudinal speed
V_y : vehicle lateral speed
a_x : vehicle longitudinal acceleration
a_y : vehicle lateral acceleration
R_e : tire effective radius
$T_{s,i}$:shaft torque of i th wheel
$T_{b,i}$:brake torque of i th wheel
ω_i : angular velocity of i th wheel

I. INTRODUCTION

The Estimation of longitudinal and lateral tire forces of a vehicle is a critical task for improving the performance of vehicle safety systems, such as electronics stability control (ESC), active front steering, and continuous damping control [1]-[4]. Since the both longitudinal and lateral tire forces of a vehicle affect the lateral and longitudinal vehicle dynamics simultaneously, the combined estimation of lateral and longitudinal tire forces is required to guarantee the performance of the above vehicle safety systems.

Several researches regarding the estimation of combined estimation for longitudinal and lateral tire forces have been introduced in the literature. In [5], the method of combined tire force estimation using the random-walk Kalman filter is suggested. Although the performance of the method was satisfactory, this algorithm has a drawback in requiring a derivative of sensor signal which needs a highly precise sensor. The method presented in [6] also showed a successful result in the performance of the combined tire force estimation. However, this algorithm had to overcome a significant computational burden due to the large matrices involved with the extended Kalman filter used in the tire force estimator.

The two different tire force estimators which do not cause a huge computational burden or require derivations of sensor signals are developed in this paper. The first method utilizes all the sensor signals available in commercial vehicles, including the brake and shaft torque signals, to estimate the vehicle tire forces. The second method uses only the lateral and longitudinal accelerations, and yaw rate signals with the steering input. The paper is organized as follows: The vehicle model used for the tire force estimation is presented in Section II. The tire force estimator using Kalman filter is developed in Section III. In Section IV, the suggested algorithm is verified with the Carsim simulation.

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Concluding remarks are given in Section V.

$$A(t)=\begin{bmatrix} & & & \mathbf{0}_{6\times 11} & & & \\ & & & -\frac{R_e}{I_w}\cdot\mathbf{I}_{4\times 4} \quad \mathbf{0}_{4\times 7} & & & \\ \frac{l_f\sin(\delta_1)-t\cos(\delta_1)}{I_z} & \frac{l_f\sin(\delta_2)+t\cos(\delta_2)}{I_z} & \frac{-l_r\sin(\delta_3)-t\cos(\delta_3)}{I_z} & \frac{-l_r\sin(\delta_4)+t\cos(\delta_4)}{I_z} & \frac{l_f\cos(\frac{\delta_1+\delta_2}{2})}{I_z} & \frac{-l_r\cos(\frac{\delta_3+\delta_4}{2})}{I_z} & \mathbf{0}_{1\times 5}\end{bmatrix} \tag{4}$$

$$B(t)=\begin{bmatrix}\mathbf{0}_{1\times 6} & \frac{T_{b,1}+T_{s,1}-R_e\mu_r F_{z,1}}{I_w} & \frac{T_{b,2}+T_{s,2}-R_e\mu_r F_{z,2}}{I_w} & \frac{T_{b,3}+T_{s,3}-R_e\mu_r F_{z,3}}{I_w} & \frac{T_{b,4}+T_{s,4}-R_e\mu_r F_{z,4}}{I_w} & 0\end{bmatrix} \tag{5}$$

$$H(t)=\begin{bmatrix}\cos(\delta_1) & \cos(\delta_2) & \cos(\delta_3) & \cos(\delta_4) & -\sin\left(\frac{\delta_1+\delta_2}{2}\right) & -\sin\left(\frac{\delta_3+\delta_4}{2}\right) & \mathbf{0}_{2\times 5}\\ \sin(\delta_1) & \sin(\delta_2) & \sin(\delta_3) & \sin(\delta_4) & \cos\left(\frac{\delta_1+\delta_2}{2}\right) & \cos\left(\frac{\delta_1+\delta_2}{2}\right) & \\ & & & \mathbf{0}_{5\times 6} & & & \mathbf{I}_{5\times 5}\end{bmatrix} \tag{6}$$

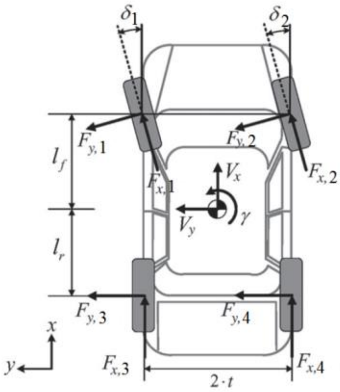

Figure 1. Planar vehicle dynamics model

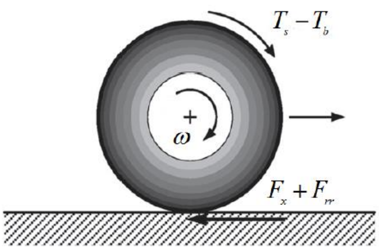

Figure 2. Wheel dynamics model

II. VEHICLE MODELS

The proposed tire force estimators are developed based on Kalman filter which requires a model of the system in a state-space form. The governing equation of the vehicle planar dynamics shown in Fig. 1 can be expressed as follow:

$$\begin{aligned} I_z\dot{r} &= l_f F_{y,f}\cos\left(\frac{\delta_1+\delta_2}{2}\right)-l_r F_{y,r}\cos\left(\frac{\delta_3+\delta_4}{2}\right)\\ &+t\left(-F_{x,1}\cos\delta_1+F_{x,2}\cos\delta_2-F_{x,3}\cos\delta_3+F_{x,4}\cos\delta_4\right)\\ &+l_f F_{x,1}\sin\delta_1+l_f F_{x,2}\sin\delta_2-l_r F_{x,3}\sin\delta_3-l_r F_{x,4}\sin\delta_4 \end{aligned} \tag{1}$$

The equation of motion for the individual wheel shown in Fig. 2 is:

$$I_w\dot{\omega}_i = T_{s,i}-R_e F_{x,i}-T_{b,i}-R_e F_{rr,i} \tag{2}$$

The subscript i =1,2,3,4 denotes the front-left, front-right, rear-left, and rear-right individual wheels, respectively. F_{rr} , i.e., the rolling resistance force [7], is defined:

$$F_{rr,i}=\mu_r F_{z,i} \tag{3}$$

where μ_r is the rolling resistance coefficient. a_x and a_y ,which are directly related to the tire forces, have the following relationships:

$$\begin{aligned} a_x &= \frac{1}{m}\left(F_{XF}+F_{XR}-F_{drag}\right)\\ a_y &= \frac{1}{m}\left(F_{YF}+F_{YR}\right)\end{aligned} \tag{7}$$

where

$$\begin{aligned} F_{XF} &= F_{x,1}\cos\delta_1+F_{x,2}\cos\delta_2+F_{y,1}\sin\delta_1+F_{y,2}\cos\delta_2\\ F_{XR} &= F_{x,3}\cos\delta_3+F_{x,4}\cos\delta_4+F_{y,3}\sin\delta_3+F_{y,4}\cos\delta_4\\ F_{YF} &= F_{x,1}\sin\delta_1+F_{x,2}\sin\delta_2+F_{y,1}\cos\delta_1+F_{y,2}\cos\delta_2\\ F_{YF} &= F_{x,3}\sin\delta_3+F_{x,4}\sin\delta_4+F_{y,3}\cos\delta_3+F_{y,4}\cos\delta_4\\ F_{drag} &= C_{av}V_x^2 \end{aligned} \tag{8}$$

F_{drag} in (8) is the aerodynamic drag force with the aerodynamic coefficient, i.e., C_{av} [7].

III. TIRE FORCE ESTIMATION USING KALMAN FILTER

A. Tire Force Estimator with Full Measuremtns

When full measurement signals including T_s and T_b are available, the wheel dynamics (2) can be taken into account for more accurate tire force estimation. Equations (1) and (2) are augmented to create the vehicle model in a state-space form with the process noise $w(t)$ and the measurement noise $v(t)$:

$$\begin{aligned}\dot{x}(t) &= A(t)x(t)+B(t)+w(t)\\ z(t) &= H(t)x(t)+v(t)\end{aligned} \tag{9}$$

where $A(t)$, $B(t)$, and $H(t)$ are defined as in (4)-(6), and thier $\mathbf{I}_{i\times k}$ and $\mathbf{0}_{i\times k}$ denote i by k identity matrix and zero matrix, respectively. The state $x(t)$ of the system is defined as follows:

$$\mathbf{x}=\begin{bmatrix}\mathbf{F} & \mathbf{\Omega} & r\end{bmatrix}^T$$

where

$$\mathbf{F}=\begin{bmatrix}F_{x,1} & F_{x,2} & F_{x,3} & F_{x,4} & F_{y,f} & F_{y,r}\end{bmatrix}$$

$$\mathbf{\Omega}=\begin{bmatrix}\omega_1 & \omega_2 & \omega_3 & \omega_4\end{bmatrix}$$

$$F_{y,f}=F_{y,1}+F_{y,2},\quad F_{y,f}=F_{y,1}+F_{y,2}$$

The measurements are:

$$\mathbf{z}=\begin{bmatrix}a_x & a_y & r & \omega_1 & \omega_2 & \omega_3 & \omega_4\end{bmatrix}$$

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The state-space system (9) can be discretized using zero-order hold so that (9) has the following form for the discrete-time Kalman filter [8]:

$$\mathrm{x}_{k+1} = A_k \mathrm{x}_k + B_k + w_k$$
$$\mathrm{z}_k = H_k \mathrm{x}_k + v_k$$

The algorithm of the discrete-time Kalman filter is:

$$\bar{\mathrm{x}}_{k+1} = A_k \hat{\mathrm{x}}_k + B_k$$
$$M_{k+1} = A_k P_k A_k^T + W_k$$
$$\hat{\mathrm{x}}_k = \bar{\mathrm{x}}_k P_k H_k^T V_k^{-1}(\mathrm{z}_k - H_k \bar{\mathrm{x}}_k)$$
$$P_k = (M_k^{-1} + H_k^T V_k^{-1} H_k)^{-1}$$

where W_k and V_k denote the covariance matrices of w_k and v_k , respectively. The state of the Kalman filter includes the equation of motion of the planar vehicle but tire forces are regarded as unknown parameters to be estimated. The values of the parameters are updated by the process noise w_k . The two front steer angles are obtained from Ackerman steering geometry using the driver's steering wheel input. The steer angles of the rear wheels are assumed to be zeros. For more details about Kalman filter, refer to [6].

B. *Simplified Tire Force Estimator*

During driving situations that do not require activations of ABSs or ESCs, the simplified estimator, to be suggested in this section, that does not require the sensor signals of the brake pressures or the engine torque shows a sufficient performance to collect the information regarding the tire forces of a vehicle. The collected tire force information in a relatively mild driving situation can be used for the identification of tire-road friction coefficient.

When most commercial vehicles accelerate, the differential gears evenly spread the torque from the engine into the driving wheels on the left- and right-hand sides of the vehicle [5]. While braking, unless wheels are locked or ABSs or ESCs are activated, the ratio of the braking forces of the front wheels and the rear wheels has a certain value depending on disk size, proportioning valve and so on. Therefore, taking the characteristics of longitudinal tire forces mentioned above into account, the four longitudinal tire forces can be expressed using one variable, i.e., F_x as follows:

During acceleration: $$F_{x,1} = F_{x,2} = F_x \qquad (13)$$

During deceleration: $$F_{x,1} = F_{x,2} = K_s F_x$$
$$F_{x,3} = F_{x,4} = (1 - K_s) F_x \qquad (14)$$

In (13), a front wheel driving vehicle is assumed. For a rear wheel driving vehicle, (13) can be modified with $F_{x,3} = F_{x,4} = F_x$. The gain K_s in the range of $(0.5,1)$ in (14) can be expressed as a function of F_x since the proportioning valve that varies the brake pressure distribution between the front and rear wheels is depending on the master cylinder pressure which is proportional to F_x during mild driving situations. K_s can be set to be a constant for vehicles that are not equipped with proportioning valves. Therefore, the Kalman filter can be reconstructed with the following simplified states and measurements as follows:

$$\mathbf{x} = \begin{bmatrix} F_x & F_{y,f} & F_{y,r} \end{bmatrix}^T$$
$$\mathbf{z} = \begin{bmatrix} a_x & a_y & r \end{bmatrix}^T$$

$A(t)$, $B(t)$, and $H(t)$ in (9) are modified for the simplified tire force estimator as in (10)-(12). Equations (1) and (7) are still augmented to create the state space system for the Kalman filter. However, since the wheel dynamics (2) are neglected in the simplified tire force estimator, four states related to wheel angular velocities, and the measurements of shaft torques and brake torques can be removed from the state-space system. Consequently, the overall computation time for the tire force estimation has greatly reduced and the tire forces of vehicles not equipped with sensors regarding shaft torque or brake pressure can also be estimated. However, the simplified tire force estimator is only applicable to a vehicle that differential brake forces are not exerted by ABS or ESC and the vehicle longitudinal dynamics is slow enough to ignore the wheel dynamics.

IV. SIMULATION RESULT

The performances of the two tire force estimators were verified and compared in the Carsim simulation whose result is presented in Fig. 4. The vehicle maneuver, speed and tire-road friction coefficient profiles are given in Fig. 3. The simulation scenarios involved both accelerating and braking in a turn on different road surfaces.

Gaussian noise was added to the simulated measurements of a_x, a_y, ω_i and r to better describe the real application scenarios. As seen in Fig.4, the estimated values of the tire forces from the two proposed estimators follow the actual tire forces successfully. Noticeable differences between the estimation performances of the simplified and full measurement tire force estimators are not found when ESC or ABS was not initiated.

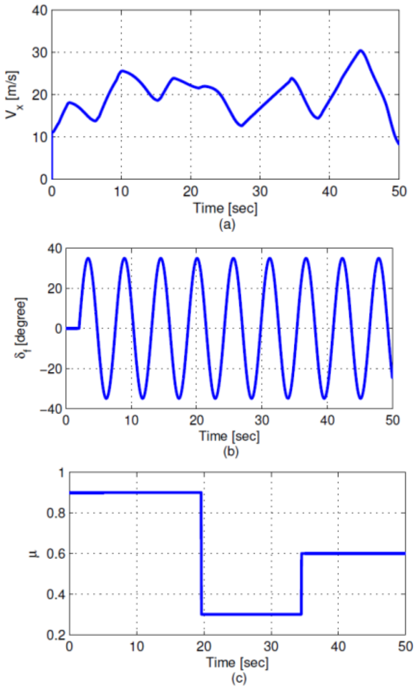

Figure 3. Simulation environment. (a) Longitudinal speed profile. (b) Steer angle profile (c) Tire-road friction coefficient.

However, when the ABS was activated at around $t = 43$ s, the simplified tire force estimator became not able to estimate the actual tire forces properly due to the neglected wheel dynamics.

V. Conclusion

The two different tire force estimators are developed based on Kalman filter. The tire force estimator using the full measurements shows the satisfactory performance in various situations. The simplified tire force estimator with the limited measurements also performs well except for the situations that ESC or ABS is activated. Though the usage of the simplified tire force estimator with the limited measurements is limited to certain conditions, it has the advantages in calculation cost and not requiring brake and shaft torque signals.

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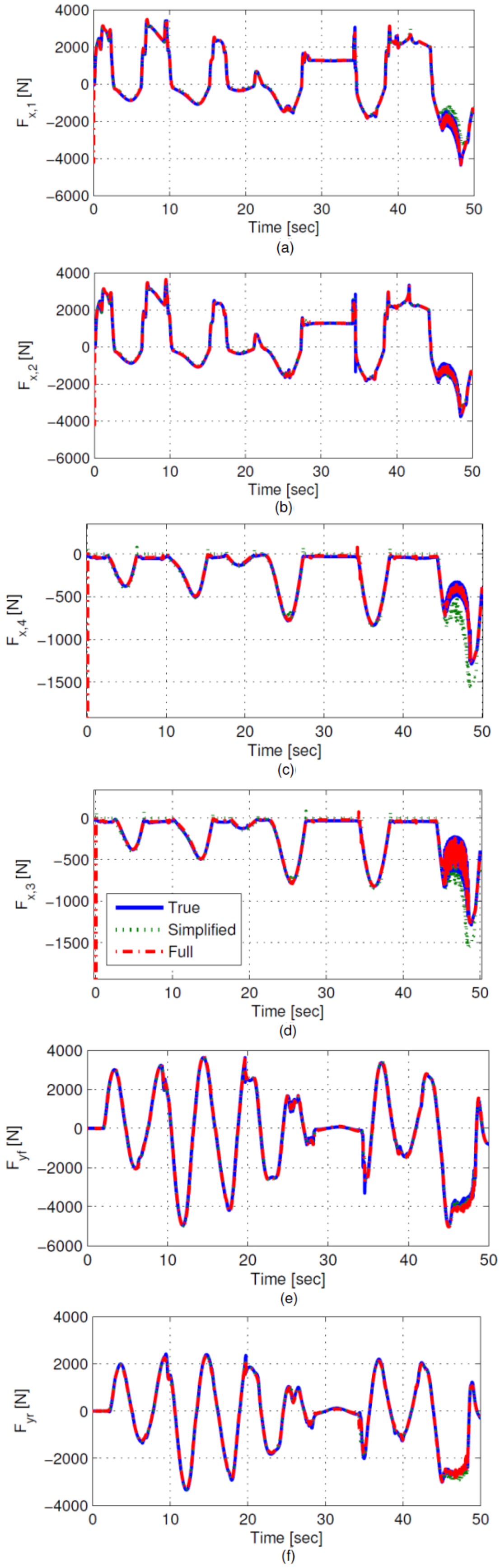

Figure 4. Estimated tire forces. (a) Front-left longitudinal tire force. (b) Front-right longitudinal tire force. (c) Rear-left longitudinal tire force.

(d) Rear-right longitudinal tire force. (e) Front axle lateral tire force. (f) Rear axle lateral tire force.

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