Model Predictive Control for Vehicle Yaw Stability

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1. Abstract

Yaw stability of an automotive vehicle in a turn is critical to the overall stability of the vehicle. In this paper, we present a method of vehicle stability control (VSC) based on Model Predictive Control (MPC). Conventional VSCs work passively as they detect excessive yaw rate or slip angle of a vehicle. However, in many cases, when excessive yaw rate or slip angle of a vehicle is detected, the vehicle is already in unstable states. Using the MPC scheme, the proposed controller can actuate brakes to generate correction moment in advance of vehicle being unstable by predicting vehicle movement of several hundred milliseconds ahead. The differences of desired vehicle states from the bicycle model and the estimated vehicle state are minimized by applying the MPC scheme. The performance of the proposed method is evaluated using the vehicle dynamics software CARSIM.

2. Keywords: Vehicle Dynamics, Yaw Stability, Model Predictive Control (MPC), Optimization

3. Introduction

Various types of Electronic Stability Control (ESC) have been developed by many researchers [1][2][3]. Basically, these ESC algorithms are activated to exert correction moment when excessive differences of actual and desired yaw rates or immoderate side slip angles are detected. However, when excessive side slip angles or excessive differences of actual and desired yaw rates are detected, in many cases, vehicles are already in unstable states. Since vehicle in unstable states tend to quickly spin out or bounce out from its desired trajectory, small delay of ESC actuation can be followed by fatal accidents. To overcome this drawback of conventional ESC, this paper focuses on development of a new ESC algorithm based on MPC scheme. First, bicycle model is designed to capture the characteristics of vehicle lateral dynamics. The initial cornering stiffness is used to generate desired yaw rate. On-line cornering stiffness adaptation is performed using the tire model [4]. The MPC scheme generates the corrective yaw moment to minimize the corrective yaw moment and the difference between the desired yaw rate and actual yaw rate.

4. Development of Yaw Stability Controller based on MPC

4.1. Vehicle Lateral Dynamics Modeling

In this section, a simple bicycle model with dynamic tire model is used to generate desired yaw rate. This bicycle model as shown in Fig.1 describes vehicle lateral dynamics.

Figure 1. Bicycle Model

The equations of motion for the bicycle model are as follows:

\[ m \dot{v} = F_f + F_r \]  \hspace{1cm} (1)

\[ I \dot{\beta} = l_F F_f - l_r F_r \]  \hspace{1cm} (2)

Where

\[ F_f = C_f \alpha_f \]
\[ F_r = C_r \alpha_r \]
\[ \alpha_f = \delta_f - \left( \beta + \frac{l_F r}{v} \right) \]
\[ \alpha_r = -\beta + \frac{l_r r}{v} \]
Dynamic tire model as defined in [5] can be obtained as follows:

\[ \tau \frac{\dot{F}_{y \_lag}}{y \_lag} + F_{y \_lag} = F_y \quad , \tau = \frac{C_u}{KV} \quad , K = \frac{\partial F}{\partial y} \bigg|_{y=0} \quad , L = \frac{C_o}{K} \approx 1.5m \]

The lagged lateral tire forces can be defined as follows:

\[ \tau \dot{F}_{y \_lag} + F_{y \_lag} = F_{sf} \]
\[ \tau \dot{F}_{yr \_lag} + F_{yr \_lag} = F_{yr} \]

Using the first order dynamic tire model and laplace transform, front and rear lateral tire force equations can be replaced by the follows:

\[ m v_x (s \beta + r) = F_{sf \_lag} + F_{yr \_lag} \]
\[ I \dot{s} r = l_f F_{sf \_lag} - l_r F_{yr \_lag} \]

As a result, the following state space form equation can be obtained

\[ \dot{X} = AX + B \delta \]

where

\[ A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{C_f + C_r}{\tau m v_x} & -\frac{1}{\tau} \left( \frac{C_l I_r - C_l I_f}{\tau m v_x} \right) & 1 & \frac{1}{\tau} \\ 0 & 0 & 0 & 1 \\ \frac{C_l I_r - C_l I_f}{\tau I_z} & 0 & -\frac{C_l I_f + C_l I_f}{\tau I_z} & -\frac{1}{\tau} \end{bmatrix} \]

\[ B = \begin{bmatrix} 0 \\ \frac{C_f}{\tau m v_x} \\ 0 \\ \frac{C_l I_f}{\tau I_z} \end{bmatrix} \]

By using the dynamic tire model, the bicycle model can be reformed to have the four states including derivatives of yaw rate and side slip angle and lagged characteristics of tire forces to better describe vehicle lateral dynamics.

4.2 Tire Model

In a regular bicycle model, the cornering stiffnesses \( C_f, C_r \) are set to be constants. However, as slip angles increase or tire longitudinal forces are applied, those cornering stiffnesses decrease. To take these phenomenon into account, the combined longitudinal and lateral brushed tire model [t] which is simple enough to be run in real-time while complicated enough to capture the tire nonlinear characteristics is used. The equations of the tire model is as follow:

\[ F_x = \frac{C_x}{f} \left( \frac{\kappa}{1+n} \right) F \]
\[ F_y = -\frac{C_o}{f} \left( \frac{\tan \alpha}{1+N} \right) F \]
Where

\[ F = \begin{cases} 
  f - \frac{1}{3\mu F_z} f^2 + \frac{1}{27\mu^2 F_z^2} f^3 & \text{if } f \leq 3\mu F_z \\
  \mu F_z & \text{else} 
\end{cases} \]

\[ f = \sqrt{C_f^2 \left( \frac{\kappa}{1 + \kappa} \right)^2 + C_a^2 \left( \frac{\tan \alpha}{1 + \kappa} \right)^2} \]

\[ \kappa = \frac{R_e \omega - V_{zt}}{V_{zt}} \]

The following parameters are identified in real-time using the linearized recursive least squares method [1] in real-time.

\[ \mu, C_f, C_a \]

4.3. Linearized Tire Model

Since we have the nonlinear tire model which is identified in real-time, the nonlinear tire characteristics can be reflected to the bicycle model by linearizing the tire force curve at current slip angle.

![Figure 2. Generic Lateral Tire Force vs Slip Angle Curve](image)

\[ F_y = C_f \alpha \] \hspace{1cm} (6)

\[ F_y = C_{f0} \alpha + F_{y0} \] \hspace{1cm} (7)

The tire force can be expressed as in (7) by linearizing the tire model at the current slip angle. Using (7) for the front and rear tire forces, the bicycle model with dynamics tire model can be modified as follow:

\[ \dot{X} = A_b X + B_b \delta + C_u + D_b M_z \] \hspace{1cm} (8)

\[ A_b = \begin{bmatrix}
  0 & 1 & 0 & 0 \\
  -\frac{C_{u1} + C_{u2}}{\tau m v_x} & -1 & \left( \frac{C_{u1} - C_{u1}}{\tau} \right) & 1 \\
  \frac{1}{\tau} & 0 & 0 & 1 \\
  \frac{C_{u1} - C_{u2}}{\tau I_z} & 0 & -\frac{C_{u1}^2 + C_{u2}^2}{\tau I_z v_x} & 1 \\
  \frac{1}{\tau} & 0 & 0 & \frac{1}{\tau} 
\end{bmatrix} \]
\[
B_n = \begin{bmatrix}
0 \\
\frac{C_{n,f}}{\mu v} \\
0 \\
\frac{C_{n,r}}{\mu v} \\
rI
\end{bmatrix}, \\
C_n = \begin{bmatrix}
0 \\
\frac{F_{n,f} + F_{n,r}}{\mu v} \\
0 \\
1 - F_{n,f} - F_{n,r} \\
rI
\end{bmatrix}, \\
D_n = \begin{bmatrix}
0 \\
0 \\
0 \\
1 - \frac{1}{\rho}
\end{bmatrix}
\]

\(M_r\) is the corrective moment. \(C_n\) is the additional term due to the linearization. (3) is used to generate the desired yaw rate, whereas (8) is used to calculate the corrective yaw moment in the MPC scheme.

4.3. Calculation of Correction Moment based on Model Predictive Control

\[
J = \sum_{n=1}^{10} M_n^2 + W(r_{n,\alpha} - r_{p,\alpha})^2
\]

subject to \(g(x) < 0\)

\(M\) is corrective yaw moment. \(r_{\alpha}, r_{p,\alpha}, W\) and \(g(x)\) are desired yaw rate from (3), yaw rate developed in (8), weighting, state constraints, respectively. State constraints \(g(x)\) can be defined as follow:

\[
\dot{\beta} - r: \quad r_{\max} + \dot{\beta}_{\max} = \begin{cases}
F_{\alpha,\max} \left(1 + \frac{b}{a}\right) \\
\frac{mv}{a} \\
\left(1 + \frac{a}{b}\right) \\
mv \\
F_{\alpha,\max} \left(1 + \frac{b}{a}\right) \\
\frac{mv}{a} \\
F_{\alpha,\max} < \frac{b}{a} F_{\alpha,\max}
\end{cases}
\]

(9)

\[
\beta - r: \quad -\beta_{\max} + \frac{b \cdot r_{\max}}{v} = \alpha_{\max}
\]

(10)

(9) is defined as in [6] but \(\dot{\beta}\) is added to the equation. (10) is determined for the vehicle not to exceed the slip angle which generate the maximum lateral tire force. For the \(\dot{\beta} - \beta\) boundary, a look-up table is used for practical reason.

4.4. Distribution of the corrective yaw moment

Fig. 3. shows a vehicle with brake force applied on the front right wheel. To generate the negative corrective yaw moment, front right and front rear wheels could be the places that brake forces to be applied. In case of applying brake force on the front right wheel on a vehicle in a left turn, lateral force of rear right wheel decreases. Diminished lateral forces of rear wheels are followed by creating positive moment which is undesirable in this case. Whereas, in case of applying brake force on the rear right wheel on a vehicle in a left turn, the lateral force of the front wheel decreases which is followed by increase of the negative corrective yaw moment. Considering friction ellipse effect of the tires, wheels that brake forces to be applied are chosen. The following equations calculate the corrective yaw moments as functions of changes of longitudinal tire forces. The change of lateral force due to additional longitudinal force applied by brake is calculated using the tire model [4].

\[
M_{r,f} = \sin \delta \cdot l_f \cdot \Delta F_y + \cos \delta \cdot t_b \cdot \Delta F_x
\]

(11)

\[
M_{r,r} = t_b \cdot \Delta F_x - l_f \cdot \Delta F_y
\]

(12)
In Fig. 4., (a) shows longitudinal and lateral tire forces along slip ratio at a constant slip angle. (b),(c) show generated corrective moments when applying brake forces on the front right and rear right respectively. As shown in (c), the excessive brake force on rear right wheel can generate corrective moment in an opposite direction due to diminished lateral force. The MPC scheme introduces in 4.3 calculates the needed corrective yaw moment. Then, Eq. (11) or Eq. (12) is used to calculate the required brake force using the Newton-Raphson method. The required brake forces are generated by applying proper brake pressure to the vehicle brake system.

5. Simulation Results
A simulation result using CARSIM is presented in Fig.6. Before the brake pedal is pushed between 4 and 5 second, the vehicle with the control follows the desired yaw rate with less error comparing to the vehicle without any control. After the brake pedal is stepped while turning, the vehicle without control loses its stability and spins out. Whereas, the vehicle with control track the desired yaw rate even when the driver steps on brake pedal while turning.

6. Conclusion
A method of vehicle stability control (VSC) based on MPC is presented. Using the MPC scheme, the proposed controller can actuate brakes to generate correction moment in advance of vehicle being unstable by predicting vehicle movement of several hundred milliseconds ahead. The differences of desired vehicle states from the bicycle model and the estimated vehicle state are minimized by applying the MPC scheme. The simulation result using CARSIM shows that proposed algorithm successfully stabilize the vehicle even brake in a turn is applied to the vehicle.
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Reference

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