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REFERENCE CLUTCH TORQUE TRAJECTORY GENERATION FOR CONTROL OF A DUAL-CLUTCH TRANSMISSION USING CONTROL ALLOCATION

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KEYWORDS – Dual-clutch transmission, Control allocation, Gearshift control, Reference clutch torque trajectory, Torque-based clutch control

ABSTRACT –
Clutch-to-clutch shift control should be carried out carefully in order to ensure a fast and comfortable gearshift. To achieve the high control performance, various studies have been conducted on torque-based control of DCTs. However, research on how to determine desired torque trajectories for control is still absent even though many papers dealt with feedback control of two clutches for a DCT to generate proper clutch torques. In this paper, a novel method for generating reference clutch torque trajectory for control of DCTs using optimization-based control allocation is proposed. Actually, the suggested method plays a role as the upper-level controller to determine desired torque trajectory of each clutch in the whole DCT control while the lower-level controller performs torque tracking control considering the dynamics of clutch actuators. The effectiveness of the method is verified through various simulations based on the proven dry DCT model.

TECHNICAL PAPER –

INTRODUCTION

As public concerns have been raised over global environmental issues, automakers have been strongly demanded to develop efficient and environmental-friendly powertrain systems. Among them, invention and development of dual-clutch transmissions (DCT) have shown significant improvements regarding the fuel efficiency. DCTs use two clutches to transmit engine torque to the transmission which can overcome drawbacks of conventional planetary-type automatic transmissions (AT) and automated manual transmissions (AMT) with one clutch. Gear shifting process of DCTs is carried out through torque transferring from one clutch to the other clutch (clutch-to-clutch shift) without a torque converter leading to more energy-efficient and comfortable shift than other types of transmissions.

High performances of the gearshift can be guaranteed only when accompanied by sophisticated clutch controls. The problems of clutch control for AMTs have been dealt with in various papers. Optimal linear quadratic controllers are developed for a dry AMT clutch considering different gear shifting performances in [1], and a clutch slip controller for the same application is designed to regulate the slip acceleration [2]. Also, a gearshift control strategy for AMTs is proposed by interpreting gearshift of AMTs as 5 different phases considering transient behaviors of the system in each phase [3]. The basic operating principle of DCTs is similar to that of AMTs, but DCTs are equipped with two clutches and transfer shafts. In the case of DCTs, gearshifts are performed through clutch-to-clutch shift, as is the case in conventional ATs, which has great potential to minimize the awkward shift impacts of AMTs. However, clutch-to-clutch shift must be realized with much greater cares than the case of AMTs by controlling two clutch actuators properly to ensure short gear shifting time and smooth gearshift at the same time. Many studies have been conducted on control of clutch-to-clutch shift for hydraulic ATs [4-7] and for DCTs [8-13]. Most of the papers are dealing with feedback control strategies in order to overcome limitations of conventional open-loop control methods. Especially in [4-7, 10-12], torque-based control strategies have been developed for clutch-to-clutch shift because clutch torques or output shaft torque directly affects shift quality. Since torque sensors are not available on the production DCTs, estimation methods of individual clutch torque are also investigated in other papers [14, 15].

Even though most previous papers have adopted tactics of torque-based feedback control for the DCT clutch control, research on how to determine desired torque of each clutch during gearshift is still missing. Therefore, a novel method of generating individual desired clutch torque using control allocation is proposed in this paper. Control allocation is an effective way to control over-actuated systems by allocating the total control demands to the individual actuators as its name indicates [16, 17]. The gear shifting process of DCTs can interpreted as the combination of two phases: one is torque phase where the torque is transferred from the engaged clutch to the on-coming clutch, and the other one is inertia phase where the on-coming clutch is being synchronized with the engine. During the gearshift of a DCT, regulation of output shaft torque, which directly determines shift comfort, is accomplished via control of the two clutches. Hence, the DCT can be interpreted as an over-actuated system with two clutches as actuators and one control output. By adopting the optimization-based control allocation method, we can produce each optimized clutch torque trajectory by simply tuning weighting factors between the two conflicting performances: short shifting time and shift comfort. The resulting torque trajectories can be utilized as reference values for various torque-based feedback controls. Actually, determining desired clutch torque values from certain set-point can be regarded as the upper-level control of the whole DCT clutch control while tracking the torque values by controlling each clutch actuator properly is referred as the lower-level one, as described in Figure 1. The lower-level control of the DCT, which have already been dealt with in many previously published papers, is out of scope in this paper. The feasibility and effectiveness of the suggested method will be verified through various simulations based on a valid DCT driveline model.

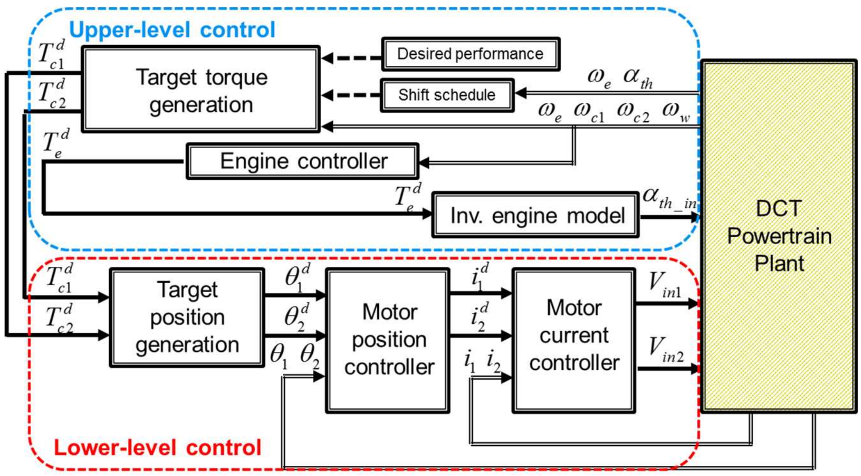


Figure 1: General structure of dry DCT powertrain controls

DCT DRIVELINE MODEL

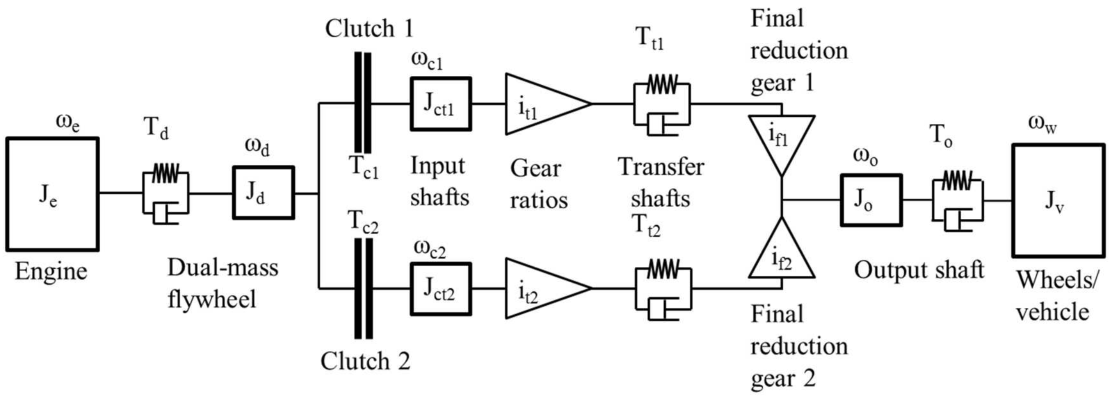


Figure 2: Schematic of the DCT driveline model

Dynamic model of a DCT driveline is briefly introduced in this section. Since this paper does not focus on detailed modeling of DCTs, for anyone interested in it, please refer to other studies, e.g., [18, 19]. Torque generated by the engine is tramitted through transmission systems to output shaft with wheels, and the DCT has two sets of input and transfer shafts between the engine and the output shaft. The driveline model consists of several angular speed dynamics based on the torque balance relations for each component. The variables J , T , ω, and θ are inertia, torque, angular speed, and rotation angle, respectively. The subscripts e , d , $c1$, $c2$, $t1$, $t2$, o, w, and v stand for engine, dual-mass flywheel, input shaft with clutch 1, input shaft with clutch 2, transfer shafts, output shaft, wheel, and vehicle. The dynamics of engine and dual-mass flywheel are expressed as (1)-(2).

$$J_e\dot{\omega}_e = T_e - T_d, \tag{1}$$
$$J_d\dot{\omega}_d = T_d - T_{c1} - T_{c2}. \tag{2}$$

where torque delivered by the engine is a function of throttle position and engine speed i.e., $T_e = f\left(\alpha_{th}, \omega_e\right)$.

In fact, many simplified models for control applications usually ignore the inertia effect of the dual-mass flywheel, and the total inertia of engine and dual-mass flywheel is regarded as one lumped engine inertia in that case. The dual-mass flywheel can be modelled as a torsional damper so that the torque transmitted through it is described as (3)

$$T_d = k_d\left(\theta_e - \theta_d\right) + b_d\left(\omega_e - \omega_d\right), \tag{3}$$

where k_d and c_d are torsional stiffness and damping coefficient of the dual-mass flywheel.

Likewise, defining effective inertias from clutch 1, 2 perspectives as J_{ct1} , J_{ct2} , dynamics of each transfer shaft can be depicted as (4)-(5), respectively,

$$J_{ct1}\dot{\omega}_{c1} = T_{c1} - \frac{T_{t1}}{i_{t1}}, \tag{4}$$
$$J_{ct2}\dot{\omega}_{c2} = T_{c2} - \frac{T_{t2}}{i_{t2}}, \tag{5}$$

where i_{t1} is the gear ratio of input and transfer shafts 1, and i_{t2} the one of input and transfer shafts 2.

Torque transferred through the clutches is modelled by (6)-(7) depending on states of the clutches:

$$T_{c1} = \begin{cases} 0 & \text{, when disengaged} \\ \mu_1 R_{c1} F_{n1} \operatorname{sgn}(\omega_d - \omega_{c1}) & \text{, when slipping} \\ T_d - T_{c2} - J_d \dot{\omega}_d & \text{, when engaged} \end{cases} \tag{6}$$

$$T_{c2} = \begin{cases} 0 & \text{, when disengaged} \\ \mu_2 R_{c2} F_{n2} \operatorname{sgn}(\omega_d - \omega_{c2}) & \text{, when slipping} \\ T_d - T_{c1} - J_d \dot{\omega}_d & \text{, when engaged} \end{cases} \tag{7}$$

where the parameters μ , R_c , F_n stand for dynamic friction coefficient, effective area, actuator normal force of each clutch.

In (4)-(5), the transfer shaft torques are represented as (8)-(9):

$$T_{t1} = k_{t1}\left(\frac{\theta_{c1}}{i_{t1}} - i_{f1}\theta_o\right) + b_{t1}\left(\frac{\omega_{c1}}{i_{t1}} - i_{f1}\omega_o\right), \tag{8}$$

$$T_{t2} = k_{t2}\left(\frac{\theta_{c2}}{i_{t2}} - i_{f2}\theta_o\right) + b_{t2}\left(\frac{\omega_{c2}}{i_{t2}} - i_{f2}\omega_o\right), \tag{9}$$

where the final reduction gear ratios are denoted as i_{f1}, i_{f2} , and k_{t1}, k_{t2}, c_{t1} and c_{t2} represent torsional stiffness, damping coefficient of each transfer shaft.

The speed dynamics of output shaft and wheel are also described using the principle of torque balance, as shown next.

$$J_o \dot{\omega}_o = i_{f1} T_{t1} + i_{f2} T_{t2} - T_o, \tag{10}$$

$$J_v \dot{\omega}_w = T_o - T_v. \tag{11}$$

The transient behavior of the output shaft can be described using the torsional spring model in the same way as the transfer shafts. Hence, the output shaft torque is calculated using (12)

$$T_o = k_o(\theta_o - \theta_w) + b_o(\omega_o - \omega_w), \tag{12}$$

where k_o and c_o stand for torsional stiffness and damping coefficient of the output shaft.

REFERENCE CLUTCH TORQUE TRAJECTORIES

Overview

A novel method to assign each desired clutch torque values as references for control of clutch-to-clutch shift is proposed in this section. This paper focuses on clutch-to-clutch shift control of DCTs, but this methodology can also be applied to gearshift control of conventional hydraulic ATs. Gearshift of transmissions should be realized quickly and also smoothly to minimize shift transients and ensure comfort, simultaneously. Desired gear shifting performances in torque and inertia phases are summarized in Table 1.

Performance	Torque phase	Inertia phase
Comfort	Minimize torque dip	Minimize lock up oscillation
Shift time	Minimize cross-shift time	Minimize slipping time
Necessary Constraints	Prevent clutch tie-up	
	Prevent engine flare	

Table 1: Control objectives of clutch-to-clutch shift in each phase

In general, fast gearshifts increase unwanted output torque oscillations, denoted as the lock-up oscillation in Table. 1. In torque phase, poor control of clutch-to-clutch shift may result in large torque dip or other undesirable phenomena such as clutch tie-up and engine flare. Examining the DCT dynamics presented in the previous section, it is easily seen that two clutch torques work as control inputs to the driveline model. An optimization-based control allocation method is adopted by considering the DCT as an over-actuated system with the clutch torques as control inputs and output shaft torque as an output. Using this method, the desired control inputs (clutch torques) are automatically determined in accordance with the desired gear shifting performances and also current states of the system.

Optimization-based control allocation

Control allocation is an effective control method for over-actuated systems where some actuator redundancy exists to distribute the total control demand into each actuator. Basic linear control allocation problems can be simply expressed as (13) [16]:

$$\begin{aligned} &Bu(t) = v(t), \\ &u_{\min} \le u(t) \le u_{\max} \end{aligned} \tag{13}$$

where $v(t) \in \mathbb{R}^k$ is the virtual control input representing the total control demand and $u(t) \in \mathbb{R}^m$ the actual control input and B is the control effectiveness matrix which defines the relationship of u and v. We can set actuator constraints of u including its rate constraints by $u_{\min}$ and $u_{\max}$. For over-actuated systems, it should be satisfied that $m > k$. The objective of control allocation is to determine the constrained actual input $u(t)$ satisfying (13) provided that the virtual input $v(t)$ is given (usually by other control law). In this paper, optimization-based control allocation using weighted least squares (WLS) is utilized for the DCT control among various control allocation methods. The WLS method is well known for its smaller computation time than other methods, and can be formulated as:

$$u_{CA} = \arg \min_{u_{\min} \le u \le u_{\max}} \left(\left\| W_u \left(u - u_d \right) \right\|^2 + \gamma \left\| W_v \left(Bu - v \right) \right\|^2 \right), \tag{14}$$

where u_d is the desired control inputs, and γ, W_u, W_v are weighting factors.

Equation (14) calculates the optimized feasible input u_{CA} that minimizes $u - u_d$ and $Bu - v$ simultaneously in accordance with the weighting factors γ, W_u, and W_v. Here, the weighting factor γ is usually fixed just expressing priority of minimizing $W_v\left(Bu - v\right)$ (assume $\gamma = 1$ in this paper).

Control allocation for clutch-to-clutch shift

Rearranging (4) and (5) with respect to T_{t1} and T_{t2} respectively, (15) and (16) are derived.

$$T_{t1} = i_{t1}\left(T_{c1} - J_{ct1}\dot{\omega}_{c1}\right), \tag{15}$$
$$T_{t2} = i_{t2}\left(T_{c2} - J_{ct2}\dot{\omega}_{c2}\right). \tag{16}$$

Substituting (15) and (16) into (10),

$$J_o\dot{\omega}_o = i_{t1}i_{f1}T_{c1} + i_{t2}i_{f2}T_{c2} - i_{t1}i_{f1}J_{ct1}\dot{\omega}_{c1} - i_{t2}i_{f2}J_{ct2}\dot{\omega}_{c2} - T_o. \tag{17}$$

Let's define the virtual input as the lumped clutch torques multiplied by gear ratios, i.e., $v \triangleq i_{t1}i_{f1}T_{c1} + i_{t2}i_{f2}T_{c2}$. Then, v indicates the total torque efforts of both clutches on the output shaft. Here, because T_{c1} and T_{c2} are the actual inputs we want to allocate, v can be represented as (18) using the notations of (13).

$$v = Bu = \begin{bmatrix} i_{t1}i_{f1} & i_{t2}i_{f2} \end{bmatrix}\begin{bmatrix} T_{c1} \\ T_{c2} \end{bmatrix}. \tag{18}$$

Then, Equation (17) is re-arranged to isolate v, as shown next.

$$v = i_{t1}i_{f1}J_{ct1}\dot{\omega}_{c1} + i_{t2}i_{f2}J_{ct2}\dot{\omega}_{c2} + J_o\dot{\omega}_o + T_o. \tag{19}$$

Given desired output torque values T_o^d, the corresponding v is calculated using (19).
The strategies of the control allocation in the case of power-on upshift from 1 to 2 are as follows.

1. The desired output torque is designed as the smooth trajectory consisting of the steady state torque values when engaged with the first and second gears.

2. Desired actual inputs $u_d = \begin{bmatrix} u_{1d} & u_{2d} \end{bmatrix} = \begin{bmatrix} T_{c1d} & T_{c2d} \end{bmatrix}$ are determined by force trajectories simply ramped up and down with desired shift time t_s. If there are many solutions of u_1 and u_2 satisfying $Bu = v$, the WLS algorithm picks the closest ones to u_{1d} and u_{2d}.

3. The input constraints and its rate constraints are as shown next:
(1) During torque phase, the off-going clutch is engaged, so its torque is constrained by $u_1 = T_e - J_{e+d}\dot{\omega}_e - u_2$ referring to Equation (6).
(2) Signs of rates of the clutch torques should not be changed which means the engaged (off-going) clutch should always move in the direction its torque is decreasing, and the on-coming one should move in opposite way, i.e., $\dot{u}_1 \le 0,\ \dot{u}_2 \ge 0$ to ensure smooth cross-shift [10].
(3) Both clutch torques should not have negative values to prevent clutch tie-up or avoid backward power recirculation, i.e., $u_1 \ge 0,\ u_2 \ge 0$.
(4) The transmitted torque through a clutch should be always smaller than its torque capacity, i.e., $u_1 \le T_{c1\_\max}$, $u_2 \le T_{c2\_\max}$.

(5) The rates of clutch torques are constrained by bandwidths of the corresponding clutch actuators, $|\dot{u}_1| \le \rho_1$, $|\dot{u}_2| \le \rho_2$.

Now, the control allocator calculates the feasible clutch torques using the WLS algorithm (14). By tuning W_u and W_v freely (weighting between fast gearshift and comfortable one), the optimized torque trajectories with desired performances are produced. The feasibility of the proposed method will be verified through model simulations in the next section.

SIMULATIONS

In this simulations, u_d is made based on the normal forces ramped up and down with cross shift time t_s. The force values are processed by a low pass filter for smoothing the trajectories, and the corresponding u_d is calculated using (6) and (7) depending on states of the clutches; that is, the clutch 1 is engaged(disengaged after torque phase) and the on-coming clutch 2 is slipping during the gearshift. This procedure is briefly illustrated in Figure 3.

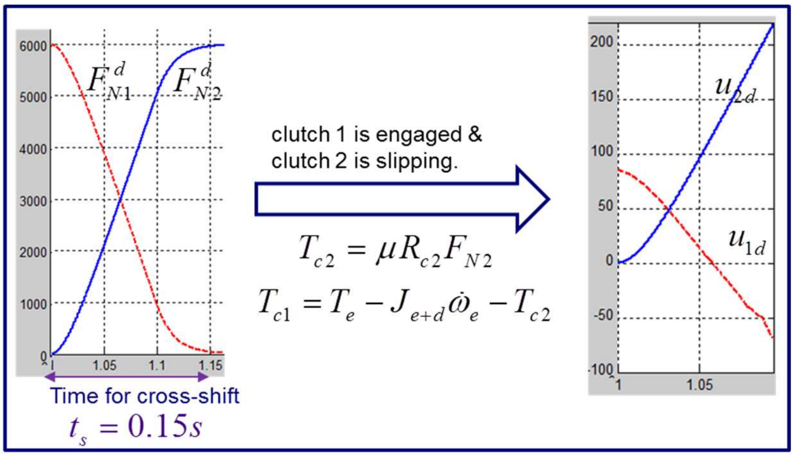
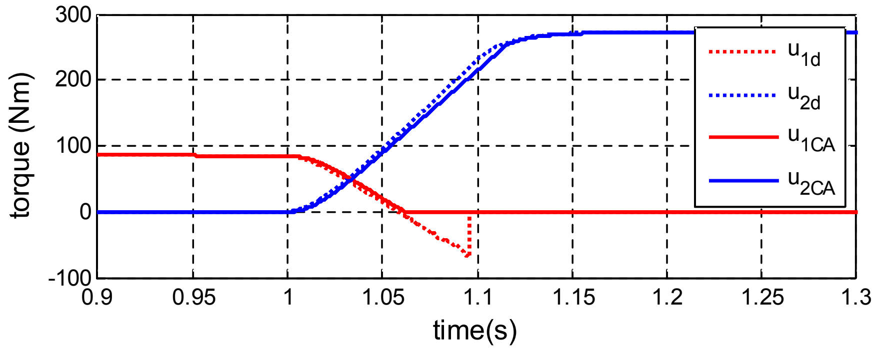


Figure 3: Design of u_d for control allocation

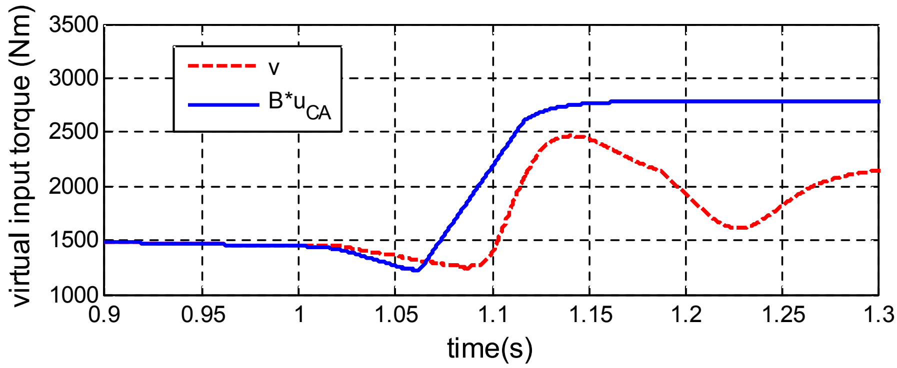
Note that we assume $\omega_e = \omega_d$ during the gearshift, and $T_{c1} = T_e - J_{e+d}\dot{\omega}_e - T_{c2}$ is used to calculate u_{1d} where J_{e+d} is the total inertia of engine and dual-mass flywheel because ω_d and T_d cannot be measured on the production transmissions. The simulation condition is 1-2 power-on upshift where the throttle position is controlled separately to reduce slip of the clutch 2 during the inertia phase.
Some simulations are conducted for different combinations of W_v and W_u to verify the feasibility of the suggested method. The DCT model for simulations consists mainly of the dynamics explained in section 2, and its accuracy is experimentally validated in [14].

<u>*Developing reference clutch torques*</u>

Case #1: $W_u = 10I$, $W_v = 1$
In the case #1, we want the control allocator to generate desired clutch torques close to u_d rather than them satisfying $v = Bu$ by assigning larger value on W_u. The results are plotted in Figure 4.



(a)



(b)

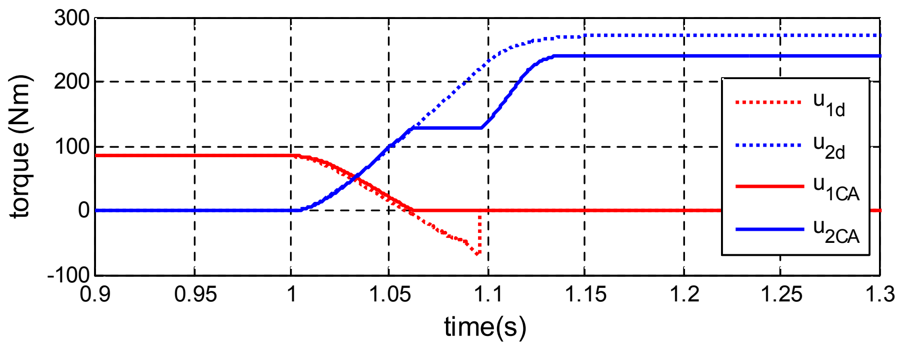
Figure 4: Control allocation results when $W_u = 10I$, $W_v = 1$

(a) Comparison of u_{CA} *and* u_d *(b) Comparison of* Bu_{CA} *and* v

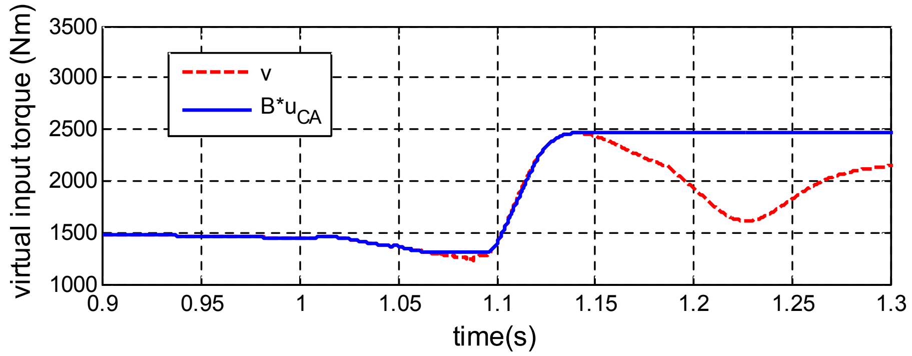
Here, good tracking of u_d indicates the resulting torque trajectories u_{CA} are close to u_d so that gearshift can be achieved as fast as the desired shift time t_s. On the other hand, if Bu_{CA} is close to v, the resulting clutch torques ensure comfortable gearshift by letting the corresponding output torque track the desired one accurately. In Figure 4, u_{1CA}, u_{2CA} are very close to u_{1d}, u_{2d}, as expected, except that u_{1d} is saturated by 0 after 1.05s due to the actuator constraints. On the other hand, Bu_{CA} does not track v well in both torque phase and inertia phase.

Case #2: $W_u = I$, $W_v = 10$

In case #2, the weighting factors are assigned in the completely opposite way to case #1. The control allocation results are shown in Figure 5.



(a)



(b)

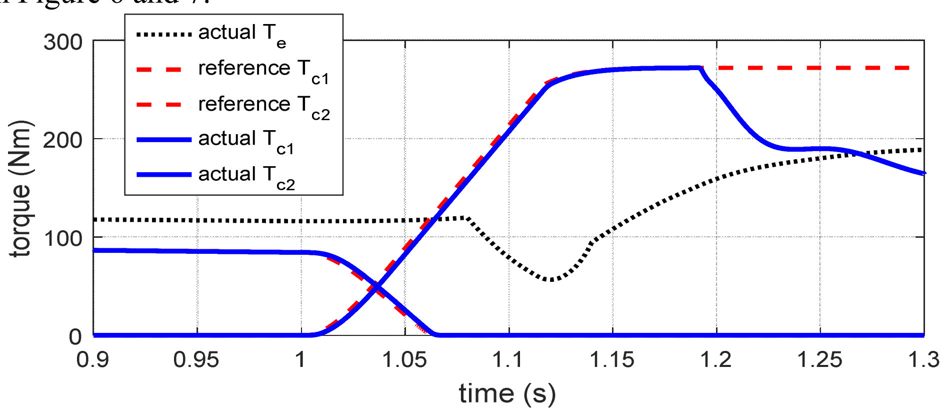
Figure 5: Control allocation results when $W_u = I$, $W_v = 10$

(a) Comparison of u_{CA} and u_d (b) Comparison of Bu_{CA} and v

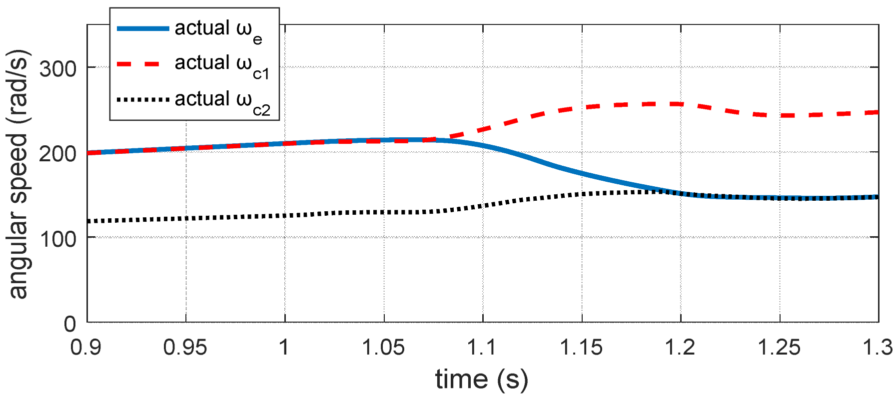
Before 1.06s, the resulting clutch torques are almost same as u_{1d} and u_{2d}, and also satisfy $Bu_{CA} = v$ at the same time. However, after that, the gap between u_{CA} and u_d is increasing while Bu_{CA} is still close to v. The torque of the on-coming clutch 2 is maintained for a while, and increases again to ensure smooth lock up, which is agreeable to the results of [10]. After 1.15s, u_{CA} cannot satisfy $Bu_{CA} = v$ any more because of the rate constraints of the clutch actuators described in the previous section indicating the corresponding desired output shaft torque to track is infeasible. Tracking it accurately is possible only when the torque of the clutch 2 is decreased again which is undesirable. Even though the shift time is increased a little, more comfortable gearshift is expected to be realized via the derived clutch torques.

<u>*Control results based on the developed torque trajectories*</u>

In this sub-section, simulations are conducted to show the actual control results of the DCT plants using the developed reference clutch torques. The reference clutch torques were converted to the corresponding force trajectories as inputs to the system. When each set of the force inputs of the case #1, 2 are applied, the resulting torque and speed responses are shown in Figure 6 and 7.

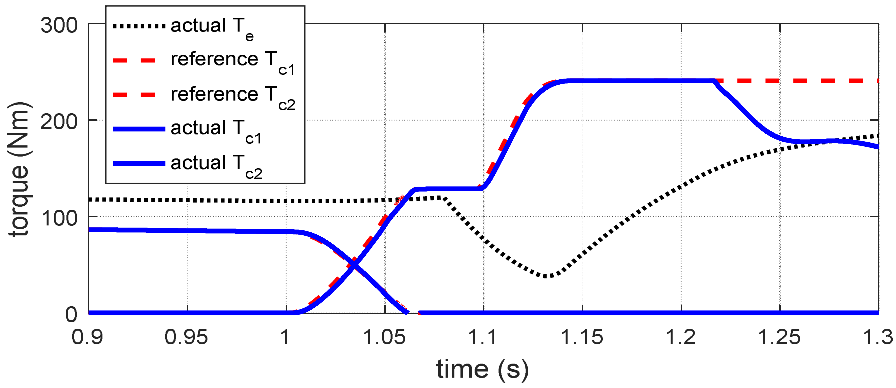


(a)

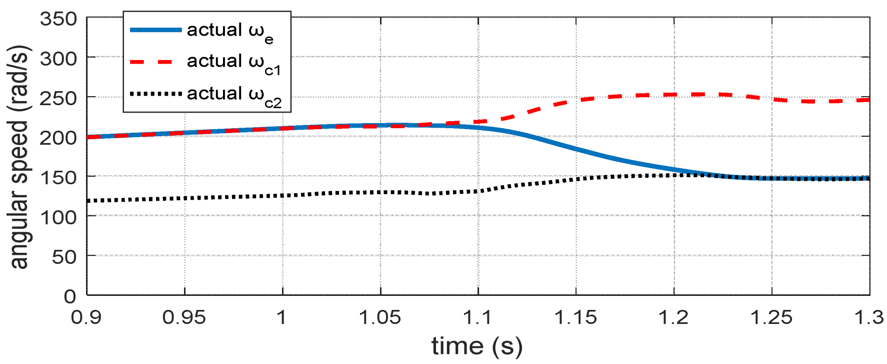


(b)

Figure 6: Actual control results in the case #1 (faster cross-shift expected)
(a) Engine, clutch torques (b) Angular speed



(a)



(b)

Figure 7: Actual control results in the case #2 (more comfortable shift expected)
(a) Engine, clutch torques (b) Angular speed

Now, the clutch torque trajectories developed in the previous sub-section are used as the reference trajectories for clutch-to-clutch shift control in the simulations (denoted as reference Tc1, 2 in Figure 6-7). The engine torque is abruptly reduced at the beginning of inertia phase by the engine speed controller to help the fast synchronization of the clutch 2. After 1.2s, the clutch 2 torque for both cases is reduced to near the engine torque as the clutch 2 is fully engaged with the engine. The maximum torque transmitted through the clutch 2 in case #2 is bigger than that of case #1, which leads to larger lock-up oscillations of the output torque. On the other hand, faster clutch-to-clutch shift is realized in case #1, as expected.

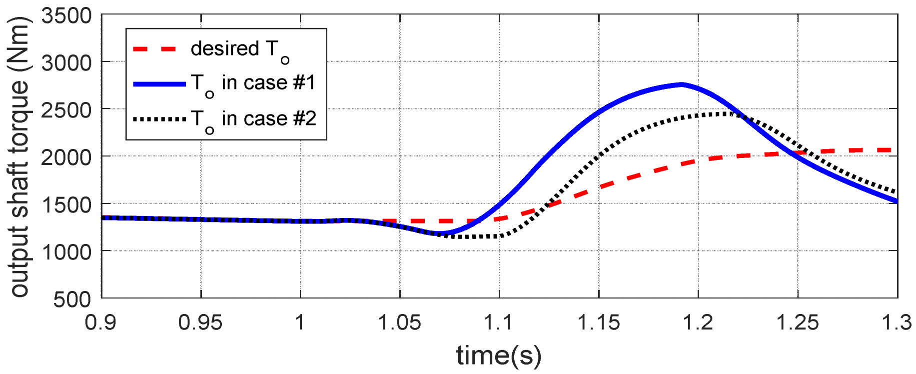


Figure 8: Comparison of output shaft torques in cases # 1, 2

Figure 8 compares responses of output shaft torques of the two cases. The desired output torque based on the steady state torque values is ideal trajectory without torque dip and lock up oscillation. It can be easily seen that output torque of case #2 shows much smaller lock up overshoot than that of case #1. In fact, it can be also seen that the torque dip of case #1 is a little smaller than that of case #2, since faster cross shift results in smaller losses of output torque in torque phase. However, much more comfortable gearshift is carried out in the case #2 with the similar amount of the torque dip and much smaller lock oscillations, compared to those of the case #1. Consequently, the clutch torque trajectories denoted as u_{1CA}, u_{2CA} are designed with desired gearshift performances by simply tuning 2 parameters, and those have a potential to be utilized as reference clutch torques for torque-based control of clutch-to-clutch shifts. Note that the output shaft torque is increased after the gearshift even if it is upshift due to the characteristic of engine operating condition (the engine torque increases as the engine speed decreases).

CONCLUSION

This paper proposes a novel method to generate reference trajectories of two clutch torques for a DCT control using control allocation. Regarding the DCT as an over-actuated system, desired torque trajectories of the two clutches during the gearshift can be developed arbitrarily by tuning only 2 parameters considering performances for gearshift with physical constraints. The control allocation method is very easy to apply and enables us to design the desired torque trajectories intuitively. The simulation results based on the valid DCT model verify that proper reference trajectories for the clutch torques are generated by the suggest method. In fact, in order to design the valid reference clutch torque trajectories, the output shaft torque must be measured in advance, which is not measurable on the production transmissions. However, there have been many papers treating estimation of the output torque for transmissions, or a sensor for it can be installed in the DCT test-bench to generate the reference trajectories offline. This paper is valuable in that it tries to deal with the optimized design of upper level controller for torque-based control of DCTs for the first time.

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REFERENCES

[1] L. Glielmo and F. Vasca, "Optimal control of dry clutch engagement," SAE Technical Paper 0148-7191, 2000.
[2] F. Garofalo, L. Glielmo, L. Iannelli, and F. Vasca, "Smooth engagement for automotive dry clutch," in *Decision and Control, 2001. Proceedings of the 40th IEEE Conference on*, 2001, pp. 529-534.
[3] L. Glielmo, L. Iannelli, V. Vacca, and F. Vasca, "Gearshift control for automated manual transmissions," *Mechatronics, IEEE/ASME Transactions on,* vol. 11, pp. 17-26, 2006.
[4] S. Bai, R. L. Moses, T. Schanz, and M. J. Gorman, "Development of a new clutch-to-clutch shift control technology," SAE Technical Paper 0148-7191, 2002.
[5] B. Z. Gao, H. Chen, K. Sanada, and Y. Hu, "Design of clutch-slip controller for automatic transmission using backstepping," *Mechatronics, IEEE/ASME Transactions on,* vol. 16, pp. 498-508, 2011.
[6] B. Gao, H. Chen, J. Li, L. Tian, and K. Sanada, "Observer-based feedback control during torque phase of clutch-to-clutch shift process," *International Journal of Vehicle Design,* vol. 58, pp. 93-108, 2012.
[7] X. Song and Z. Sun, "Pressure-based clutch control for automotive transmissions using a sliding-mode controller," *Mechatronics, IEEE/ASME Transactions on,* vol. 17, pp. 534-546, 2012.
[8] M. Goetz, M. Levesley, and D. Crolla, "Dynamics and control of gearshifts on twin-clutch transmissions," *Proceedings of the institution of mechanical engineers, Part D: Journal of Automobile Engineering,* vol. 219, pp. 951-963, 2005.
[9] M. Kulkarni, T. Shim, and Y. Zhang, "Shift dynamics and control of dual-clutch transmissions," *Mechanism and Machine Theory,* vol. 42, pp. 168-182, 2007.
[10] J. Kim, K. Cho, and S. B. Choi, "Gear shift control of dual clutch transmissions with a torque rate limitation trajectory," in *American Control Conference (ACC), 2011*, 2011, pp. 3338-3343.
[11] P. D. Walker, N. Zhang, and R. Tamba, "Control of gear shifts in dual clutch transmission powertrains," *Mechanical Systems and Signal Processing,* vol. 25, pp. 1923-1936, 2011.
[12] K. van Berkel, T. Hofman, A. Serrarens, and M. Steinbuch, "Fast and smooth clutch engagement control for dual-clutch transmissions," *Control Engineering Practice,* vol. 22, pp. 57-68, 2014.
[13] Y. Liu, D. Qin, H. Jiang, and Y. Zhang, "Shift control strategy and experimental validation for dry dual clutch transmissions," *Mechanism and Machine Theory,* vol. 75, pp. 41-53, 2014.
[14] J. J. Oh, S. B. Choi, and J. Kim, "Driveline modeling and estimation of individual clutch torque during gear shifts for dual clutch transmission," *Mechatronics,* vol. 24, pp. 449-463, 2014.
[15] J. J. Oh and S. B. Choi, "Real-time estimation of transmitted torque on each clutch for ground vehicles with dual clutch transmission," *Mechatronics, IEEE/ASME Transactions on,* vol. 20, pp. 24-36, 2015.
[16] O. Härkegård, "Backstepping and control allocation with applications to flight control," 2003.
[17] O. HäRkegåRd and S. T. Glad, "Resolving actuator redundancy—optimal control vs. control allocation," *Automatica,* vol. 41, pp. 137-144, 2005.
[18] E. Galvagno, M. Velardocchia, and A. Vigliani, "Dynamic and kinematic model of a dual clutch transmission," *Mechanism and Machine Theory,* vol. 46, pp. 794-805, 2011.
[19] Y. Liu, D. Qin, H. Jiang, and Y. Zhang, "A systematic model for dynamics and control of dual clutch transmissions," *Journal of Mechanical Design,* vol. 131, p. 061012, 2009.