Direct Adaptive Longitudinal Control of Vehicle Platoons

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***Abstract*—An important aspect of an automated highway system design is the synthesis of an automatic vehicle following system. Associated with automatic vehicle following systems is the problem of the stability of a string of vehicles, i.e., the problem of spacing error propagation, and in some cases, amplification upstream from one vehicle to another, due to some disturbance at the head of the string. Realistic vehicle following designs must also address parametric uncertainties such as mass of the vehicle, aerodynamic drag, and tire drag. The mass of the vehicle varies with the number of passengers. At small intervehicular separations, aerodynamic drag force changes significantly with the distance to be maintained. In this paper, we address the problem of stability of a vehicle string in the presence of parametric uncertainty and present a Lyapunov-based decentralized adaptive control algorithm to compensate for such parametric variations. We examine this direct adaptive control algorithm for platoon performance and parameter convergence. We present the simulation results to demonstrate the effectiveness of the adaptive controller.**

***Index Terms*—Advanced cruise-control systems, decentralized adaptive control, direct adaptive control, intelligent vehicle highway system (IVHS), longitudinal control.**

I. INTRODUCTION

IN AUTOMATIC vehicle following systems, vehicles are dynamically coupled by feedback control laws. The stability of the string of automated vehicles depends on the information available for feedback and on how such information is processed in the synthesis of a vehicle following controller. The problem of investigating the stability of a vehicle string (also called platoon) under automatic control has attracted significant research in the last three decades. For a good overview of the efforts in the area of automatic vehicle following, the reader is referred to [16], [2], and [15]. For a good overview of the analysis of string stability, the reader is referred to [1], [5], [3], [9], [7], and [17].

The design of an automatic vehicle following controller consists of a specification of the desired following distance as a function of the speed and the design of a control system that regulates the speed of the vehicle in accordance with the given spacing policy. The specification of the desired spacing as a function of the speed of the vehicle is referred to as the specification of a spacing policy for an automatic vehicle following system. A spacing policy employed by a controlled vehicle is called a constant spacing policy if the desired following distance is independent of its speed. A spacing policy is a variable spacing policy if it is not a constant spacing policy.

The tracking requirement with a variable spacing policy is not stringent. For this reason, the stability of a string of controlled vehicles employing a variable spacing policy can be guaranteed without any intervehicular communication. Variable spacing policies, though not addressed in this paper, are employed in adaptive cruise-control schemes. The reader is referred to [5], [7], [17] for the synthesis of control laws based on variable spacing policies. The reader is referred to [8] for an adaptive vehicle following control scheme based on a variable spacing policy.

The tracking requirement in a constant spacing policy is stringent. The string stability of such a platoon of vehicles can be guaranteed if information of a reference vehicle is fed back in the vehicle following control law [4], [16], [9].

This paper is concerned with the design of an adaptive controller when a constant spacing policy is employed by an automated vehicle. With a constant spacing policy, the desired following distance of every vehicle in the platoon is constant and is independent of the speed of the vehicle.

In [10], an indirect decentralized adaptive control algorithm is designed for a platoon of vehicles employing a constant spacing policy. In this paper, we present a direct, decentralized adaptive control algorithm, which satisfies the same performance objectives. The advantage of such a direct scheme is the ease of its on-line implementation.

The reader is referred to [12], [14], and [11] for an introduction to the adaptive control of nonlinear systems. The reader is referred to [13] for the input–output properties of linear feedback systems.

This paper is organized as follows: In Section II, we investigate the effect of parametric uncertainty on the platoon performance. In Section III, we present the adaptive control algorithm. In Section IV, we examine the platoon performance with the adaptive longitudinal controller and discuss heuristically the conditions for parameter convergence. In Section V, we discuss the simulation results. In Section IV, we present our conclusions and suggest directions for further research.

II. EFFECT OF PARAMETRIC UNCERTAINTY ON THE PLATOON PERFORMANCE

In Fig. 1, a string of automated vehicles is shown. Vehicle 0 is the lead vehicle in the string, and vehicle i is its ith following

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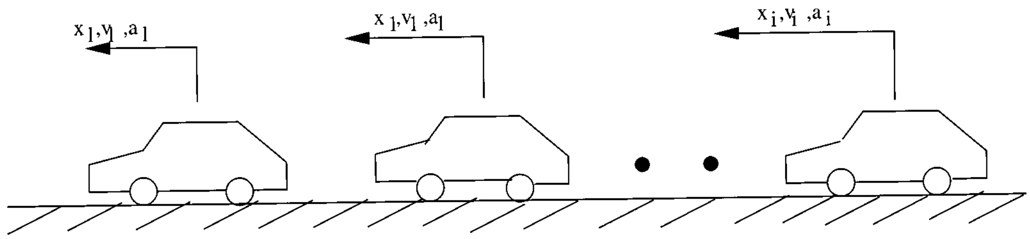


Fig. 1. Spacing errors in a platoon.

vehicle. The position, velocity, and acceleration of the ith following vehicle are, respectively, x_i, v_i, a_i. The corresponding quantities for the lead vehicle are x_l, v_l, a_l, respectively.

In the above figure, L_i is the desired intervehicular distance of the ith following vehicle and ϵ_i is the deviation in the intervehicular distance from its desired value. ϵ_i is referred to as the spacing error of the ith vehicle and is given by

$$\epsilon_i := x_i - x_{i-1} + L_i.$$

The following model of the longitudinal dynamics of a member vehicle in the platoon is used for designing the control algorithm:

$$\ddot{x}_i = \frac{u_i - c_i \dot{x}_i^2 - f_i}{M_i} \tag{1}$$

where x_i, u_i, c_i, f_i, M_i are the position, control effort, effective aerodynamic drag coefficient, rolling resistance friction, and effective inertia of the ith following vehicle, respectively.

Any vehicle following controller must be designed to satisfy the following performance objectives.

1) Individual Vehicle Stability: Individual vehicle stability is the ability of any member vehicle in the platoon to track any bounded acceleration and velocity profile of its predecessor with bounded spacing and velocity errors.

2) String Stability: It is desired that the errors in spacing and velocity must not amplify upstream from one vehicle to another. In the presence of parametric uncertainty in each vehicle, this objective is relaxed here. We will require that the errors in spacing and velocity be bounded in time, and uniformly in vehicle index. More precisely, we adopt the following definitions in our analysis.

Definition II.1 (String Stability): A platoon [whose member vehicles are governed by (1)] is string stable if, given any $\gamma > 0$, $\exists\, \delta > 0$ such that

$$\sup_i \max\{|\epsilon_i(0)|, |\dot{\epsilon}_i(0)|\} < \delta$$
$$\Rightarrow \sup_i \sup_{t \ge 0} \max\{|\epsilon_i(t)|, |\dot{\epsilon}_i(t)|\} < \gamma.$$

Definition II.2 (Uniform Boundedness of Errors): The spacing errors of member vehicles in a platoon are uniformly bounded if, for some $\delta > 0$, $\exists\, \gamma > 0$ such that

$$\sup_i \max\{|\epsilon_i(0)|, |\dot{\epsilon}_i(0)|\} < \delta$$
$$\Rightarrow \sup_i \sup_{t \ge 0} \max\{|\epsilon_i(t)|, |\dot{\epsilon}_i(t)|\} < \gamma.$$

3) Zero Steady-State Spacing Errors: Finally, we require that $\epsilon_i(t), \dot{\epsilon}_i(t) \rightarrow 0$ as $t \rightarrow \infty$. This helps to maintain a reliable traffic capacity.

From the definitions, it is clear that any platoon consisting of a finite number of member vehicles will be string stable if the (individual) stability of the member vehicles is guaranteed. For designing controllers for member vehicles, it is assumed that every platoon has infinite member vehicles. This assumption can be made without any loss of generality because the information of vehicles ahead is only used in the synthesis of automatic vehicle following control algorithms. Such an assumption makes it convenient to study the asymptotic properties of the propagation of errors in a vehicle platoon.

In the presence of parametric uncertainty, the requirement of string stability is very stringent. In the presence of parametric uncertainty, there is no guarantee that the errors in spacing and velocity are bounded. Even if the errors in spacing and velocity are bounded, the bound is dependent on the initial spacing and velocity errors and the initial parameter estimation errors. As a result, given any γ, δ, one can determine a set of initial estimates of the parameter, such that the string stability definition is not satisfied.

From a practical standpoint, uniform boundedness of spacing errors is a reasonable performance objective. Hence, we adopt this criterion in the design of adaptive longitudinal controller.

A. Effect of Uncertainty in Mass of the Vehicle

Define an auxillary error, $S_{1i}, \; i = 1, 2, \ldots$, given in [6], as

$$S_{1i} = \dot{\epsilon}_i + q_1 \epsilon_i + q_3 (v_i - v_l) + q_4 \left(x_i - x_l + \sum_{j=0}^{i} L_j \right).$$

Here, q_1, q_3, and $q_4$are control parameters and will be chosen later. The subscript l refers to the corresponding quantities for the lead vehicle in the string.

The control effort u_i is chosen to make $\dot{S}_{1i} + \lambda S_{1i} = 0$ and is given by

$$u_i = c_i \dot{x}_i^2 + f_i + M_i u_{\text{isl}} \tag{2}$$

$$u_{\text{isl}} = \frac{1}{1+q_3} \left[\ddot{x}_{i-1} + q_3 \ddot{x}_l - q_1 \dot{\epsilon}_i - q_4 (v_i - v_l) - \lambda S_{1i} \right]. \tag{3}$$

For individual vehicle stability, it is sufficient that $q_1, \; q_3, \; q_4, \; \lambda > 0$. In the presence of uncertainty in the mass

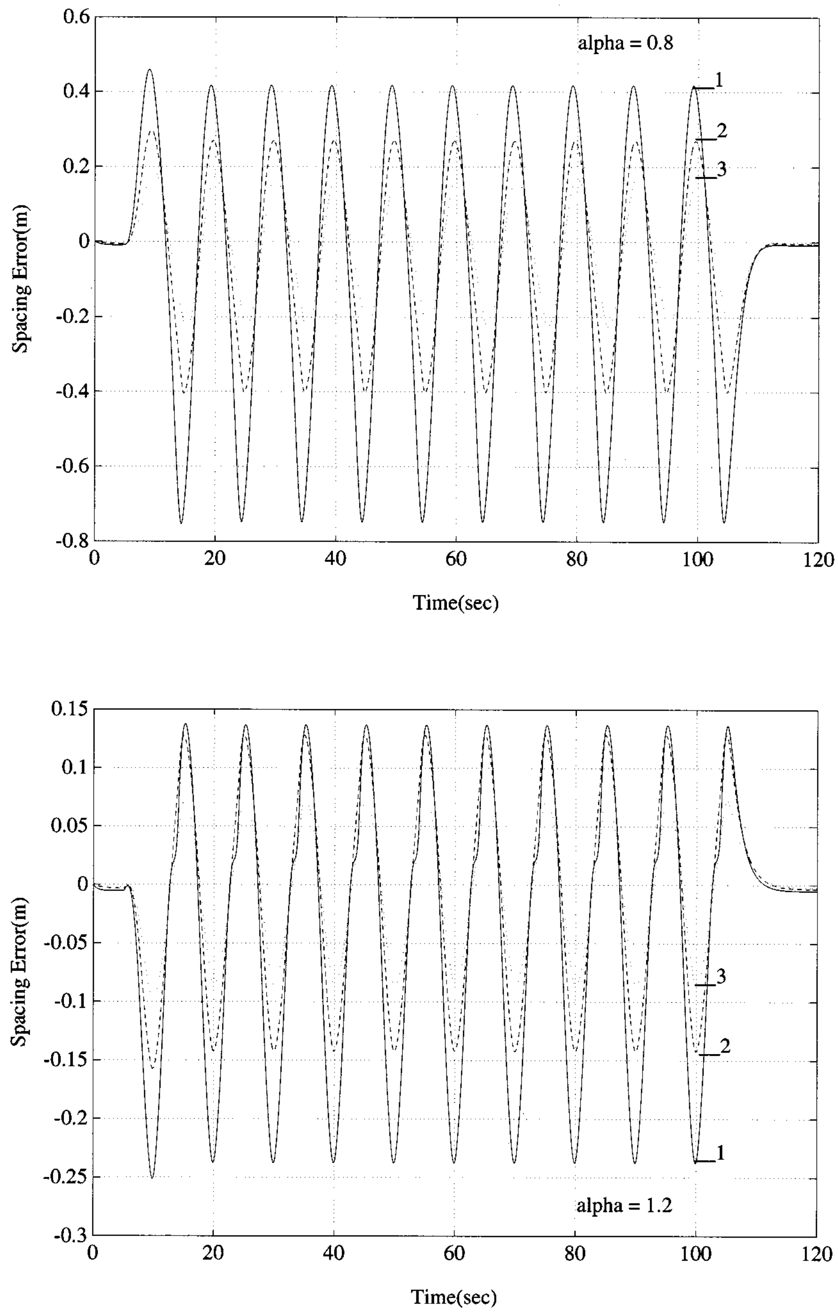


Fig. 2. Effect of uncertainty in mass on the platoon performance.

f the vehicle

$$u_i = c_i \dot{x}_i^2 + f_i + \hat{M}_i u_{\mathrm{isl}}$$

so that

$$\ddot{x}_i = \frac{\hat{M}_i}{M_i} u_{\mathrm{isl}}.$$

In the above equations, $\hat{M}_i$ is the estimate of the mass of the vehicle.

Let $\hat{G}(s)$ denote the transfer function that relates the spacing error in the first following vehicle to the acceleration of the lead vehicle. Let $\hat{H}(s)$ denote the transfer function that relates the spacing error of any vehicle to the spacing error of its immediate predecessor. Then

$$\hat{G}(s) := \frac{\hat{\epsilon}_1}{\hat{a}_l}(s) = \frac{\alpha - 1}{s^2 + \alpha\left[\left(\lambda + \frac{q_1 + q_4}{1 + q_3}\right)s + \frac{\lambda(q_1 + q_4)}{1 + q_3}\right]}$$

$$\hat{H}(s) := \frac{\hat{\epsilon}_i}{\hat{\epsilon}_{i-1}}(s) = \frac{\alpha}{(1 + q_3)} \frac{(s + q_1)(s + \lambda)}{\left[s^2 + \alpha\left[\left(\frac{q_1 + q_4}{1 + q_3} + \lambda\right)s + \frac{\lambda(q_1 + q_4)}{1 + q_3}\right]\right]}$$

where $\alpha = \hat{M}_i / M_i$.

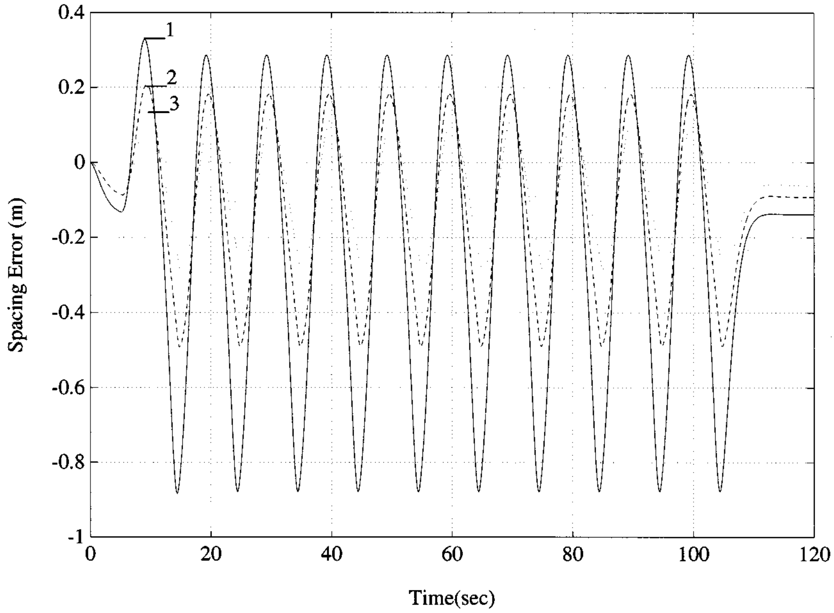


Fig. 3. Effect of uncertainty in rolling resistance and mass on the platoon performance.

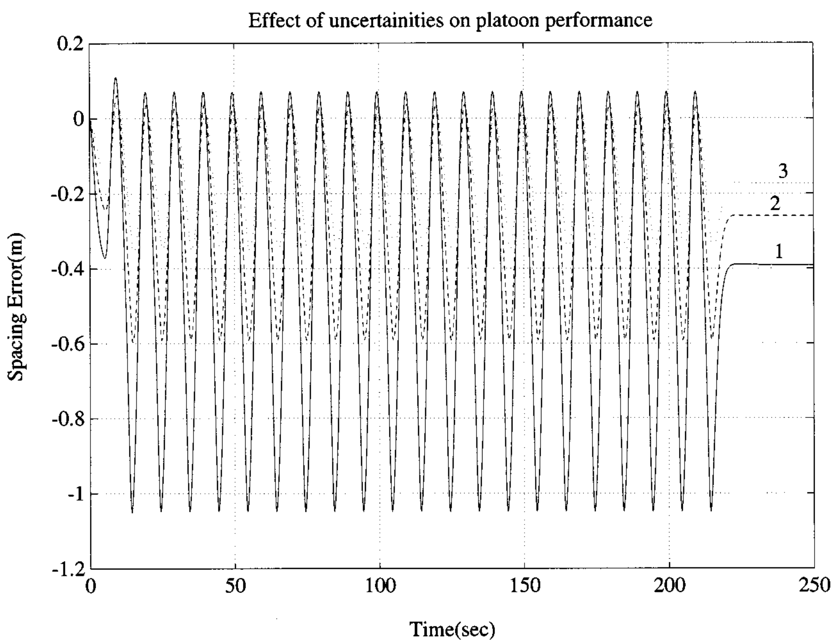


Fig. 4. Effect of uncertainty in all parameters on the platoon performance.

Proposition II.1: Given $\hat{H}(s)$ and that $q_1 > (q_1+q_4)/(1+q_3)$, $\lambda \neq q_1$, $\lambda \neq (q_1+q_4)/(1+q_3)$, there exist two constants $\beta_l < 1$, $\beta_h > 1$ such that $\forall \alpha \in [\beta_l, \beta_h]$, $h(t) > 0$, and, consequently, $||h||_1 = |\hat{H}(0)| = q_1/(q_1+q_4)$.

The intuition is that the impulse response $h(t)$ is a continuous function of α. If $h(t) > 0$ for $\alpha = 1$, then $h(t) > 0$ in some neighborhood of $\alpha = 1$. Therefore, $||h||_1$ does not change for small perturbations in α around unity. The poles of $\hat{H}(s)$ should be simple so that $h(t) > 0 \ \forall t \geq 0$. We can also guarantee that $\epsilon_i \to 0$ asymptotically whenever the lead vehicle reaches a steady velocity after a maneuver in finite time.

Claim: Suppose $r_1, r_2 > 0$. The impulse response of $(s^2+p_1 s+p_2)/(s^2+r_1 s+r_2)$ is positive iff $r_1^2-4r_2 \geq 0, p_1-r_1 \geq 0$, and $2(p_2-r_2) \geq (p_1-r_1)(r_1-\sqrt{r_1^2-4r_2})$.

Proof: Unless $r_1^2 - 4r_2 > 0$, the impulse response will be oscillatory and will change sign an infinite number of times. Write

$$\frac{s^2+p_1 s+p_s}{s^2+r_1 s+r_2} = 1 + \frac{(p_1-r_1)s+(p_2-r_2)}{s^2+r_1 s+r_2}.$$

An application of the initial value theorem indicates that unless $(p_1-r_1) \geq 0$, the impulse response is negative in the vicinity of $t=0$. Finally, the impulse response of $(s+\zeta)/((s+\eta)(s+\nu))$ is positive iff the impulse response of $(s+\zeta-\eta)/(s(s+\nu-\eta))$ is positive. Unless $\zeta > \min\{\eta, \nu\}$, the impulse response always changes sign. The proof of the claim follows from this observation.

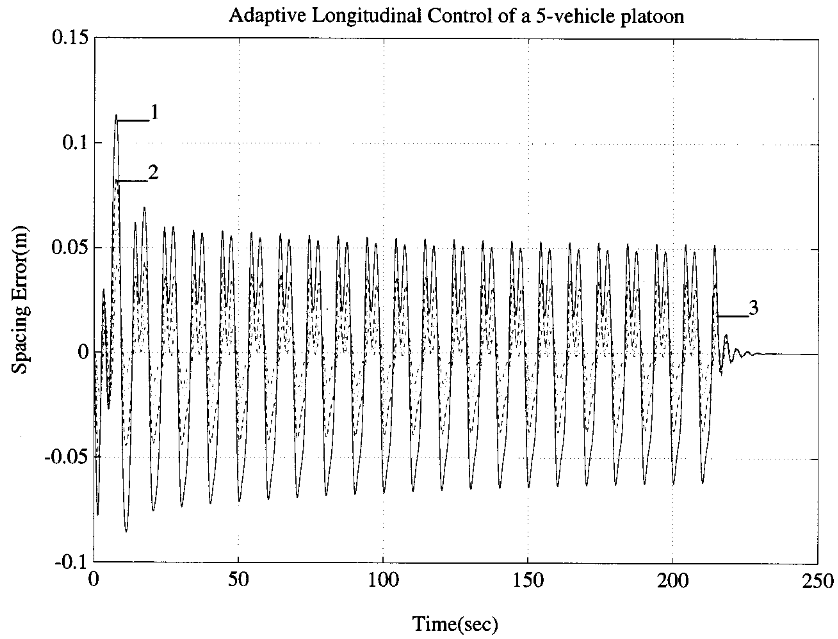


Fig. 5. Platoon performance with adaptation.

If r_1, r_2 are continuous functions of α, define $F_1(\alpha) := r_1^2 - 4r_2$, $F_2(\alpha) := p_1 - r_1$, $F_3(\alpha) := (p_1 - r_1)((r_1 - \sqrt{F_1(\alpha)})/2)$. Clearly, F_1, F_2, F_3 are continuous functions and F_3 is well defined if $F_1 \geq 0$. Suppose $F_1(\alpha_0)$, $F_2(\alpha_0)$, $F_3(\alpha_0) > 0$ and $\alpha_0 > 0$. Then, from the continuity of the functions, there exist two constants $\beta_l, \beta_h > 0$ such that $\alpha_0 \in [\beta_l, \beta_h]$ and $\forall\, \alpha \in [\beta_l, \beta_h]$, $F_1(\alpha)$, $F_2(\alpha)$, $F_3(\alpha) > 0$. The result of the proposition, then, follows immediately from this observation.

Proposition II.2: If the conditions in Proposition II.1 hold, the decentralized control law given by (2) ensures the uniform boundedness of spacing errors of member vehicles in the platoon.

Proof: Let

$$\Delta(s) = s^2 + \alpha\left[\left(\frac{q_1+q_4}{1+q_3} + \lambda\right)s + \frac{\lambda(q_1+q_4)}{1+q_3}\right].$$

Let β_1, β_2 be the minimum and maximum absolute values of the roots of $\Delta(s)\ \forall\, \alpha \in [\beta_l, \beta_h]$. Let

$$\begin{aligned} C_1 &= \epsilon_i(0) - \frac{\alpha}{1+q_3}\epsilon_{i-1}(0) \\ C_2' &= \dot{\epsilon}_i(0) - \frac{\alpha}{1+q_3}\dot{\epsilon}_{i-1}(0) \\ &\quad + \alpha\left(\left(\frac{q_1+q_4}{1+q_3} + \lambda\right)\epsilon_i(0) - \frac{q_1+\lambda}{1+q_3}\epsilon_{i-1}(0)\right) \\ \hat{\epsilon}_i(s) &= \hat{H}(s)\hat{\epsilon}_{i-1}(s) + \frac{sC_1 + C_2'}{\Delta(s)}. \end{aligned}$$

Define

$$C_2 = C_2' - \beta_2 C_1.$$

Therefore

$$\begin{aligned} \hat{\epsilon}_i(s) &= \hat{H}(s)\epsilon_{i-1}(s) \\ &\quad + \frac{(s+\beta_2)C_1(\epsilon_i(0), \dot{\epsilon}_i(0)) + C_2(\epsilon_i(0), \dot{\epsilon}_i(0))}{\Delta(s)}. \end{aligned}$$

It now follows that

$$\sup_{t\geq 0} |\epsilon_i(t)| \leq \frac{q_1}{q_1+q_4}\sup_{t\geq 0}|\epsilon_{i-1}(t)| + \frac{|C_1|}{\beta_1} + \frac{(1+q_3)|C_2|}{\alpha\lambda(q_1+q_4)}.$$

Since C_1, C_2 are linear functions of $\epsilon_i(0)$ and $\dot{\epsilon}_i(0)$, uniform boundedness of spacing errors can be guaranteed.

The error in the first following vehicle is governed by $\hat{G}(s)$. Because of a mismatch in the estimation of parameters, the maximum error in the first vehicle is dependent on the magnitude and frequency content of the lead vehicle maneuver. Therefore, string stability cannot be assured.

β_l, β_h indicate the degree of robustness in string stability to variations in mass. If $q_1 = 3$, $q_3 = 1$, $q_4 = 1$, $\lambda = 4$, $\beta_l < 0.9$, $\beta_h \geq 1.166$. With this choice of control gains, we have robustness in string stability to a 10% variation in mass. The proof of uniform boundedness relies on the fact that $q_4 \neq 0$, i.e., the availability of the relative position of the lead vehicle to every controlled vehicle.

B. Effect of Uncertainty in Rolling Resistance and Mass of the Vehicle

With uncertainty in mass of the vehicle and rolling resistance moment, the control effort u_i is given by

$$u_i = \hat{M}_i u_{\text{isl}} + c_i\dot{x}_i^2 + \hat{f}_i.$$

Hence, for $i \geq 2$

$$\begin{aligned} \hat{\epsilon}_i(s) &= \hat{H}(s)\hat{\epsilon}_{i-1}(s) + \frac{\hat{\phi}_{1i}(s)}{\Delta(s)} \\ \Delta(s) &= s^2 + \alpha\left[\left(\frac{q_1+q_4}{1+q_3} + \lambda\right)s + \frac{\lambda(q_1+q_4)}{1+q_3}\right]. \end{aligned} \tag{4}$$

where

$$\phi_{1i}(t) = \frac{\tilde{f}_i}{M_i} - \frac{\tilde{f}_{i-1}}{M_{i-1}}.$$

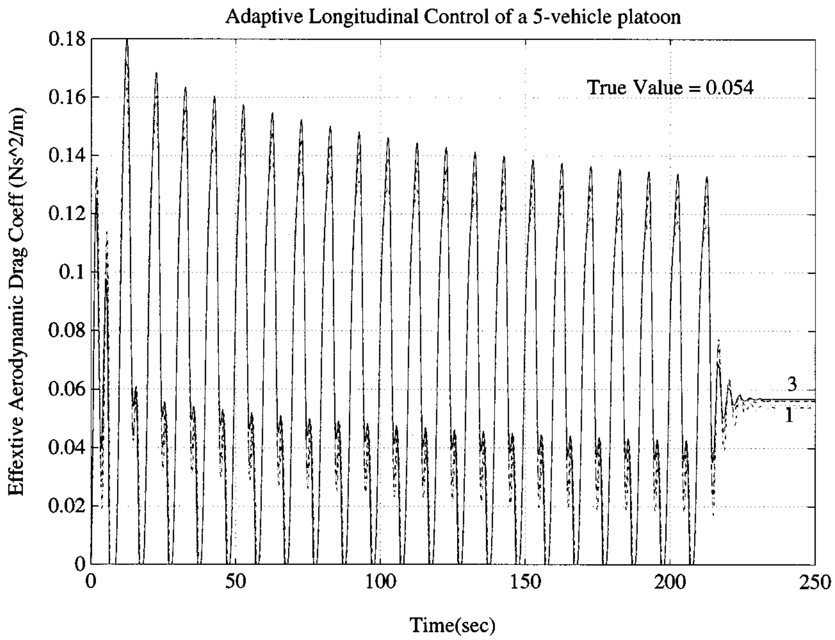


Fig. 6. Behavior of aerodynamic drag coefficient during adaptation.

For $i = 1$

$$\ddot{\epsilon}_1 + \alpha\left[\left(\frac{q_1+q_4}{1+q_3}+\lambda\right)\dot{\epsilon}_1 + \frac{\lambda(q_1+q_4)}{1+q_3}\epsilon_1\right] = (\alpha-1)\ddot{x}_l + \frac{\tilde{f}_1}{M_1}. \quad (5)$$

Proposition II.3: If:

1) the conditions in Proposition II.1 hold;

2) $\sup_i |(\tilde{f}_i/M_i)(0)|, \sup_i |\epsilon_i(0)|, \sup_i |\dot{\epsilon}_i(0)|$ are bounded;

then $\sup_i \sup_{t\geq 0} \max\{|\epsilon_i(t)|, |\dot{\epsilon}_i(t)|\}$ is bounded. (That is, the spacing errors are uniformly bounded in time and vehicle index.)

Proof: From (4) and (5) and from the definition of $\hat{G}(s)$, $\hat{H}(s)$, it follows that

$$\sup_{t\geq 0}|\epsilon_1(t)| \leq \frac{1+q_3}{\lambda(q_1+q_4)}\left[(\alpha-1)\|\ddot{x}_l\| + \sup_{t\geq 0}\left|\frac{\tilde{f}_1}{M_1}(t)\right| + (\beta_1|\epsilon_1(0)| + |\dot{\epsilon}_1(0)|)\right] + \frac{\beta_1}{\beta_2}|\epsilon_1(0)|$$

where β_1, β_2 are the minimum and maximum absolute values of the roots of $\Delta(s)\ \forall\,\alpha \in [\beta_l, \beta_h]$. For all $i \geq 2$

$$\sup_{t\geq 0}|\epsilon_i(t)| \leq \frac{q_1}{q_1+q_4}\sup_{t\geq 0}|\epsilon_{i-1}(t)| + \frac{2(1+q_3)}{\lambda(q_1+q_4)}\sup_i\sup_{t\geq 0}\left|\frac{\tilde{f}_i(t)}{M_i}\right| + C$$

where C is the constant associated with the initial conditions. Hence, $\sup_i \sup_{t\geq 0}|\epsilon_i(t)|$ is bounded.

Although the spacing errors are uniformly bounded in the presence of uncertainty in the rolling resistance moment, steady-state spacing errors are nonzero. One way to avoid the problem of steady-state errors is to incorporate integral action in the definition of auxillary error given by the first equation in Section II-A.

If there is any mismatch in the aerodynamic drag coefficient, individual vehicle stability cannot be guaranteed in the large and hence uniform boundedness of spacing errors or string stability cannot be assured. Here, we resort to parameter adaptation to improve the robustness of the control algorithm.

III. Direct Adaptive Control Algorithm

We assume that the lead vehicle performs a bounded velocity, acceleration, and jerk maneuver. In the presence of parametric uncertainty, the control effort is given by

$$u_i = \hat{c}_i\dot{x}_i^2 + \hat{f}_i + \hat{M}_i u_{\text{isl}}. \quad (6)$$

Hence

$$\dot{S}_{1i} + \lambda S_{1i} = \frac{1+q_3}{M_i}[\tilde{M}_i u_{\text{isl}} + \tilde{c}_i\dot{x}_i^2 + \tilde{f}_i]. \quad (7)$$

Define a Lyapunov function candidate

$$V_i = \frac{M_i}{1+q_3}\frac{S_{1i}^2}{2} + \gamma_1\frac{\tilde{M}_i^2}{2} + \gamma_2\frac{\tilde{c}_i^2}{2} + \gamma_3\frac{\tilde{f}_i^2}{2}. \quad (8)$$

Choose the adaptation laws as follows:

$$\dot{\hat{M}}_i = -\frac{1}{\gamma_1}S_{1i}u_{\text{isl}} \quad (9)$$

$$\dot{\hat{c}}_i = -\frac{1}{\gamma_2}S_{1i}\dot{x}_i^2 \quad (10)$$

$$\dot{\hat{f}}_i = -\frac{1}{\gamma_3}S_{1i}. \quad (11)$$

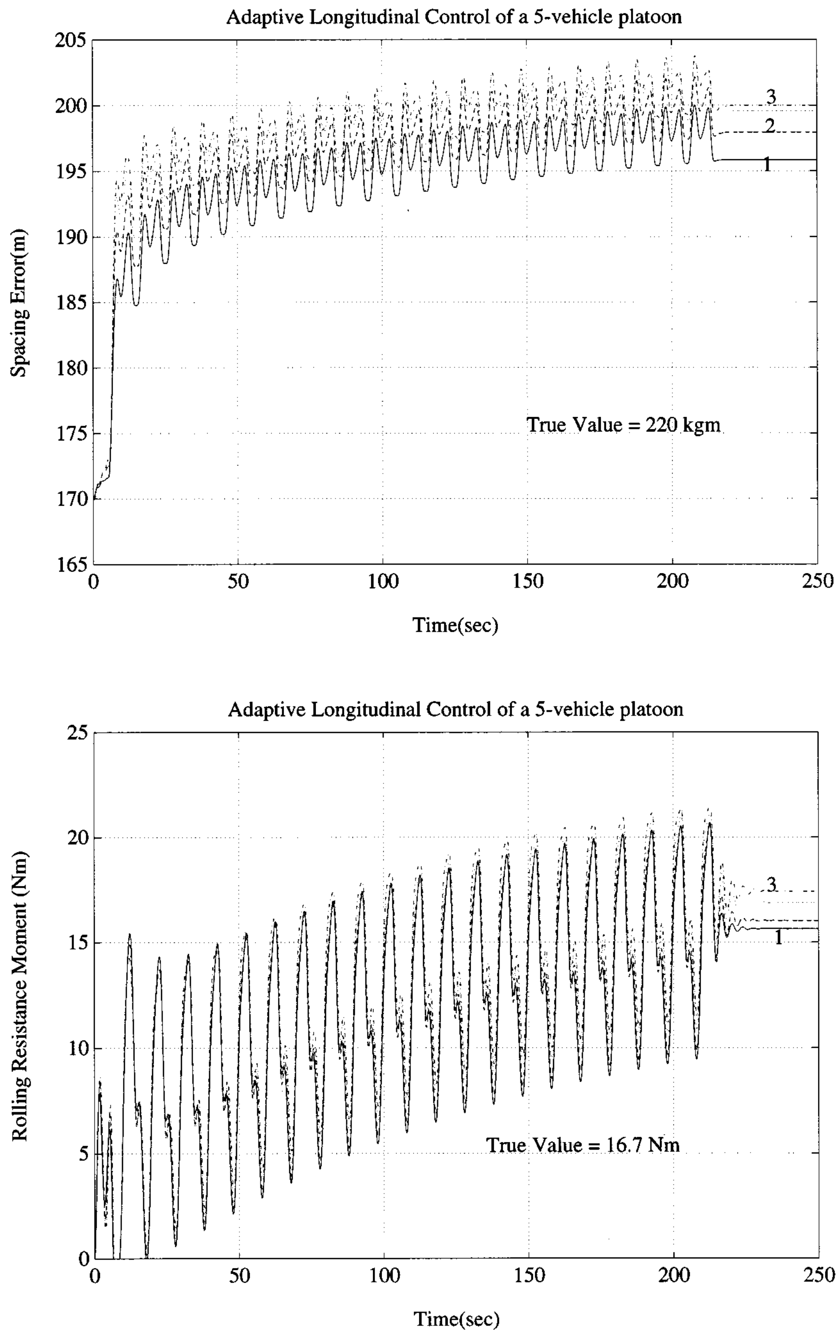


Fig. 7. Behavior of mass and rolling resistance parameters during adaptation.

The reader is referred to [12], [14], and [11] for gradient and other adaptive laws.

IV. Analysis for Uniform Boundedness of Spacing Errors and Parameter Convergence

With the choice of adaptation laws and control in the previous section, we obtain

$$\dot{V}_i = -\frac{\lambda M_i}{1+q_3} S_{1i}^2 \leq 0 \tag{12}$$

$$\ddot{x}_i = \frac{\hat{M}_i u_{\text{isl}} + \tilde{c}_i \dot{x}_i^2 + \tilde{f}_i}{M_i}. \tag{13}$$

From (8) and (12), it follows that $S_{1i} \in L_\infty \cap L_2$; $\tilde{M}_i,\ \tilde{c}_i,\ \tilde{f}_i \in L_\infty$.

A. Uniform Boundedness of Spacing Errors

Proposition IV.1: If:

1) $\sup\{|\tilde{M}_i(0)|,\ \sup_i |\tilde{c}_i(0)|,\ \sup_i |\tilde{f}_i(0)|,\ \sup_i |\epsilon_i(0)|,\ \sup_i |\dot{\epsilon}_i(0)|,\ \sup_i |\sum_{j=1}^{i} |\epsilon_j(0)|,\ \sup_i |\sum_{j=1}^{i} |\dot{\epsilon}_j(0)|$ exist;

2) $\ddot{x}_l(t) \in L_\infty$;

then the control law given by (6) together with the adaptation laws given by (9)–(11) ensure that:

1) $\sup_{t\geq 0} \sup_i \max\{|\tilde{M}_i(t)|, |\tilde{c}_i(t)|, |\tilde{f}_i(t)|\}$ is bounded;

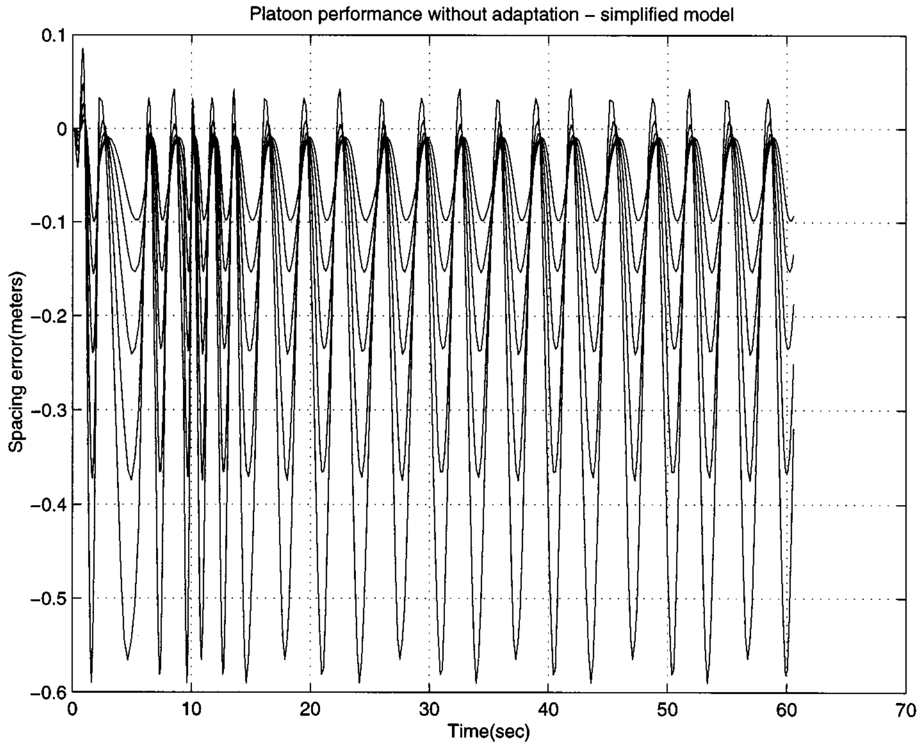


Fig. 8. Simplified vehicle model: performance of the platoon without adaptation.

2) $\sup_{t\geq 0}\sup_i \max\{|\epsilon_i(t)|, \quad |\dot{\epsilon}_i(t)|, \quad |\sum_{j=1}^{i}\epsilon_j(t)|, |\sum_{j=1}^{i}\dot{\epsilon}_j(t)|\}$ is bounded;
3) $\epsilon_i,\ \dot{\epsilon}_i \to 0$ as $t\to\infty$;
4) if, in addition, $(d\ddot{x}_l(t)/dt)\in L^\infty$, then $\ddot{\epsilon}_i\to 0$ as $t\to\infty$.

Proof: In the following proof, $S_{1j}(t)\equiv 0$ if $j\leq 0$.

1) By hypothesis, $\sup_i |V_i(0)|$ exists. Since $V_i(t)$ is decreasing, $\sup_i \sup_{t\geq 0}|V_i(t)|\leq \sup_i |V_i(0)|$. Therefore, $\sup_i \max\{|S_{1i}(t)|, |\tilde{M}_i(t)|, |\tilde{c}_i(t)|, |\tilde{f}_i(t)|\} < C^*$, where C^* is a real and positive constant.
2) $\sup_{t\geq 0}|S_{1i}(t)-S_{1(i-1)}(t)|\leq 2\sup_{t\geq 0}|S_{1i}(t)|$ and hence, $\sup_i\sup_{t\geq 0}|S_{1i}(t)-S_{1(i-1)}(t)|$ exists. Since

$$(1+q_3)\dot{\epsilon}_1+(q_1+q_4)\epsilon_1=S_{11} \tag{14}$$

$$(1+q_3)\dot{\epsilon}_i+(q_1+q_4)\epsilon_i=\dot{\epsilon}_{i-1}+q_1\epsilon_{i-1}+S_{1i}-S_{1i-1}$$

it follows that

$$\sup_{t\geq 0}|\epsilon_1(t)|\leq \frac{\sup_i\sup_{t\geq 0}|S_{1i}(t)|+(1+q_3)\sup_i|\epsilon_i(0)|}{q_1+q_4}$$

$$\sup_{t\geq 0}|\dot{\epsilon}_1(t)|\leq \frac{2}{1+q_3}\left[\sup_i\sup_{t\geq 0}|S_{1i}(t)| + (1+q_3)\sup_i|\epsilon_i(0)|\right]$$

$$\sup_{t\geq 0}|\epsilon_i(t)|\leq \frac{q_1}{q_1+q_4}\sup_{t\geq 0}|\epsilon_{i-1}(t)|+\sup_i\sup_{t\geq 0} \cdot\frac{|S_{1i}-S_{1(i-1)}(t)|+(2+q_3)\sup_i|\epsilon_i(0)|}{q_1+q_4}.$$

Hence, $\sup_i\sup_{t\geq 0}|\epsilon_i(t)|$ is bounded. Rewriting (14)

$$\dot{\epsilon}_i=\frac{1}{1+q_3}\dot{\epsilon}_{i-1}+\frac{q_1\epsilon_{i-1}-(q_1+q_4)\epsilon_i+S_{1i}-S_{1i-1}}{1+q_3}$$

$$\Rightarrow \sup_{t\geq 0}|\dot{\epsilon}_i(t)|\leq \frac{1}{1+q_3}\sup_{t\geq 0}|\dot{\epsilon}_{i-1}(t)|+(2q_1+q_4)\sup_i\sup_{t\geq 0} \cdot\frac{|\epsilon_i(t)|+\sup_i\sup_{t\geq 0}|S_{1i}(t)-S_{1(i-1)}(t)|}{1+q_3}.$$

Hence, $\sup_i\sup_{t\geq 0}|\dot{\epsilon}_i(t)|$ is bounded.

3) If $\sup_i\sup_{t\geq 0}\max\{|\tilde{M}_i(t)|, \ |\tilde{c}_i(t)|, \ |\tilde{f}_i(t)|\}$, and $\sup_i |S_i(0)|$ are sufficiently small so that for some $q<1$, $\hat{M}_i/(M_i(1+q_3))\leq q$, then u_{isl}, $\ddot{x}_i$ are uniformly bounded in time and vehicle index

$$S_{1i}=\dot{\epsilon}_i+q_1\epsilon_i+q_3r_i+q_4p_i$$

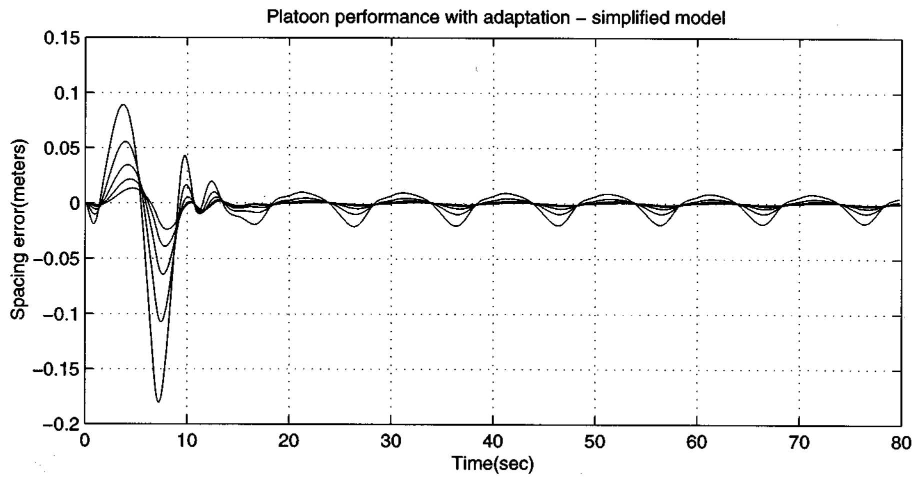
where

$$r_i(t)=\dot{x}_i-\dot{x}_l$$

$$p_i(t)=x_i-x_l+\sum_1^i L_j.$$

Since $\sup_i\sup_{t\geq 0}\max\{|S_{1i}(t)|, |\epsilon_i(t)|, |\dot{\epsilon}_i(t)|\}$ exists, it follows that

$$\sup_{t\geq 0}|p_i(t)|\leq \frac{1}{q_4}\left(\sup_i\sup_{t\geq 0}|S_{1i}(t)|+\sup_i\sup_{t\geq 0}|\dot{\epsilon}_i(t)| + q_1\sup_i\sup_{t\geq 0}|\epsilon_i(t)|\right)$$



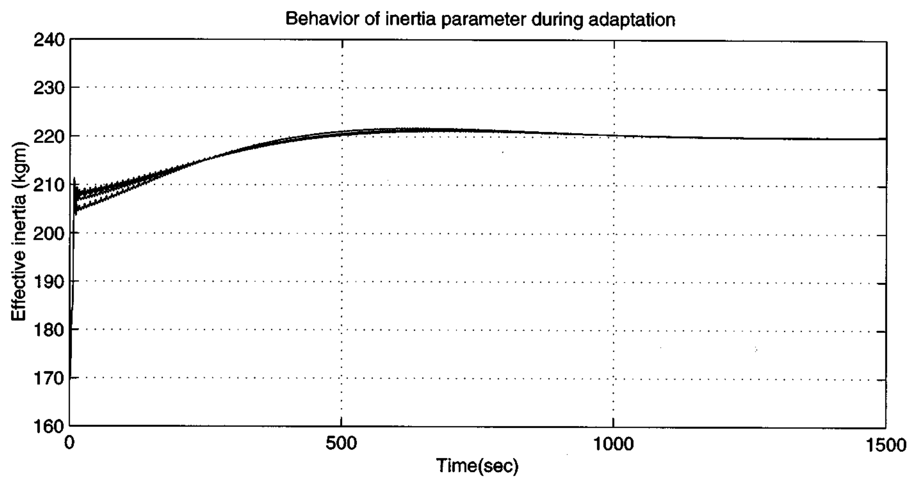


Fig. 9. Simplified vehicle model: performance of the platoon with adaptation and behavior of inertia parameter.

and hence $\sup_i \sup_{t\geq 0} \max\{|r_i(t)|, |p_i(t)|\}$ exists

$$u_{\text{isl}} = \frac{1}{1+q_3}\left[\frac{\hat{M}_i}{M_i}u_{i-1sl} + \frac{\tilde{c}_i\dot{x}_i^2 + \tilde{f}_i}{M_i} - q_1\dot{\epsilon}_i - q_4(v_i - v_l) - \lambda S_{1i}\right].$$

By hypothesis, $\hat{M}_i/(M_i(1+q_3)) < q < 1$. Since all the other terms on the right-hand side are uniformly bounded, it follows that u_{isl} is bounded uniformly in time and vehicle index. From (13), $\ddot{x}_i$ is also bounded uniformly in time and vehicle index. From (7), it follows that $\sup_i \sup_{t\geq 0} |\dot{S}_{1i}(t)|$ is bounded. Hence, by Barbalat's lemma, $S_{1i} \to 0$ asymptotically for all i.

4) ϵ_i, $\dot{\epsilon}_i \to 0$ asymptotically.

Proof: For $i = 1$

$$(1+q_3)\dot{\epsilon}_1 + (q_1+q_4)\epsilon_1 = S_{11}.$$

Since $S_{11} \to 0$ as $t \to \infty \Rightarrow \dot{\epsilon}_1$, $\epsilon_1 \to 0$ as $t \to \infty$. Therefore, $r_1(t)$, $p_1(t) \to 0$ as $t \to \infty$. Assume ϵ_j, $\dot{\epsilon}_j$, $p_j(t)$, $r_j(t) \to 0$ as $t \to \infty$ $\forall j \leq i-1$. Then

$$(1+q_3)\dot{\epsilon}_i + (q_1+q_4)\epsilon_i = S_{1i} - \sum_{j=1}^{i-1}(q_3\dot{\epsilon}_j + q_4\epsilon_j).$$

Therefore, $\dot{\epsilon}_i$, $\epsilon_i \to 0$ asymptotically. Consequently, $p_i(t)$, $r_i(t) \to 0$ asymptotically.

5) $\dot{S}_{1i}(t)$ is uniformly continuous if $(d\ddot{x}_l/dt)$, $\ddot{x}_l$, $\dot{x}_l$ are bounded and continuous.

From (3) and (13), we have

$$\dot{u}_{\text{isl}} = \frac{1}{1+q_3}\left[\ddot{x}_{i-1} + q_3\ddot{x}_l - q_1\ddot{\epsilon}_i - q_4(\ddot{x}_i - \ddot{x}_l) - \lambda\dot{S}_{1i}\right] \quad (15)$$

$$\dddot{x}_i = \frac{\dot{\hat{M}}_i u_{\text{isl}} + \dot{\hat{c}}_i\dot{x}_i^2 + \dot{\hat{f}}_i + \hat{M}_i\dot{u}_{\text{isl}} + 2\tilde{c}_i\dot{x}_i\ddot{x}_i}{M_i}. \quad (16)$$

From the above equations

$$\dddot{x}_i = \frac{\hat{M}_i}{(1+q_3)M_i}\dddot{x}_{i-1} + \frac{\hat{M}_i\left(q_3\dddot{x}_l - q_1\ddot{\epsilon}_i - q_4\left(\dddot{x}_i - \dddot{x}_l\right) - \lambda\dot{S}_i\right)}{(1+q_3)M_i} + \frac{\dot{\hat{M}}_i u_{\text{isl}} + \dot{\hat{c}}_i\dot{x}_i^2 + \dot{\hat{f}}_i + 2\tilde{c}_i\dot{x}_i\ddot{x}_i}{M_i}.$$

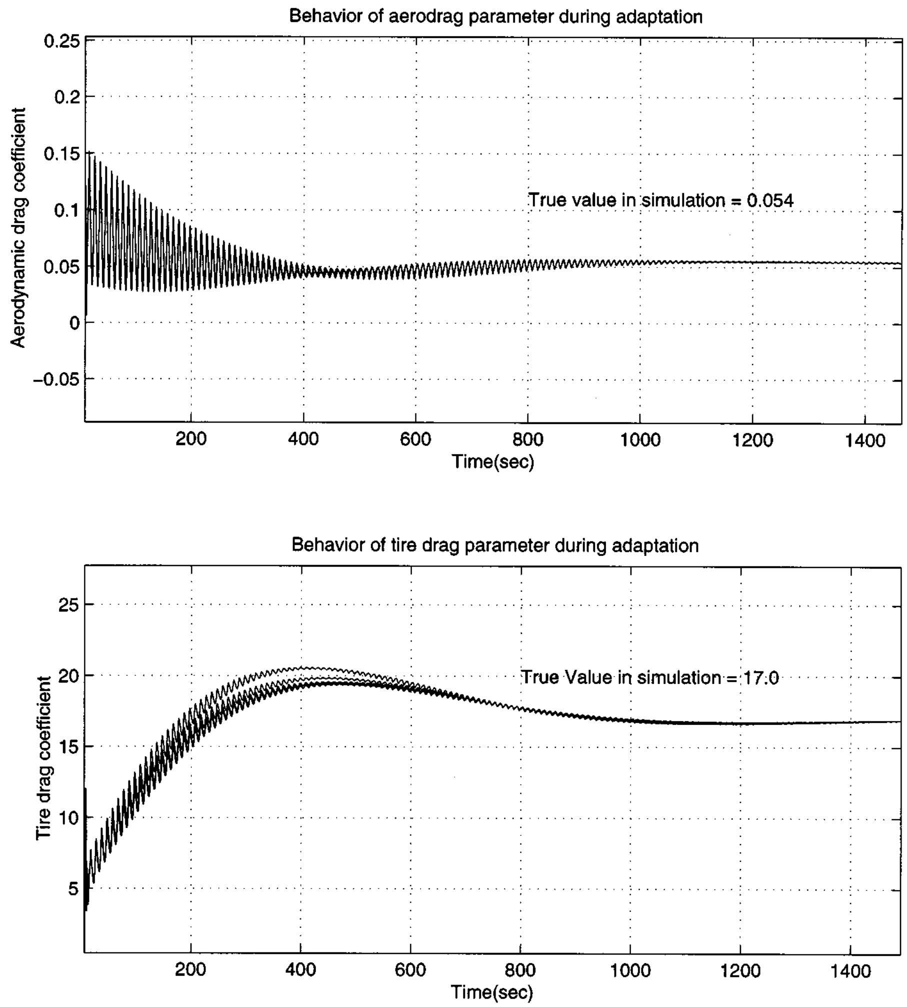


Fig. 10. Simplified vehicle model: behavior of aerodrag and tire drag coefficients during adaptation.

Since $\hat{M}_i/((1+q_3)M_i) < q < 1$, and all the terms on the right-hand side of the above equation are uniformly bounded, it follows that $\ddot{x}_i$ and, hence, $\dot{u}_{\text{isl}}$ are uniformly bounded. Since

$$\ddot{S}_{1i} = -\lambda\dot{S}_{1i} + \frac{1+q_3}{M_i} \cdot \left[\dot{\hat{M}}_i u_{\text{isl}} + \dot{\hat{c}}_i\dot{x}_i^2 + \dot{\hat{f}}_i + \tilde{M}_i\dot{u}_{\text{isl}} + 2\tilde{c}_i\dot{x}_i\ddot{x}_i\right]$$

it follows that $\ddot{S}_{1i}$ is uniformly bounded. Since S_{1i} and the right-hand side of (7) is continuous, it follows that $\dot{S}_{1i}$ is continuous, and hence, $\dot{S}_{1i}$ is uniformly continuous. Since

$$\lim_{t\to\infty}\left|\int_0^t \dot{S}_{1i}(\tau)\,d\tau\right| = |S_{1i}(0)| \le \|S_1\|_\infty$$

and $\dot{S}_{1i}$ is uniformly continuous, it follows, by Barbalat's lemma, that $S_{1i} \to 0$ asymptotically. Therefore, from the definition of $S_{1i}(t)$, $\ddot{\epsilon}_i \to 0$ asymptotically.

B. Convergence of Parameters

A heuristic argument is provided for the convergence of parameters in this section. Since $\dot{S}_{1i} + \lambda S_{1i} \to 0$ asymptotically, $\tilde{M}_i u_{\text{isl}} + \tilde{c}_i\dot{x}_i^2 + \tilde{f}_i \to 0$ asymptotically. For the convergence of parameters, the persistence of excitation condition must be satisfied. Let $W_i = [u_{\text{isl}} \;\; \dot{x}_i^2 \;\; 1]^T$. Then, there must exist three positive constants δ, γ_1, γ_2 such that

$$\gamma_2 I \ge \int_t^{t+\delta} W_i W_i^T\,d\tau \ge \gamma_1 I \qquad \forall\, t \ge 0.$$

Since ϵ_i, $\dot{\epsilon}_i$, $\ddot{\epsilon}_i \to 0$ asymptotically, $u_{\text{isl}} \approx \ddot{x}_l$, $\dot{x}_i^2 \approx \dot{x}_l^2$. If $\dot{x}_l = A_0 + A_1\sin(wt)$, where $A_0 > A_1 > 0$, choosing $\delta = 12\pi/w$, we have the equation shown at the top of the next page. The matrix below is positive definite $\forall\, A_1 > 0$. For this reason, we expect that the parameter estimates converge to their true values.

V. Simulation Results

Simulations are performed for a five-vehicle platoon. The vehicle plant model in the first set of simulations, from Figs. 1–5, considers the effects of slip between the tire and the ground, slip across the torque converter, manifold air dynamics, and the lag in the brake torque, all of which are neglected in developing the controller. In all of the simulations, all of the vehicles in the platoon start with zero initial position velocity errors. The lead ve-

$$\int_t^{t+\delta} W_i W_i^T \, d\tau \approx \begin{pmatrix} 6\pi A_0^2 w & 0 & 0 \\ 0 & \left(\left(A_0^2 + \frac{A_1^2}{2}\right)^2 + \frac{A_1^4}{8} + 2A_0^2 A_1^2\right) \frac{12\pi}{w} & \left(A_0^2 + \frac{A_1^2}{2}\right) \frac{12\pi}{w} \\ 0 & \left(A_0^2 + \frac{A_1^2}{2}\right) \frac{12\pi}{w} & \frac{12\pi}{w} \end{pmatrix}$$

hicle in the platoon makes the following acceleration maneuver:

$$a_l(t) = \begin{cases} 0, & 0 \le t \le 5 \\ -1.2 \sin\left(\frac{2\pi(t-5)}{10}\right), & 5 \le t \le 5 + 10N \\ 0, & t \ge 5 + 10N \end{cases}$$

where N is a positive integer. The following gains are used for simulating the five-vehicle platoon: $q_1 = 1$, $q_3 = 1$, $q_4 = 0.5$, $\lambda = 1.0$, $\gamma_1 = 28.0$, $\gamma_2 = 0.008$, $\gamma_3 = 30.0$. The set of gains chosen are representative of the gains chosen in the experiments conducted at Berkeley. However, in the experiments, $q_4 = 0$ (i.e., relative position information of the lead vehicle in the platoon was never used). In all the figures that follow, the number "i" on the figure represents the plot for the ith following vehicle. Fig. 2 illustrates the effect of uncertainty in mass of the vehicle on the platoon performance. α, shown in the figure, denotes the ratio of the estimated mass used in the controller and the true mass. As expected, the peak spacing errors decrease geometrically at a ratio of 2/3 with vehicle index. The spacing errors decay to zero. Fig. 3 depicts uniform boundedness of spacing errors in the presence of uncertainty in the mass and rolling resistance. The controller's mass estimate is 20% less than the actual estimate, and the rolling resistance estimate is 0.0 Nm. We have nonzero steady-state spacing errors in this case. Since all the vehicles are identical (including the estimates), $\phi_{1i}(t) \equiv 0 \ \forall i \ge 2$. Hence, by (4), the steady-state spacing errors decrease with vehicle index.

Fig. 4 describes how uncertainty in aerodynamic drag coefficient affects the performance of the platoon. For this simulation, the controller has no knowledge of the aerodynamic drag coefficient and the rolling resistance moment, i.e., $\hat{c}_i = 0$, $\hat{f}_i = 0$. As in the previous case, the estimate of mass is 20% less than its true value. The maximum spacing errors and the steady-state spacing errors are higher than before due to the additional uncertainty. Fig. 5 demonstrates the effectiveness of the adaptive controller. It can be seen that the errors go to zero and the maximum spacing errors are significantly smaller compared to the nonadaptive case. Also, the peak spacing errors decrease monotonically with vehicle index. Figs. 6 and 7 show how the parameters behave during adaptation. Parameters do not converge, but oscillate in the neighborhood of their true values. This is due to the fact that the controller model neglects four states associated with torque converter, manifold air dynamics, slip between the tire and the wheels, and lag in delivering the desired brake torque.

The vehicle plant model considered in the latter set of simulations (from Figs. 6–10) is a simplified model described by (1) in Section II. The corresponding plots of performance with and without adaptation are shown in Figs. 6 and 7. The plots depicting parameter behavior are shown in Figs. 7 and 8.

The parameters converge to their true values when the simplified model is used as a vehicle plant model. These simulation results are in concurrence with the heuristic argument.

VI. Conclusion

In this paper, we have investigated the effect of parametric uncertainty on the platoon performance. In order to improve the performance of the platoon in the presence of model uncertainty, we developed a decentralized adaptive control algorithm. It guarantees zero steady-state spacing errors and uniform boundedness of spacing errors under some mild assumptions. The estimated parameters do not converge to their true values when a detailed vehicle plant model is used, even if the lead vehicle trajectory is persistently exciting. This is due to the fact that the design of the controller is based on a simplified model. To clarify this claim, we conducted simulations with the simplified model as the vehicle plant model. In this case, the parameters converge to their true values, in concurrence with the heuristic argument, if the lead vehicle trajectory is persistently exciting.

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